Linked List

Set Data Structure

- Operations:
    - Add an element
    - Remove an element
    - Answer question about containment
- Implemented as a singly linked list

Linked List

- Adding threads should not <u>lower</u> throughput
  - Contention effects
  - Fixed by queue locks
- Should increase throughput
  - Not possible if inherently sequential
  - But surprising things are parallelizable

Linked List

- Coarse-Grained synchronization
  - Each method locks the object
    - Avoid contention using queue locks
    - Easy to reason about
    - But "Sequential Bottleneck"
      - Threads stand in line
      - So adding more threads does not improve throughput
      - In fact, could make things worse

Linked List

- Instead of using a single lock:
    - Use fine-grained synchronization
        - Split object into
            - independently synchronized components
        - Methods conflict only:
            - When they access the same component at the same time

Linked List

- Use optimistic synchronization
  - Search without locking
  - If you find it, lock, and check that it did not change
- In general, optimistic synchronization
  - Is good when it works
  - But mistakes are expensive

Linked List

- Lazy synchronization
  - Postpone hard work
  - Removing components is tricky
    - So use *logical* removal:
      - Mark the component as deleted instead of deleting it
    - Followed by *physical* removal:
      - Delete the component

Linked List

- Lock-free Synchronization
  - Don't use locks at all
    - Use Compare-And-Set and relatives
    - Needs no scheduler assumptions or support
    - But is complex and can have high overhead

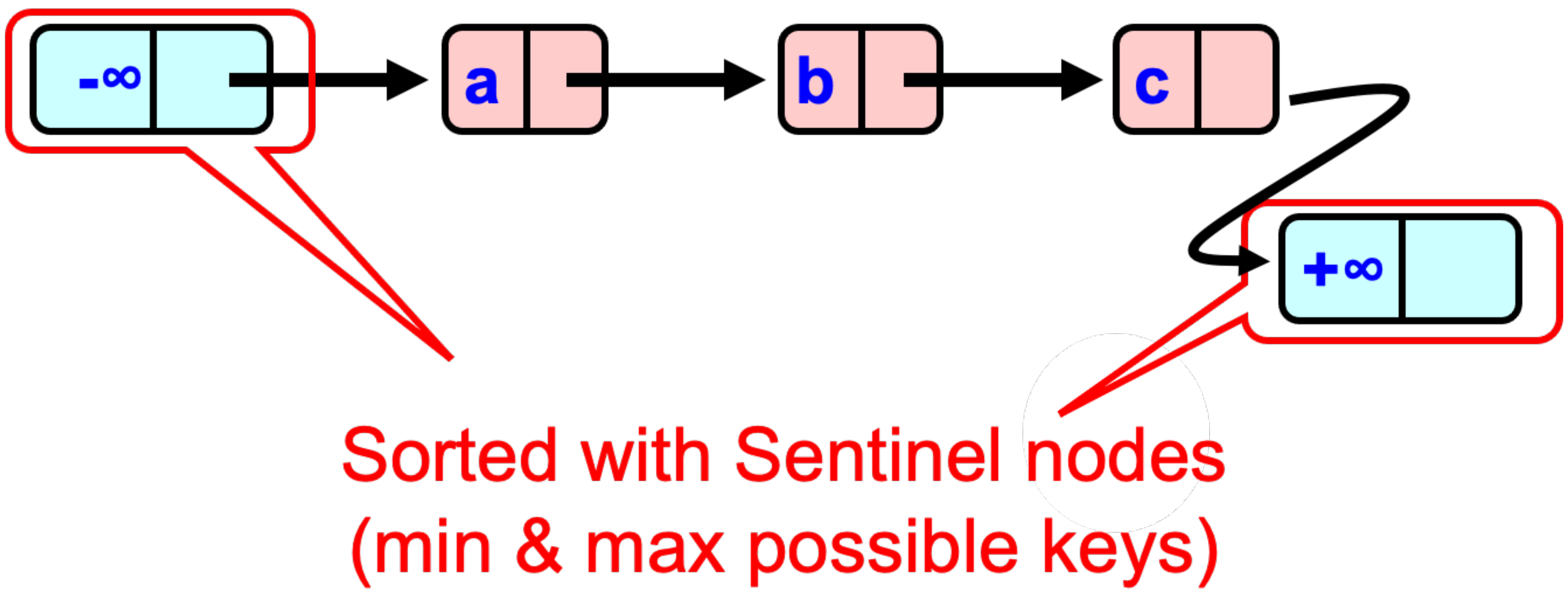
Linked List

- Singly linked list:
  - Use a List Node class

```
public class Node {
   public T item;
   public int key;
   public volatile Node next;
```

