# Homework 8 — Advanced Algorithms

**Problem 1:**

The *underlying* graph of a multigraph with self-loops $G = (V, E)$ is a graph $G_u = (V, E_u)$ with the same set of vertices and self-loops removed and multiple edges between two vertices replaced by a single edge. Notice that $E$ is a multi-set. We can define $E_u$ as

$$(x, y) \in E_u \Leftrightarrow \left(x \neq y \text{ and there is an edge from } x \text{ to } y \text{ in } E\right).$$

Find an $O(|V| + |E|)$ algorithm that calculates the adjacency list representation of the underlying graph of a multigraph given by an adjacency list.

**Problem 2:**

Consider a directed Graph $G = (V, E)$. A *sink* is a vertex $v \in V$ such that

$$\forall u \in V,\ u \neq v \ :\ (u, v) \in E,$$

i.e. such that there is an edge from all other vertices to $v$. A sink $v \in V$ is *universal* if there is no edge from $v$ to any other node. If $V = \{v_1, v_2, \ldots, v_n\}$, then the graph is given by an adjacency matrix $A = \left(a_{i,j}\right)_{1 \leq i,j \leq n}$ with

$$a_{i,j} = \begin{cases} 1 & \text{if } (i, j) \in E \\ 0 & \text{if } (i, j) \notin E \end{cases}.$$

Find an $O(n)$ algorithm that given an adjacency matrix of size $n \times n$ determines whether the graph has a universal sink.

(**Hint:** Your algorithm has to be able to skip a row / column by looking at a single element).

You need to (a) specify the algorithm and (b) argue that your algorithm is $O(n)$.