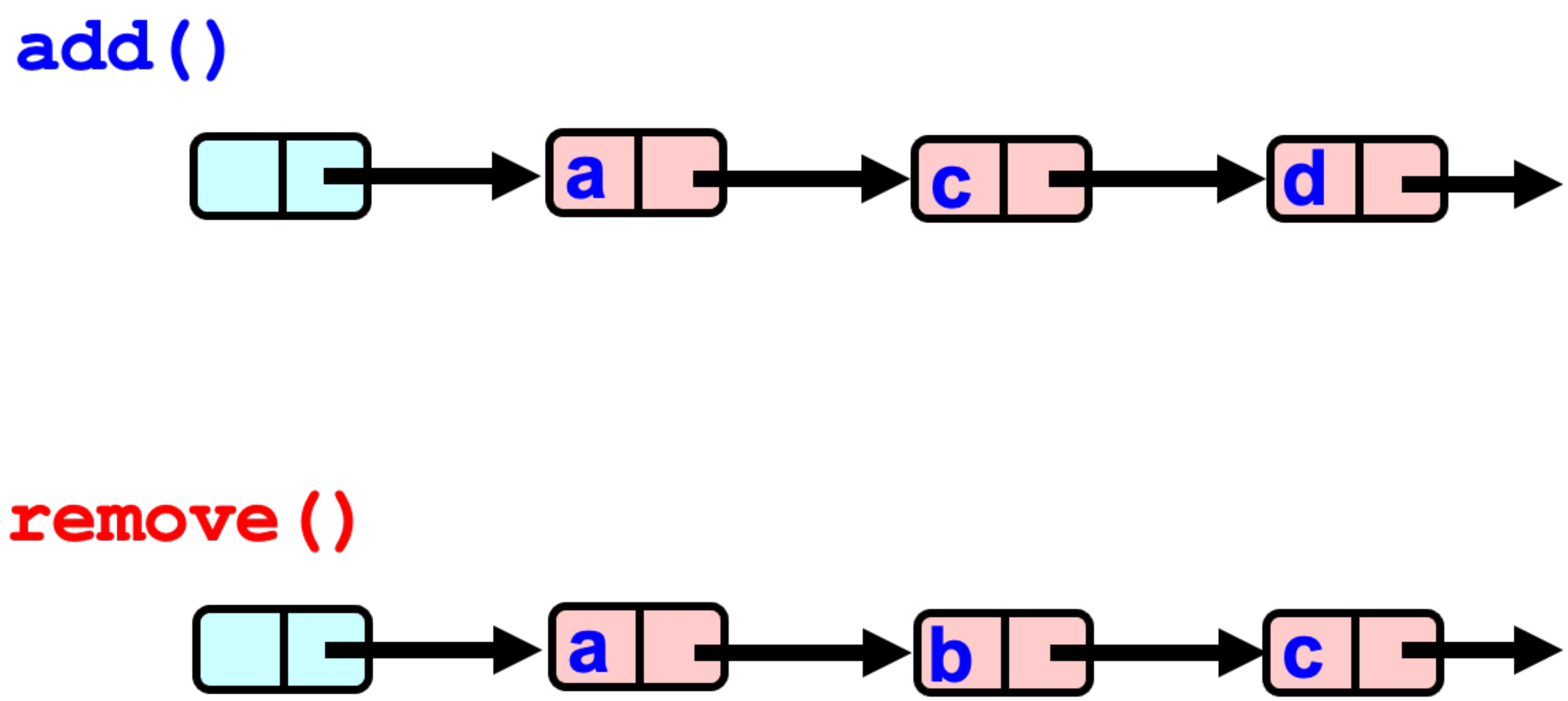
Linked List

- Use *Sentinel Nodes*



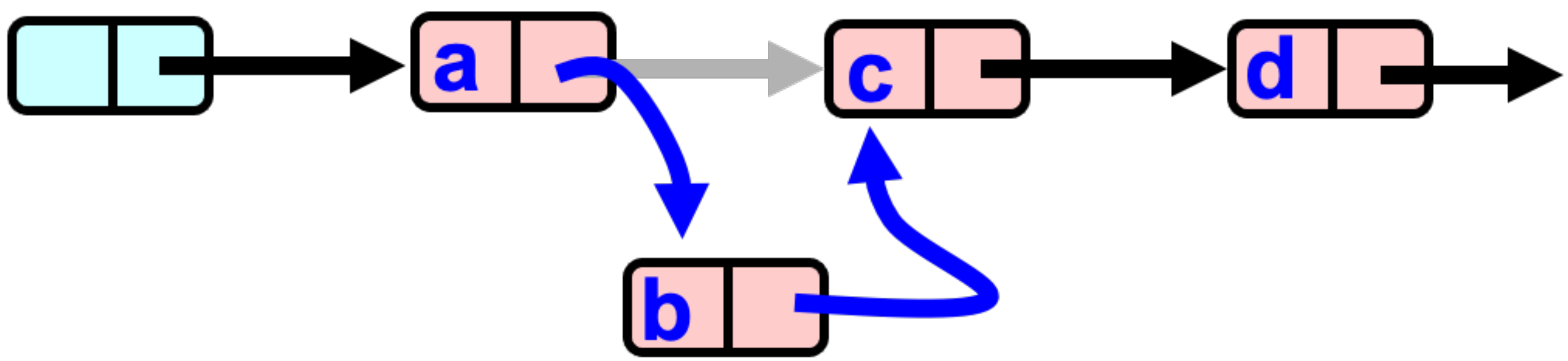
Linked List

- Operations involve pointer chasing

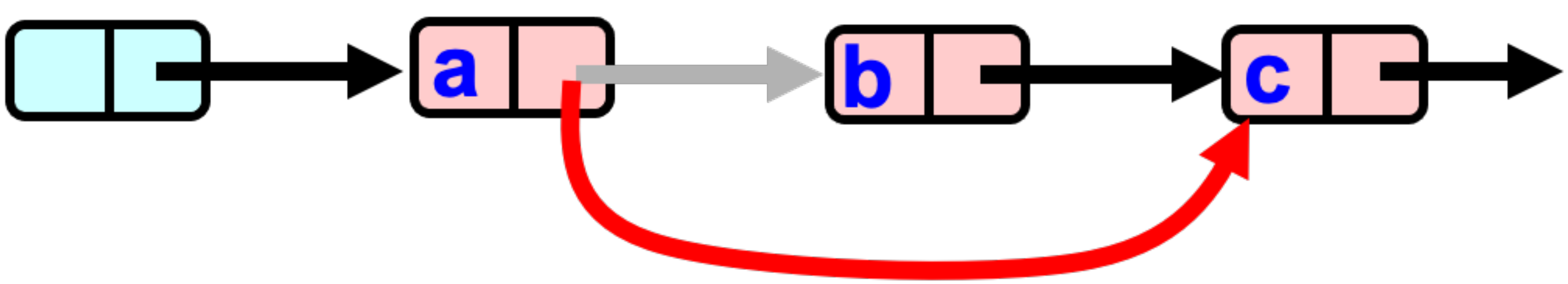


Linked List

add()

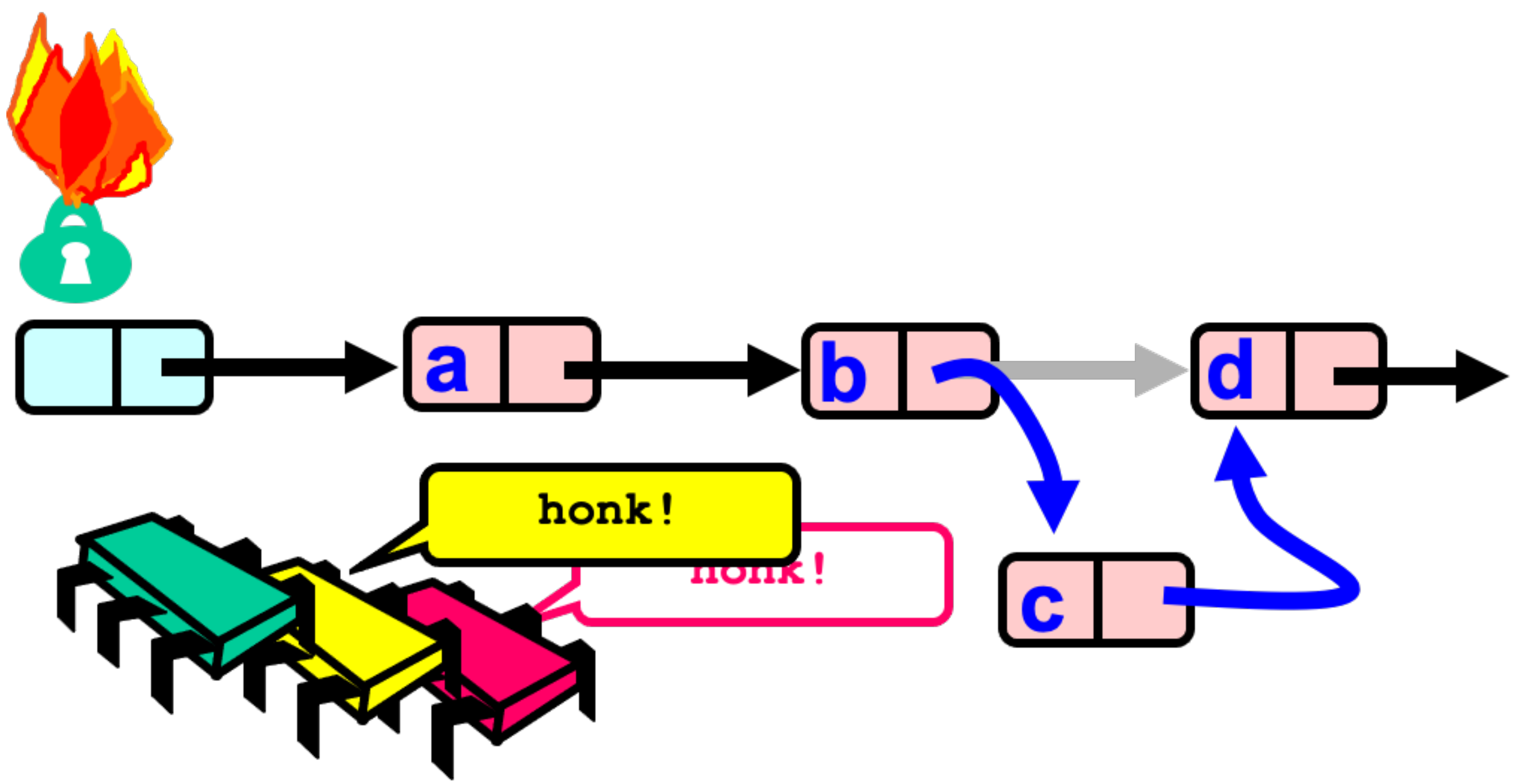


remove()



Coarse Grained Locking

- Coarse Grained Locking
  - Single hotspot + bottleneck leads to convoys

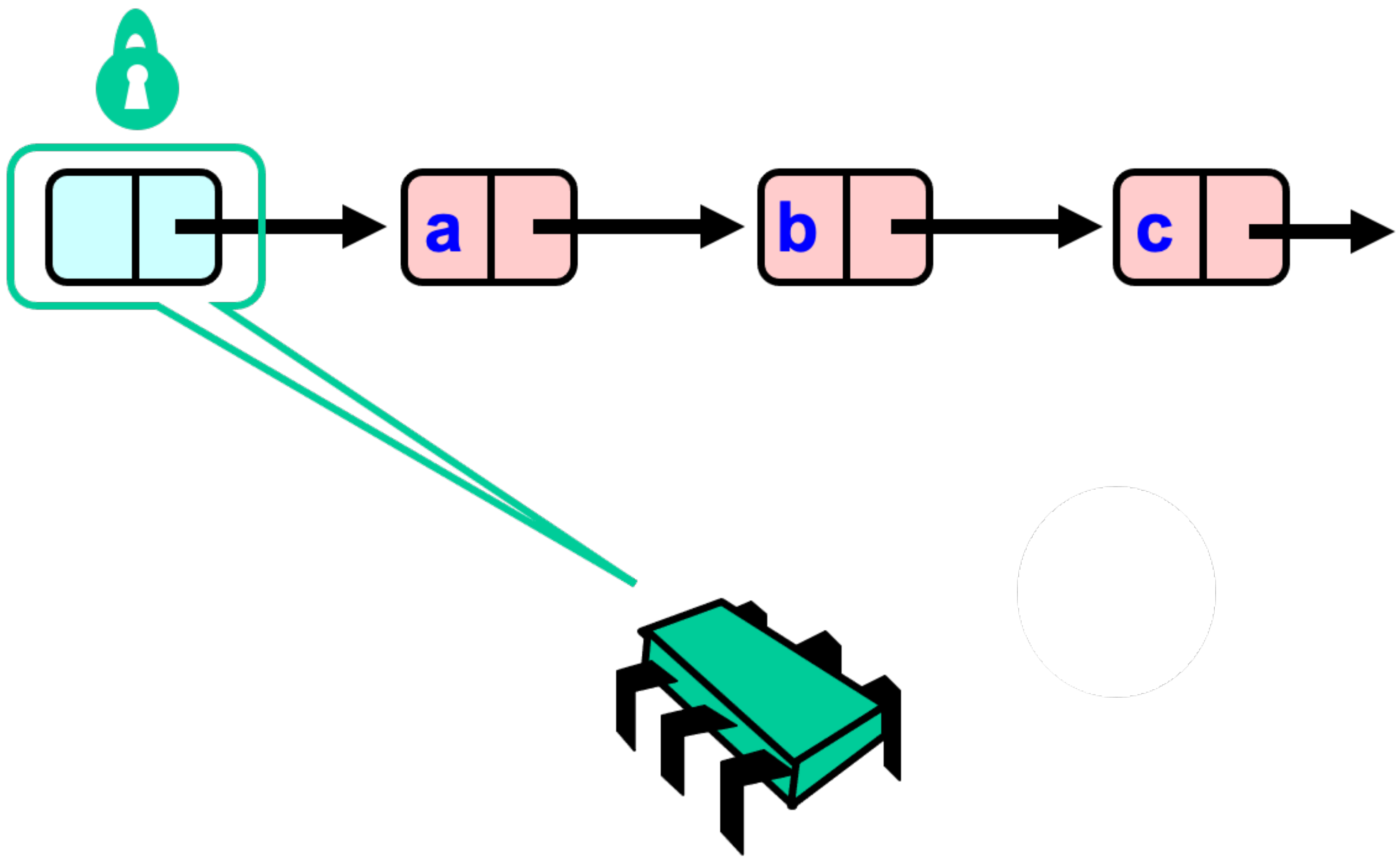


Fine Grained Locking

- Fine-grained locking
  - Requires care
  - Split object into pieces
    - Each piece has its own lock
    - Methods that work on disjoint set of pieces do not exclude each other

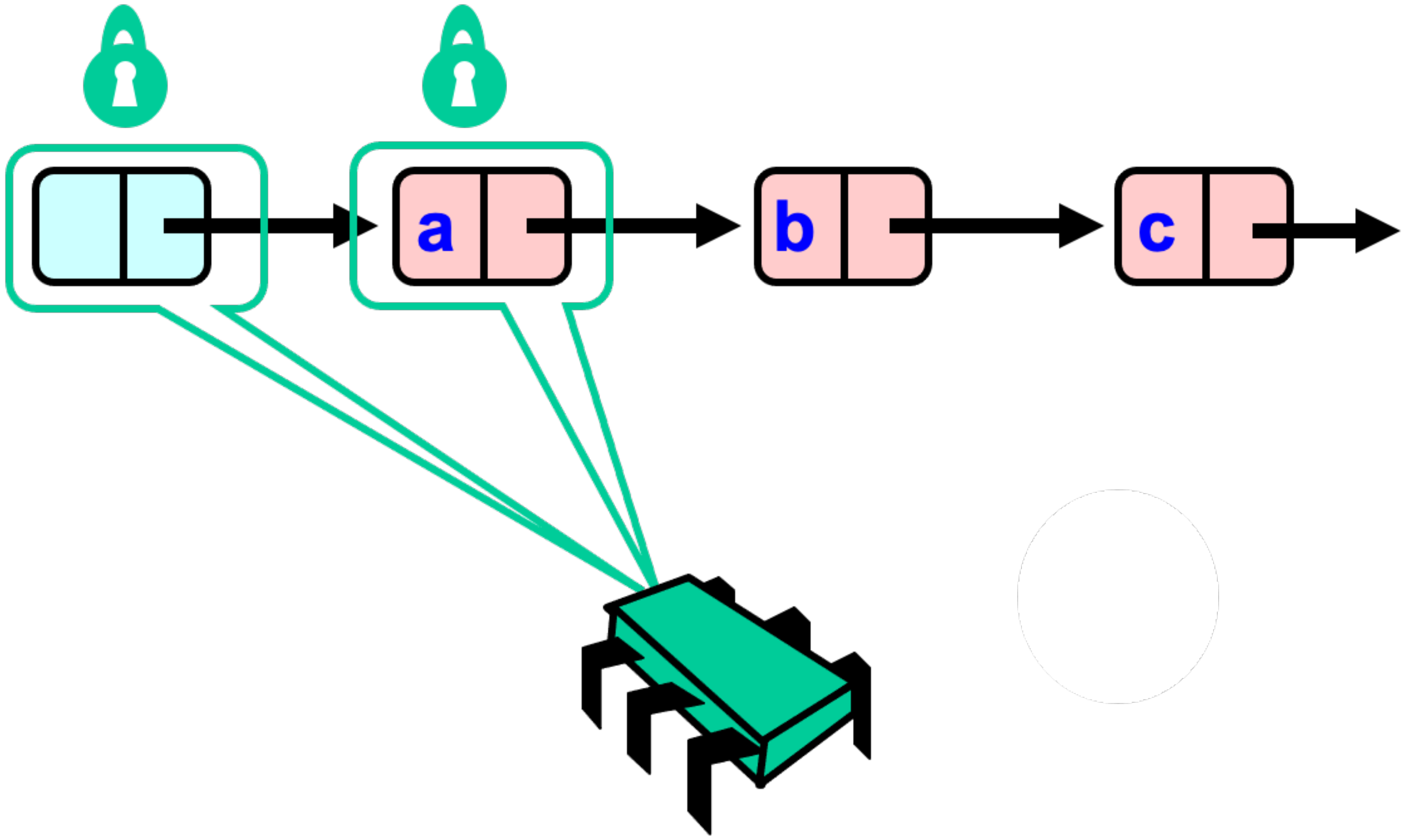
Fine Grained Locking

- Hand-over-Hand locking



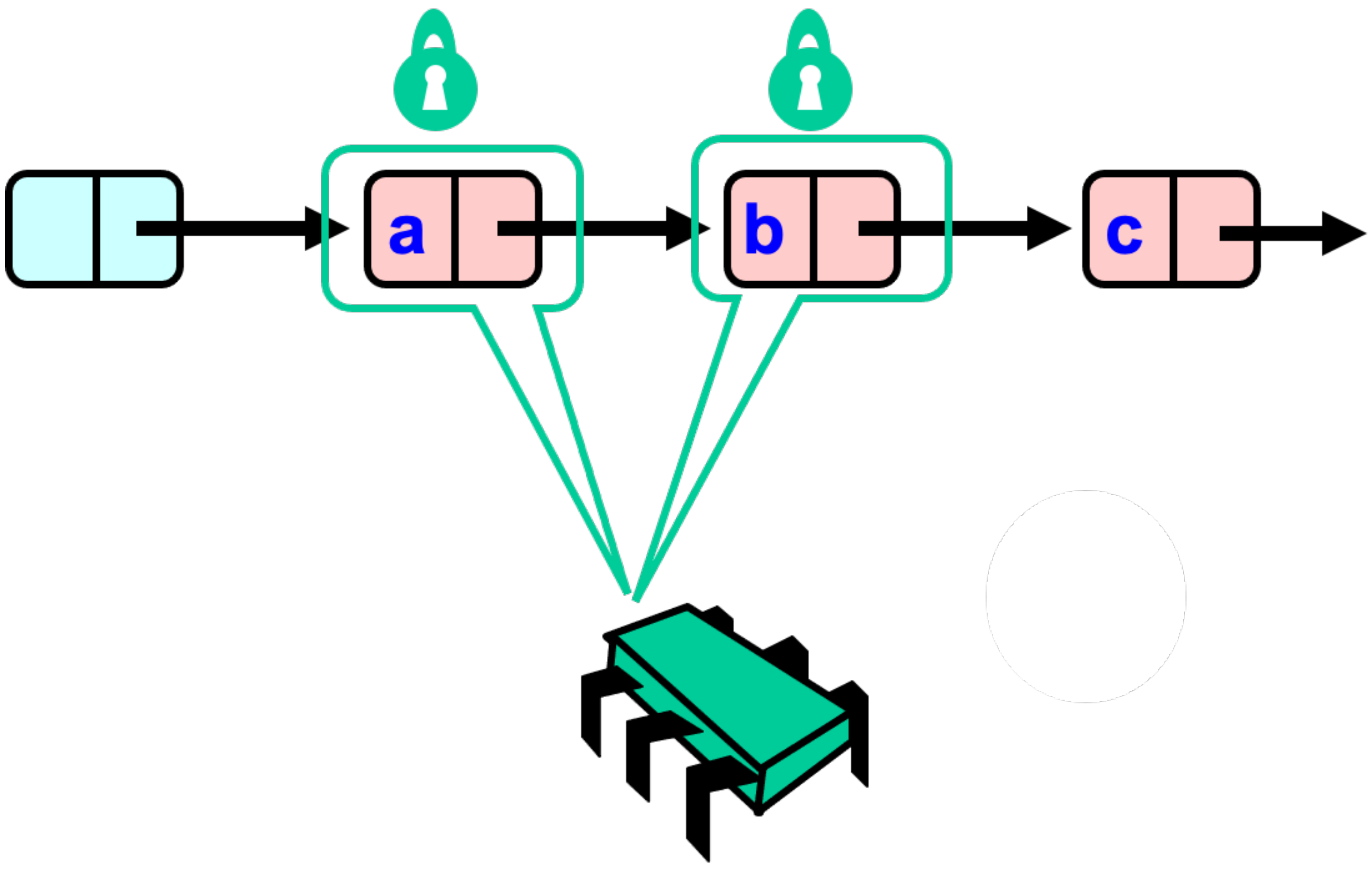
Fine Grained Locking

- Hand-over-Hand locking



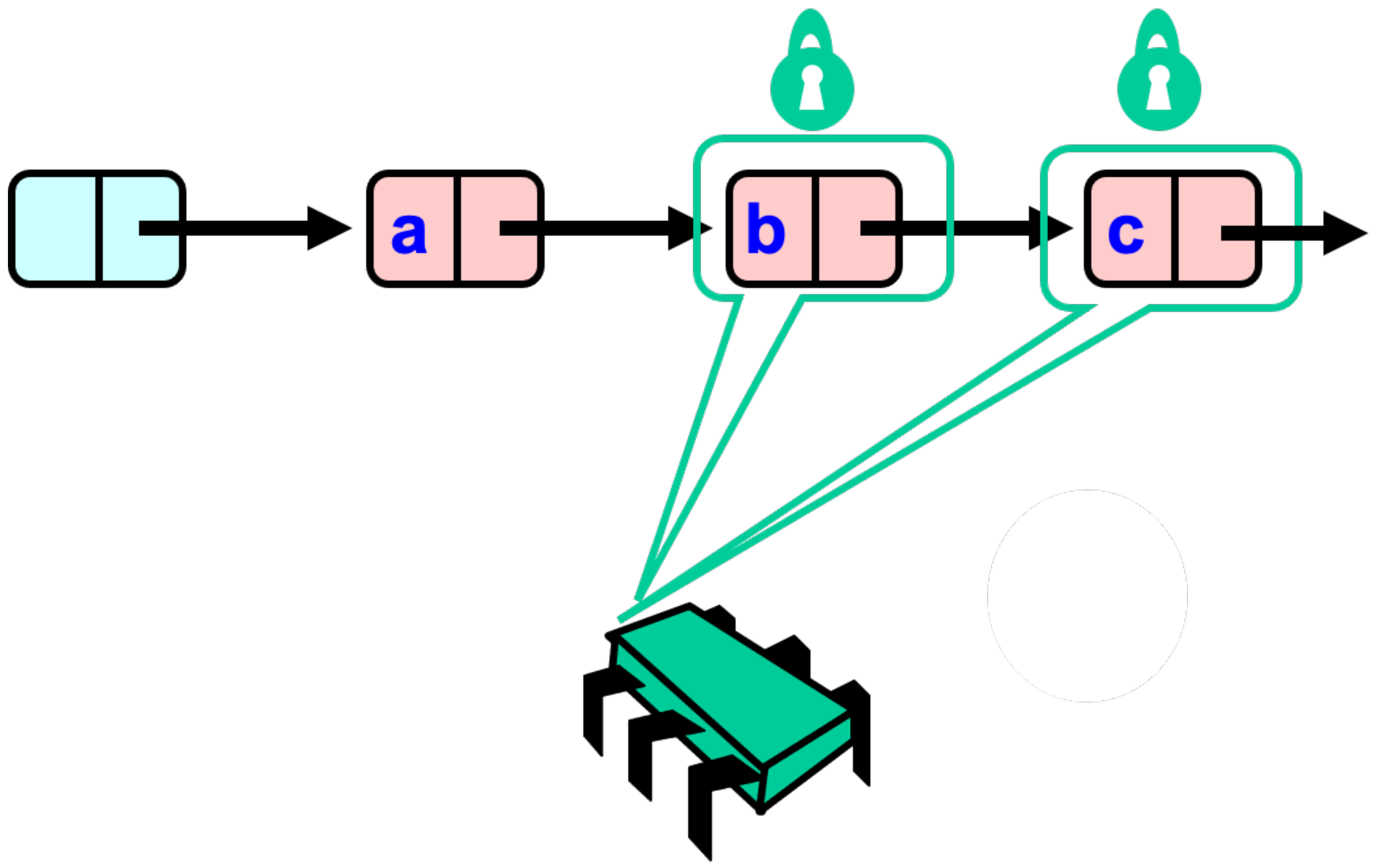
Fine Grained Locking

- Hand-over-Hand locking



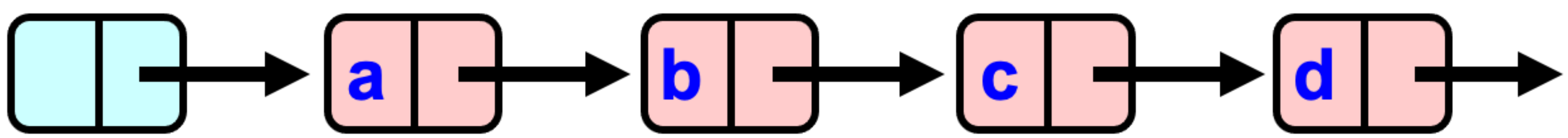
Fine Grained Locking

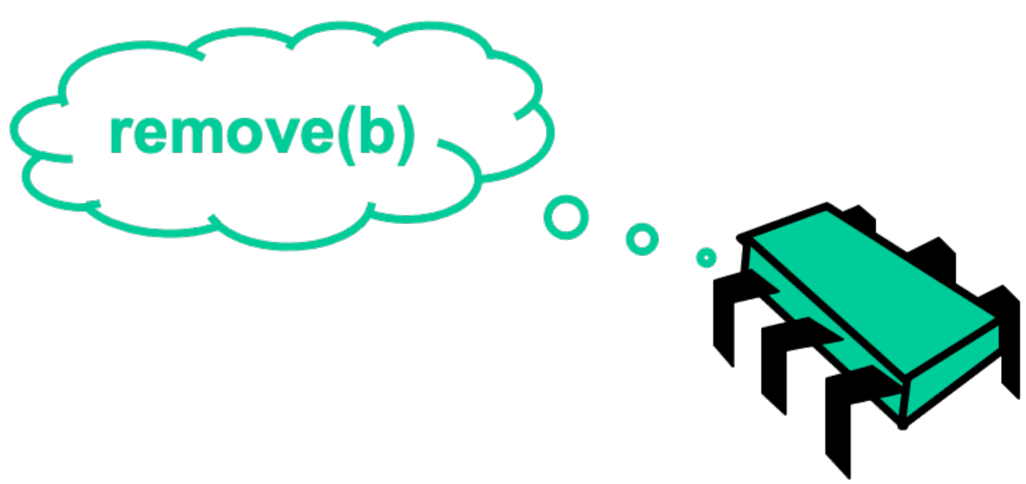
- Hand-over-Hand locking



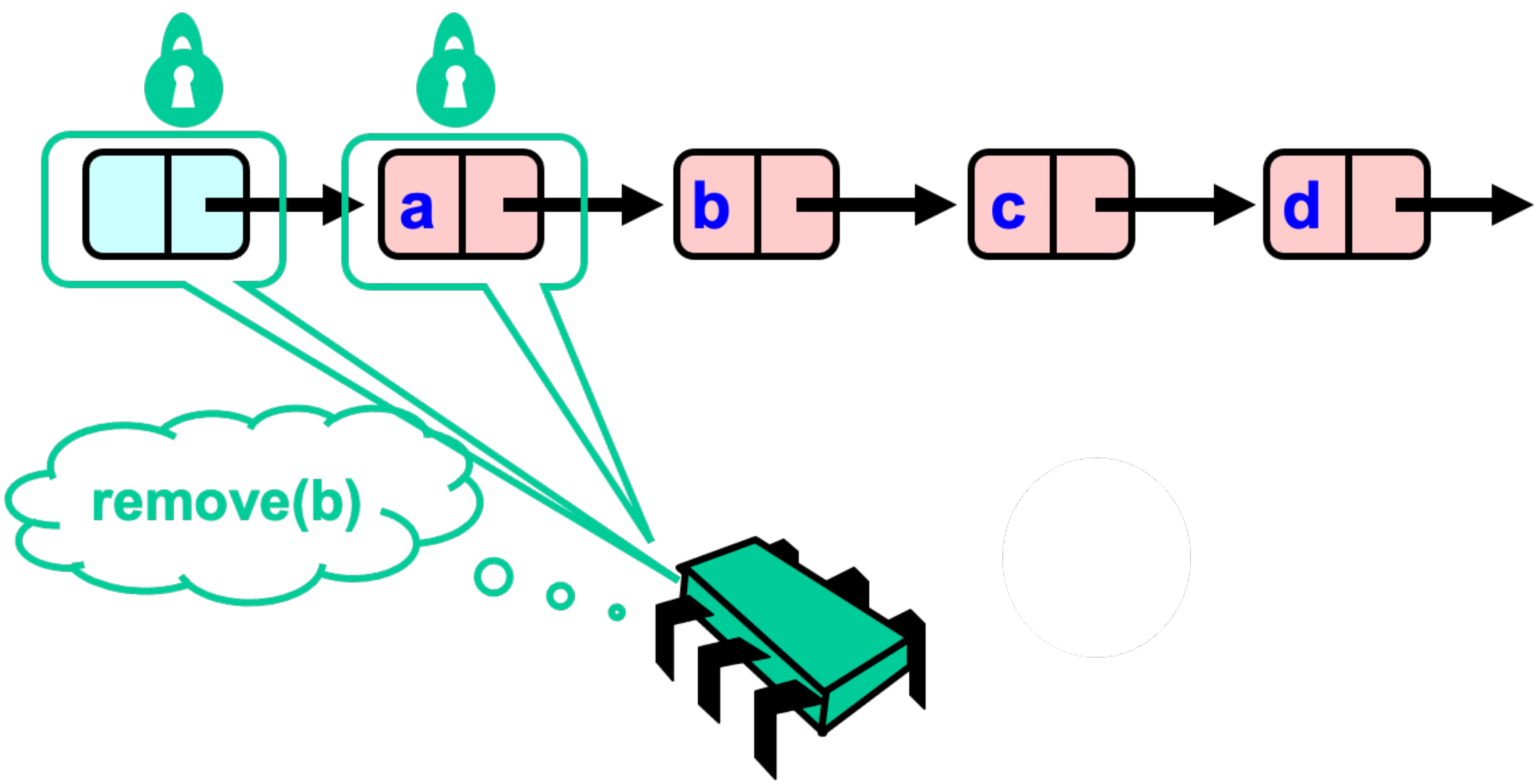
Fine Grained Locking

- Implementing remove
  - Problem arise when other threads try to access an adjacent node

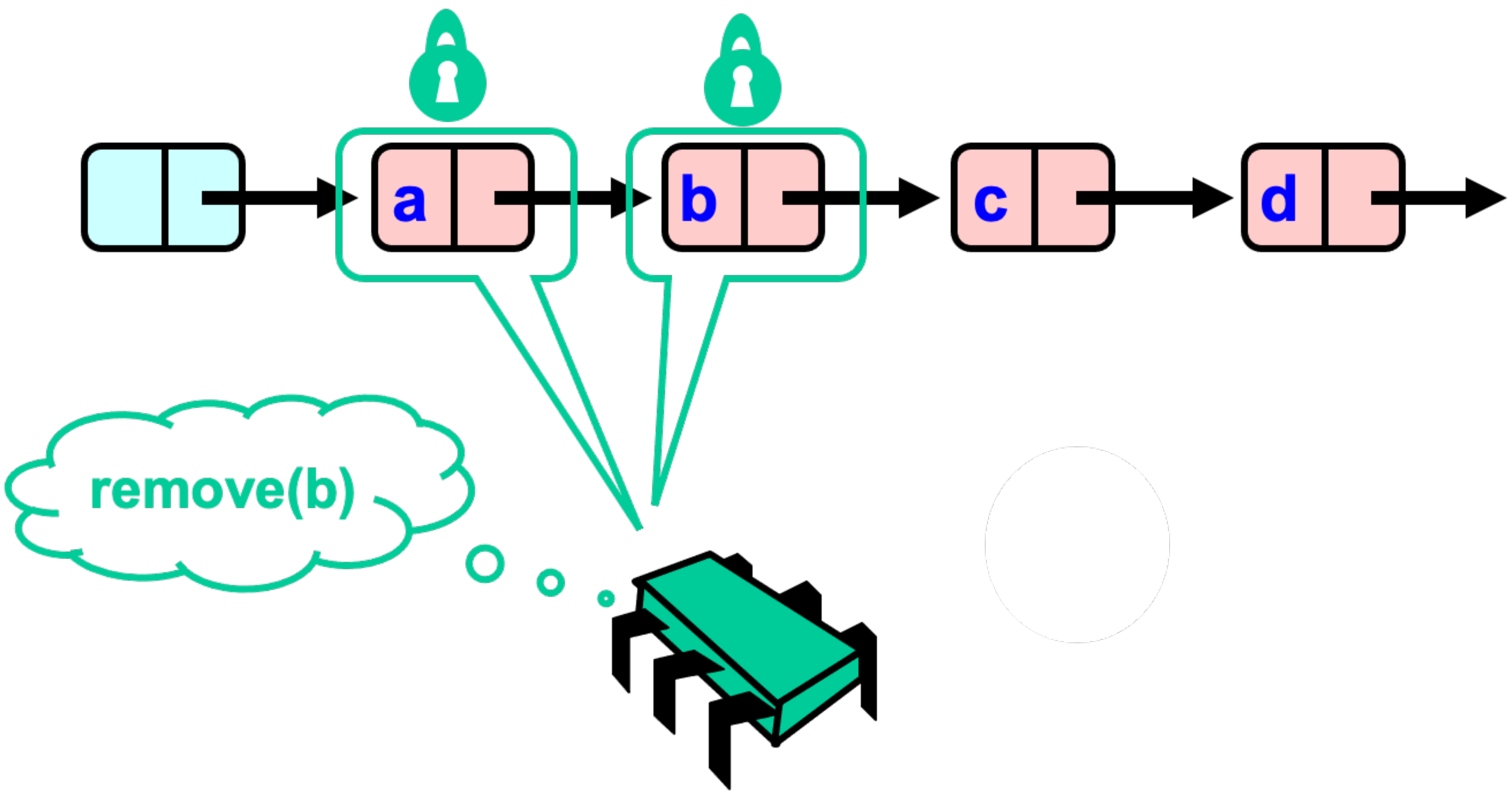




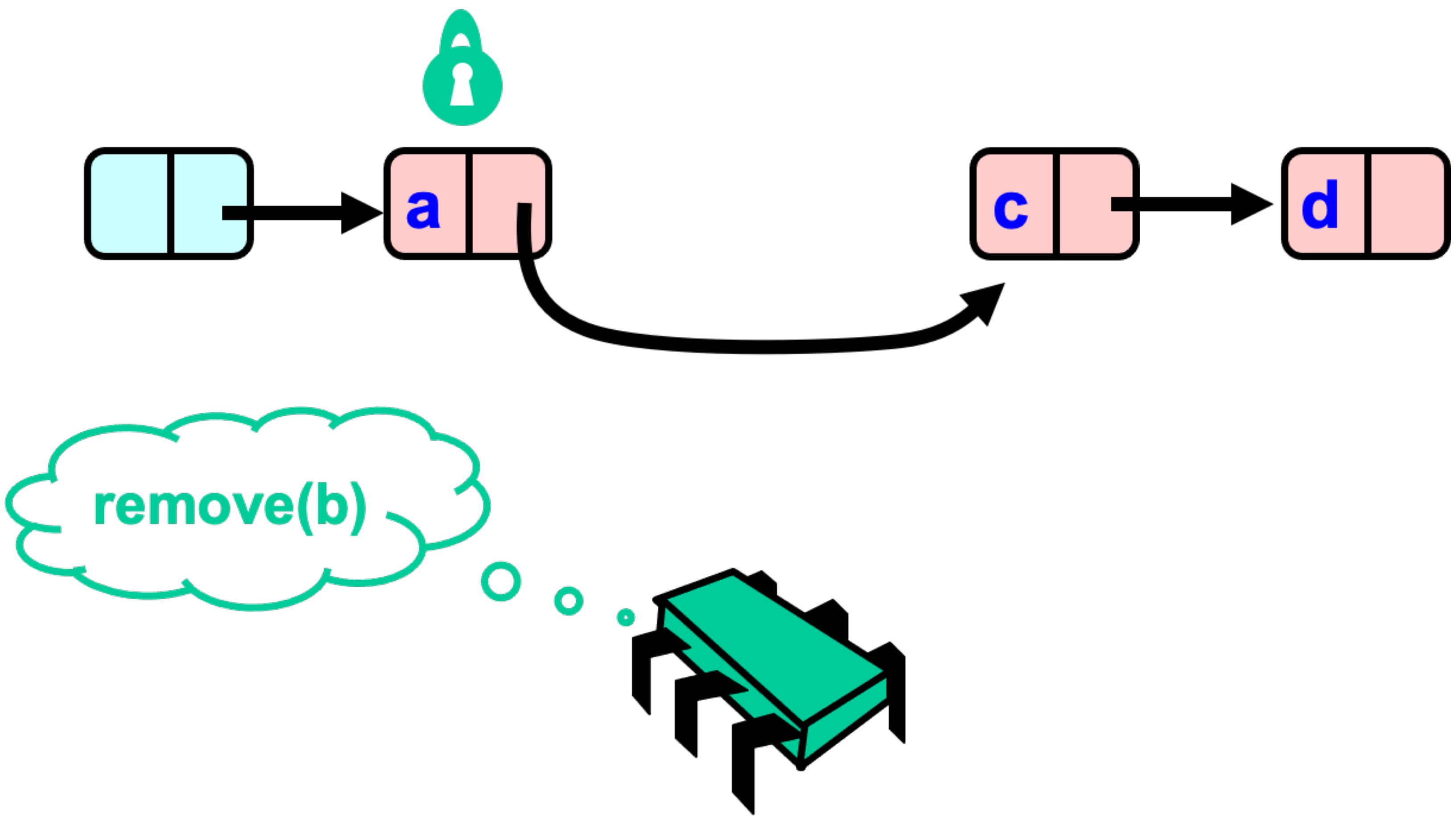
Fine Grained Locking

- Hand-to-hand locking assures that a thread that tries a competitive operation has a lock conflict
- 

Fine Grained Locking

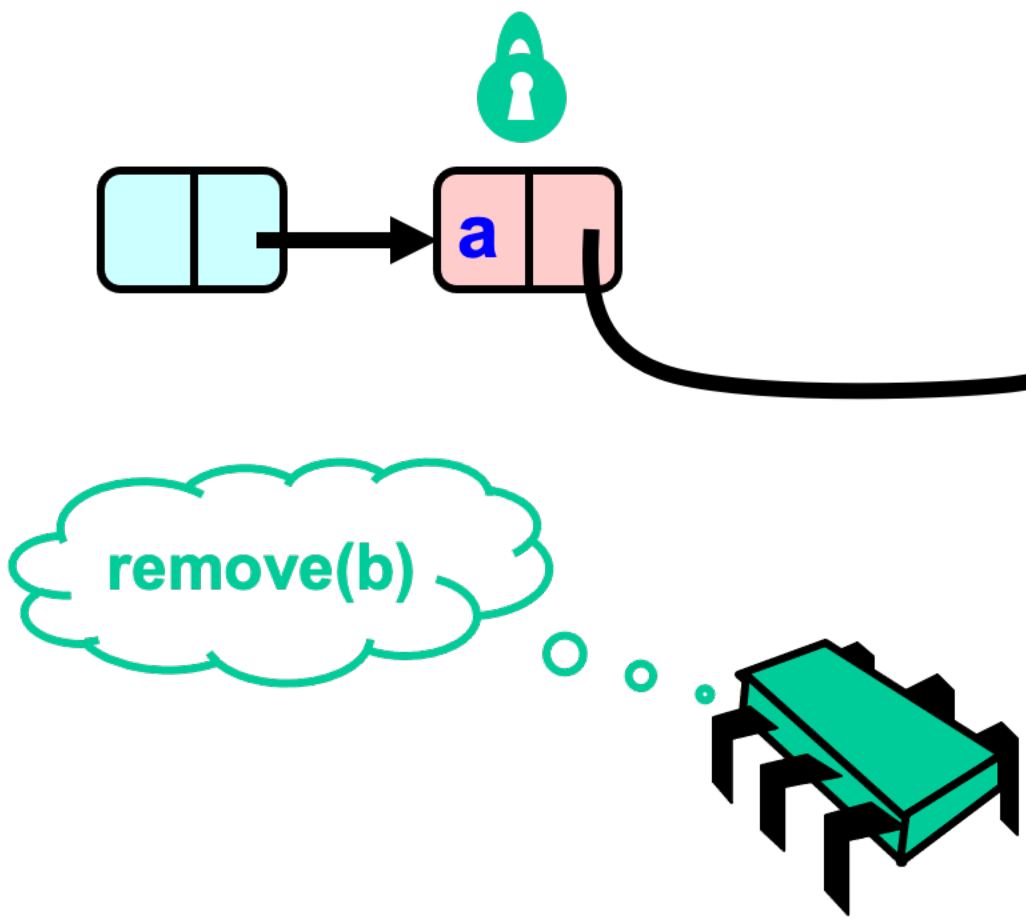


Fine Grained Locking



Fine Grained Locking

- Why lock the victim node?

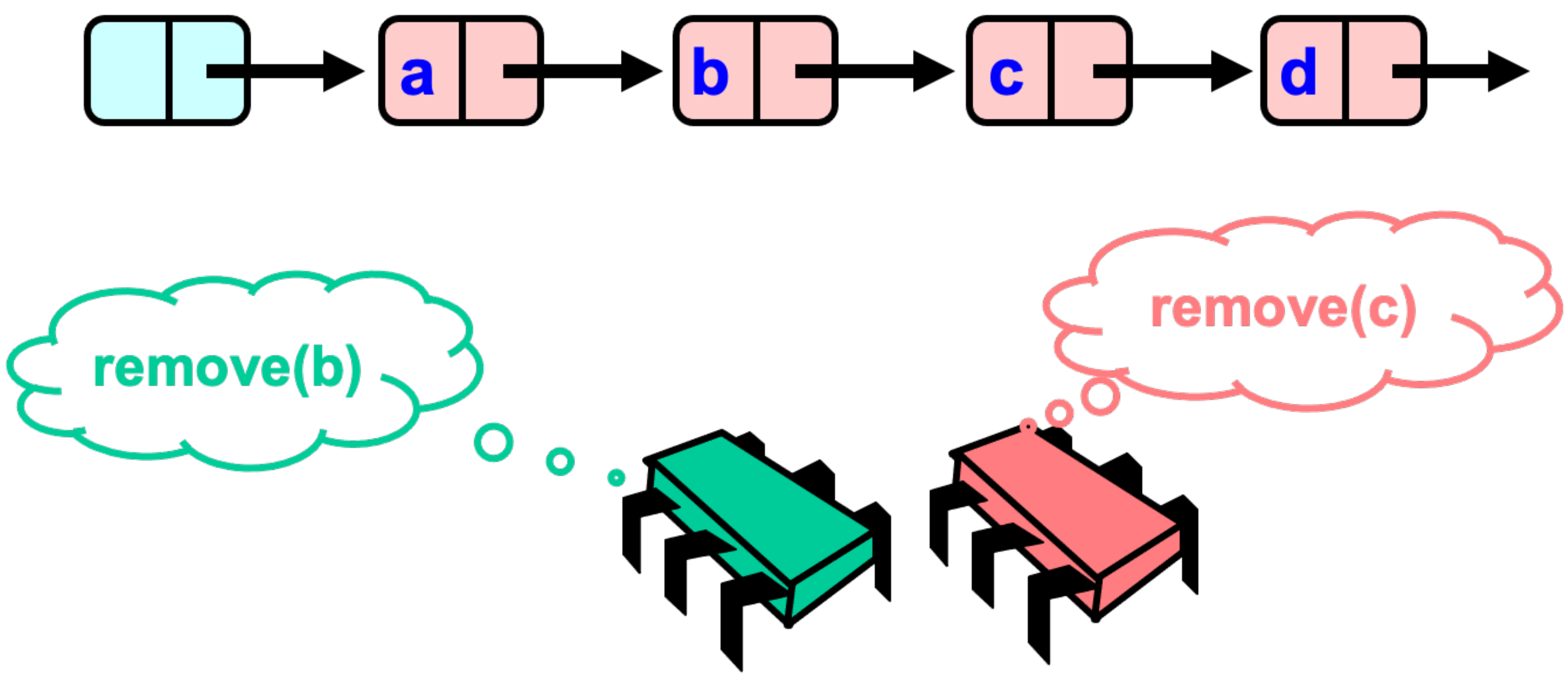


Fine Grained Locking

- Another thread might want to add after b
    - Homework 3

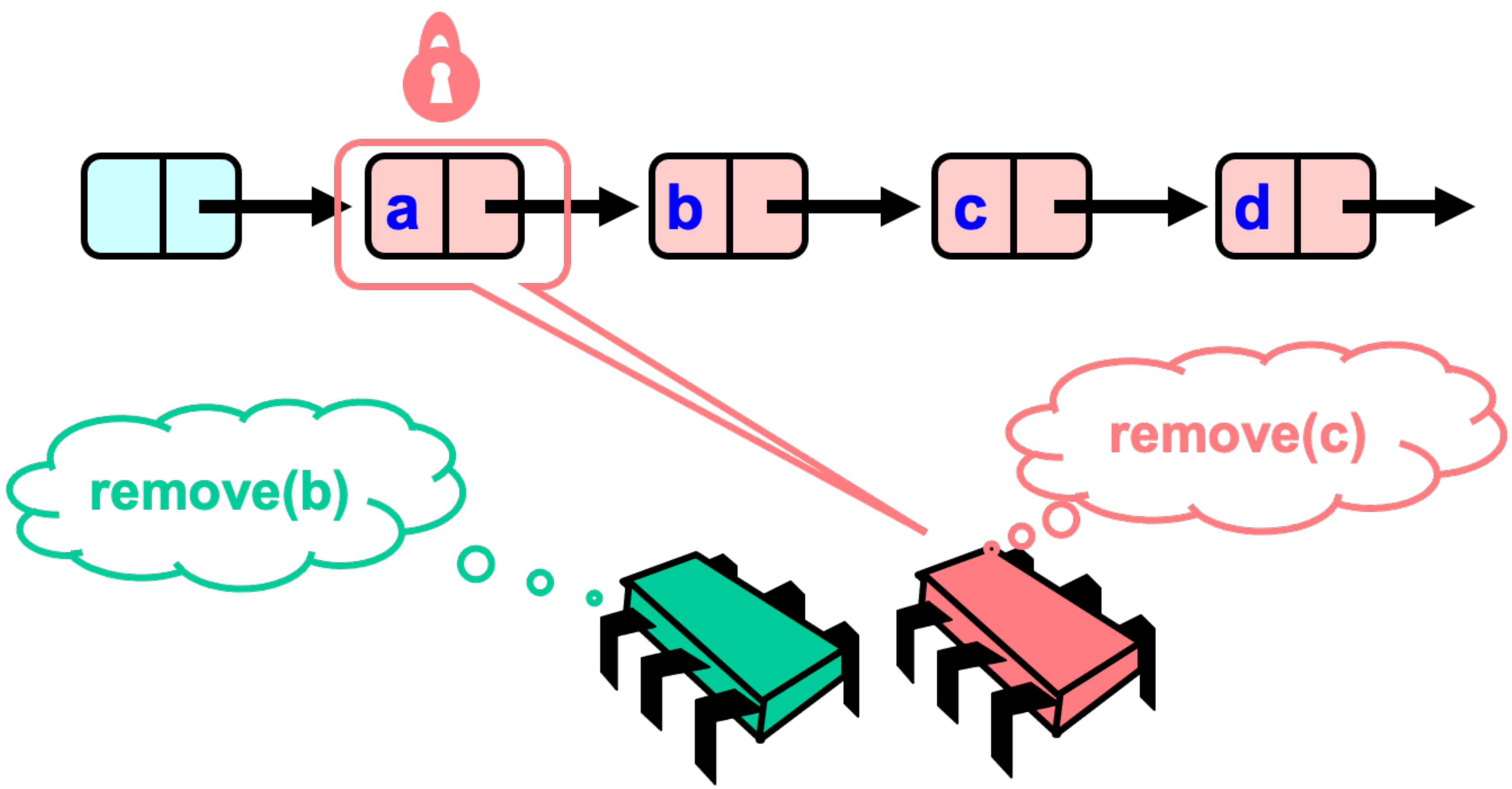
Fine Grained Locking

- Concurrent Removal



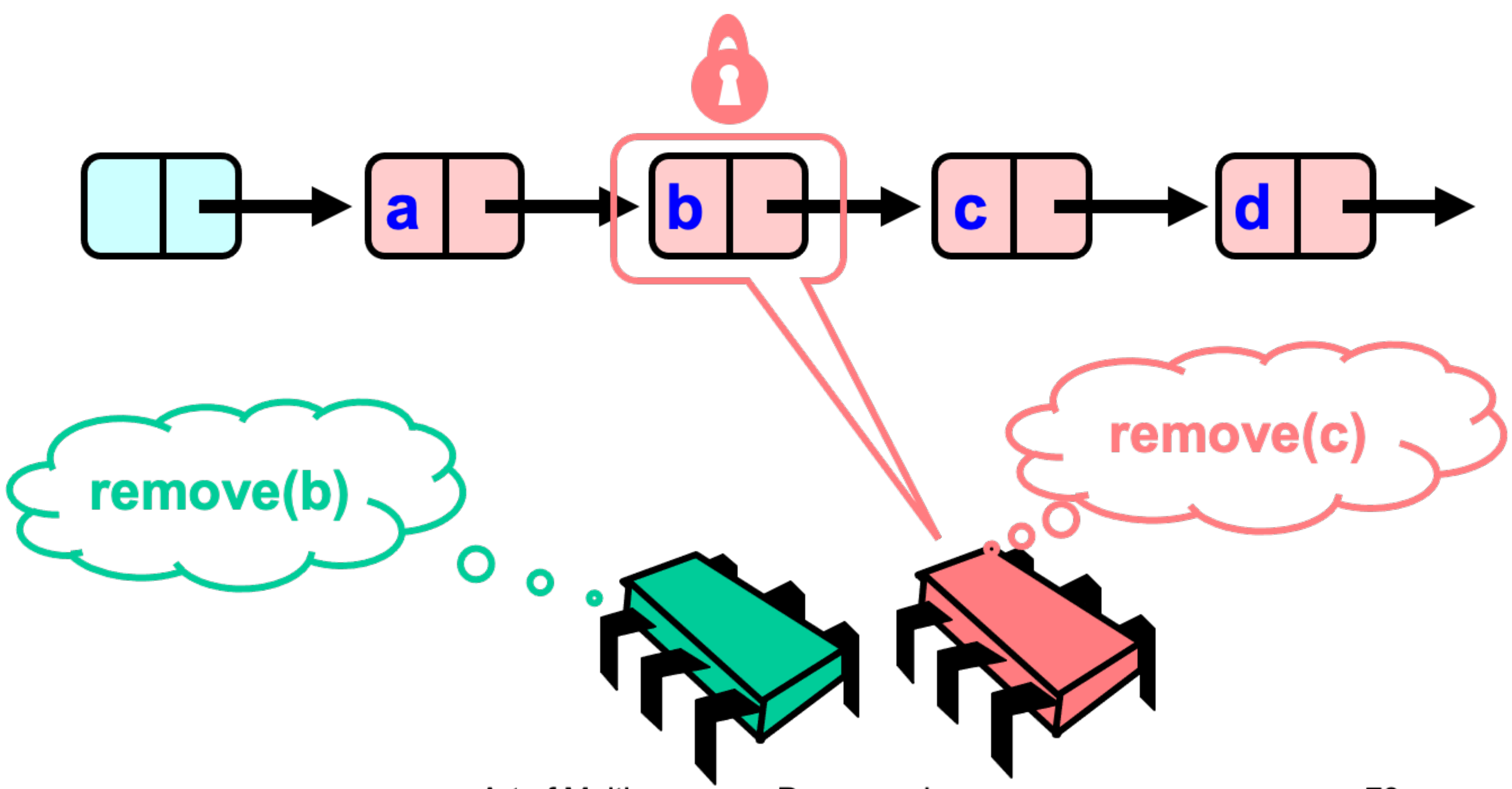
Fine Grained Locking

- Concurrent Removal



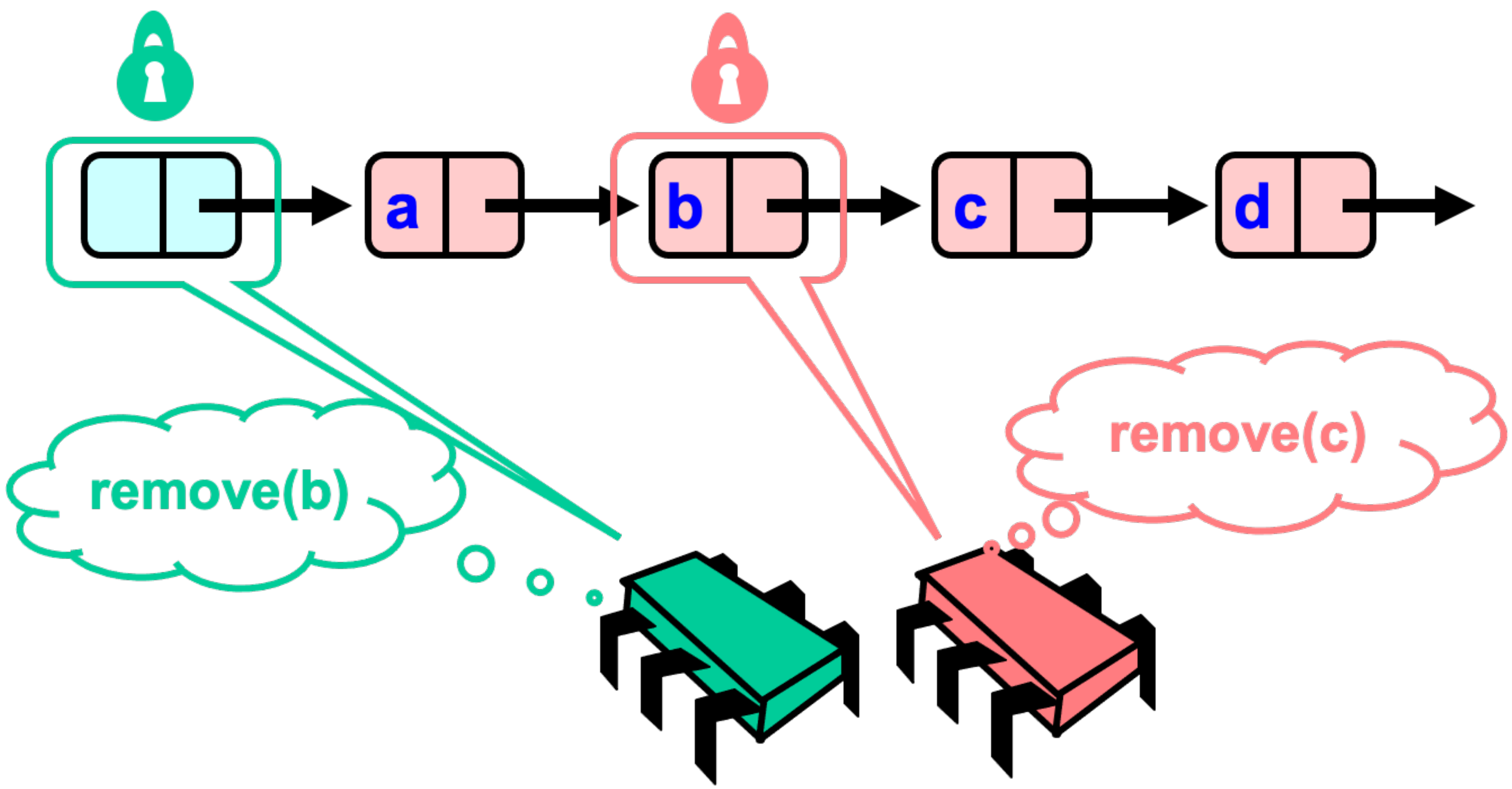
Fine Grained Locking

- Concurrent Removal



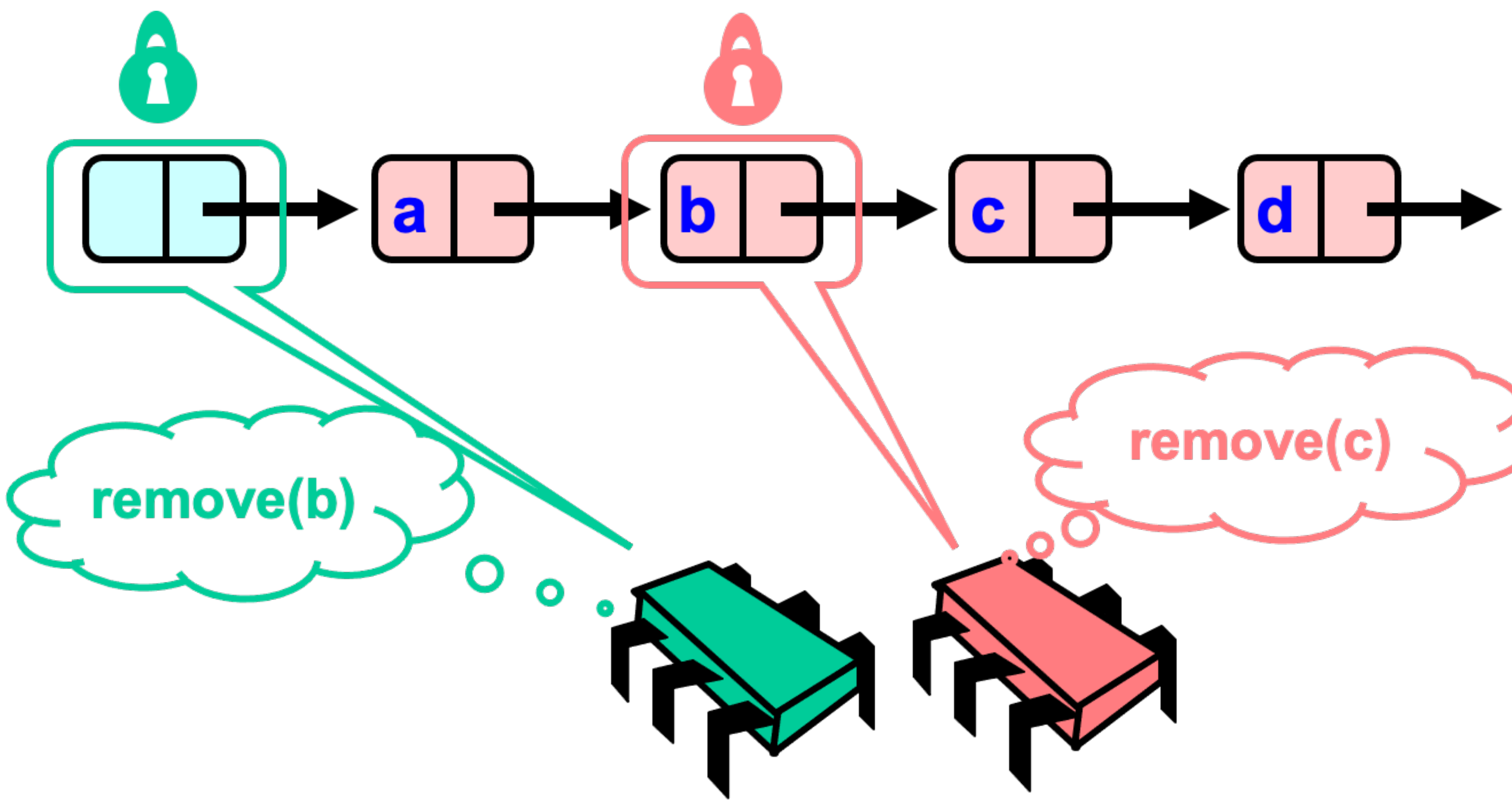
Fine Grained Locking

- Concurrent Removal



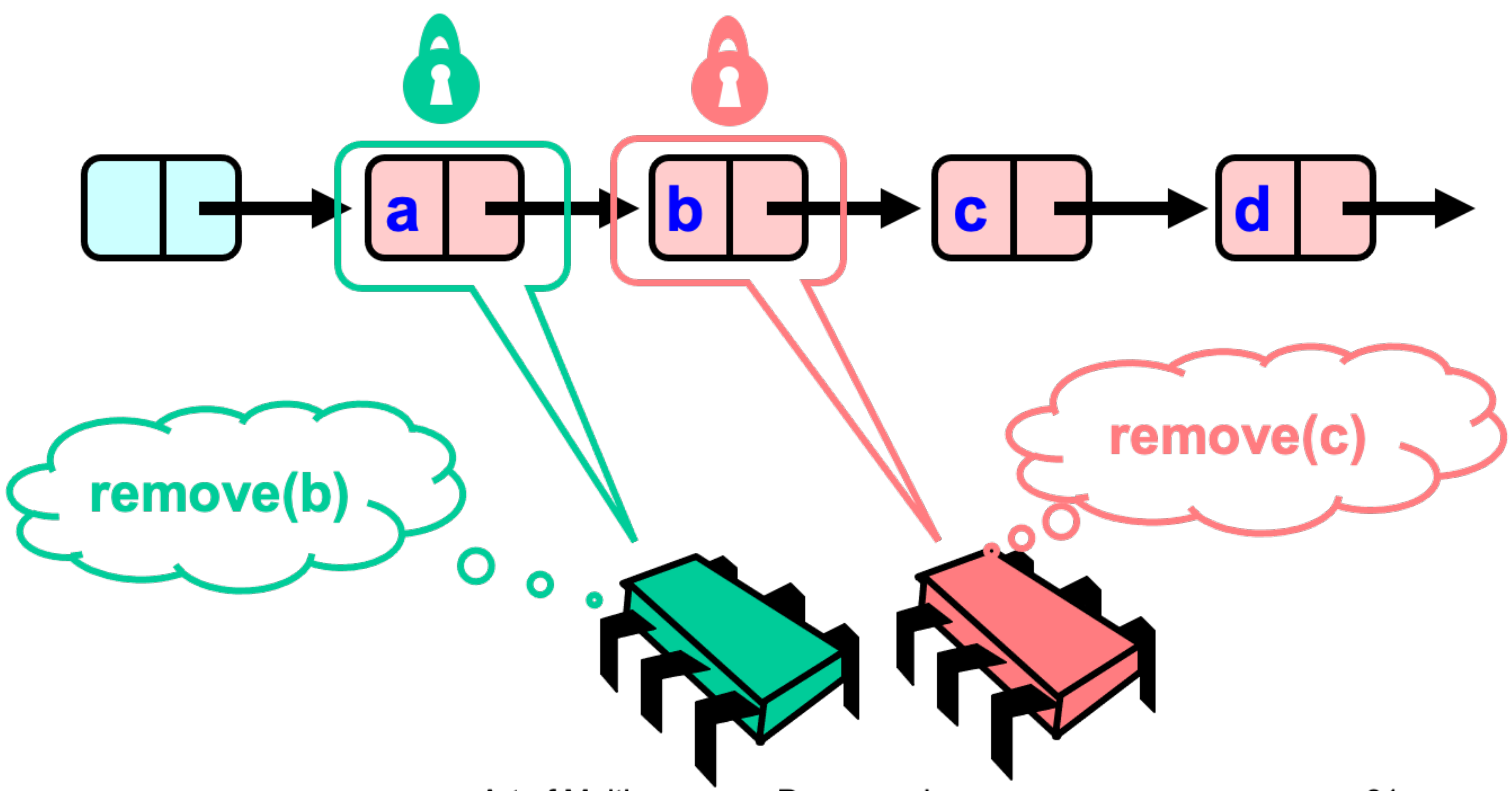
Fine Grained Locking

- Concurrent Removal



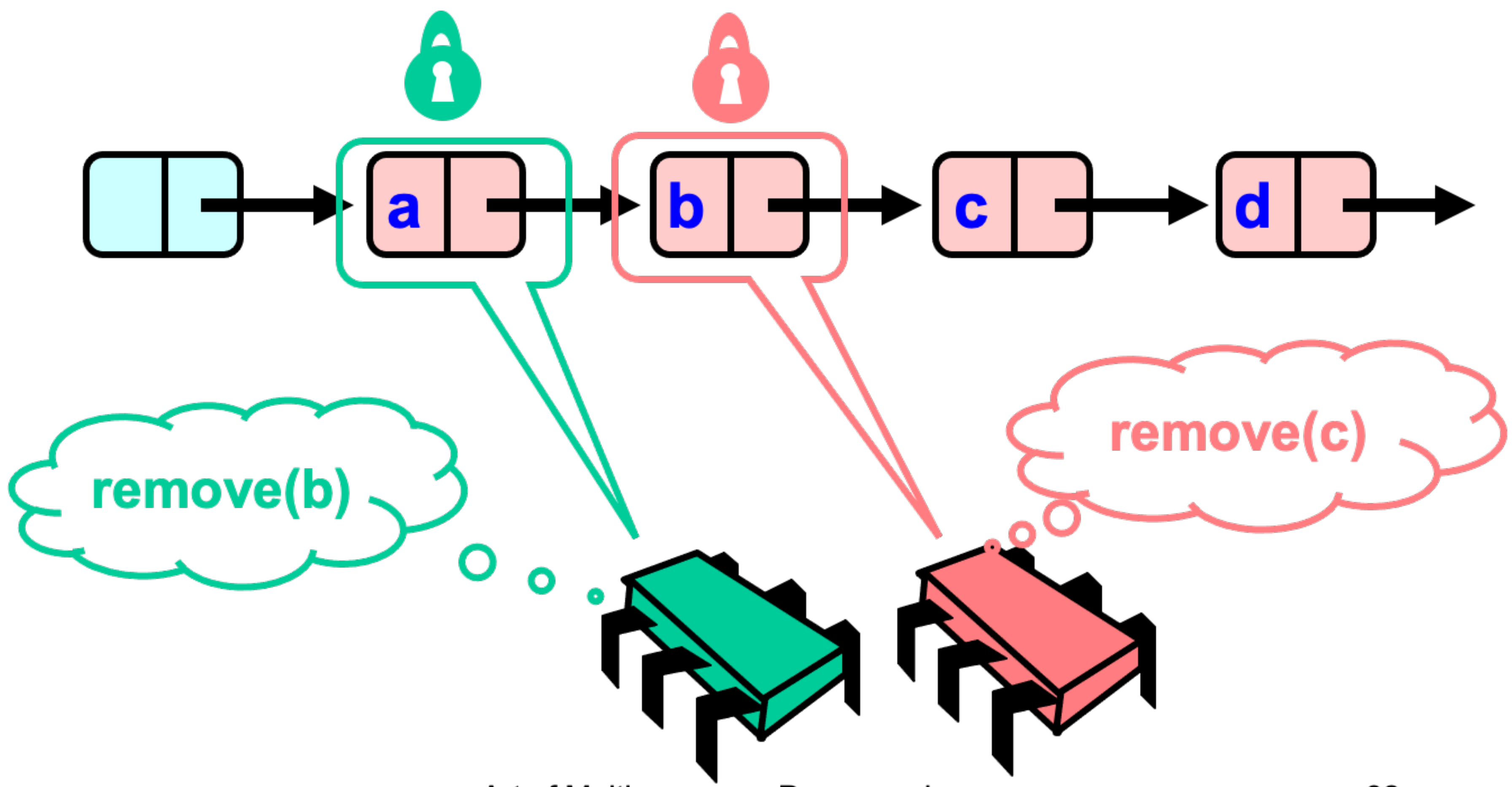
Fine Grained Locking

- Concurrent Removal



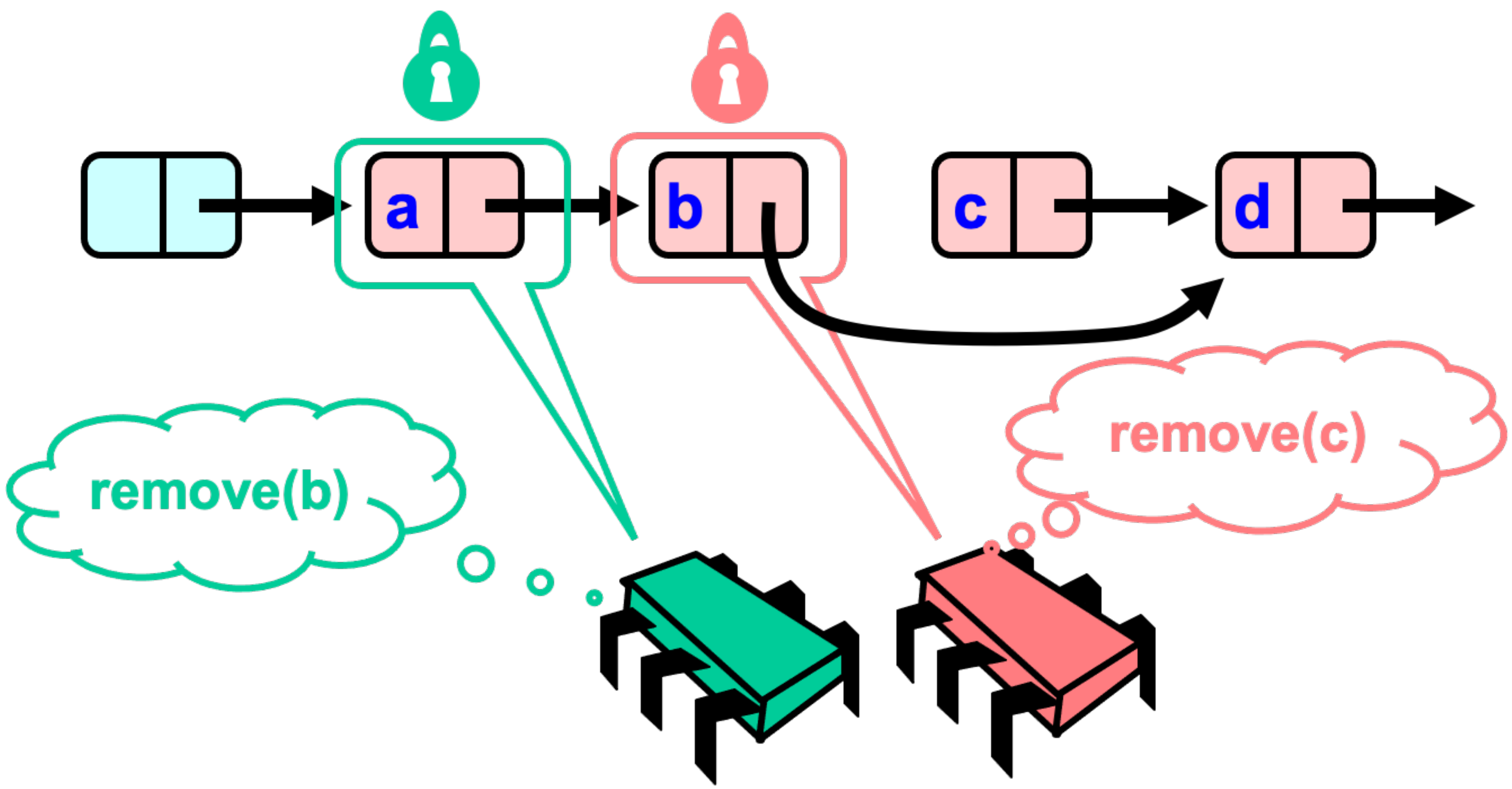
Fine Grained Locking

- Concurrent Removal



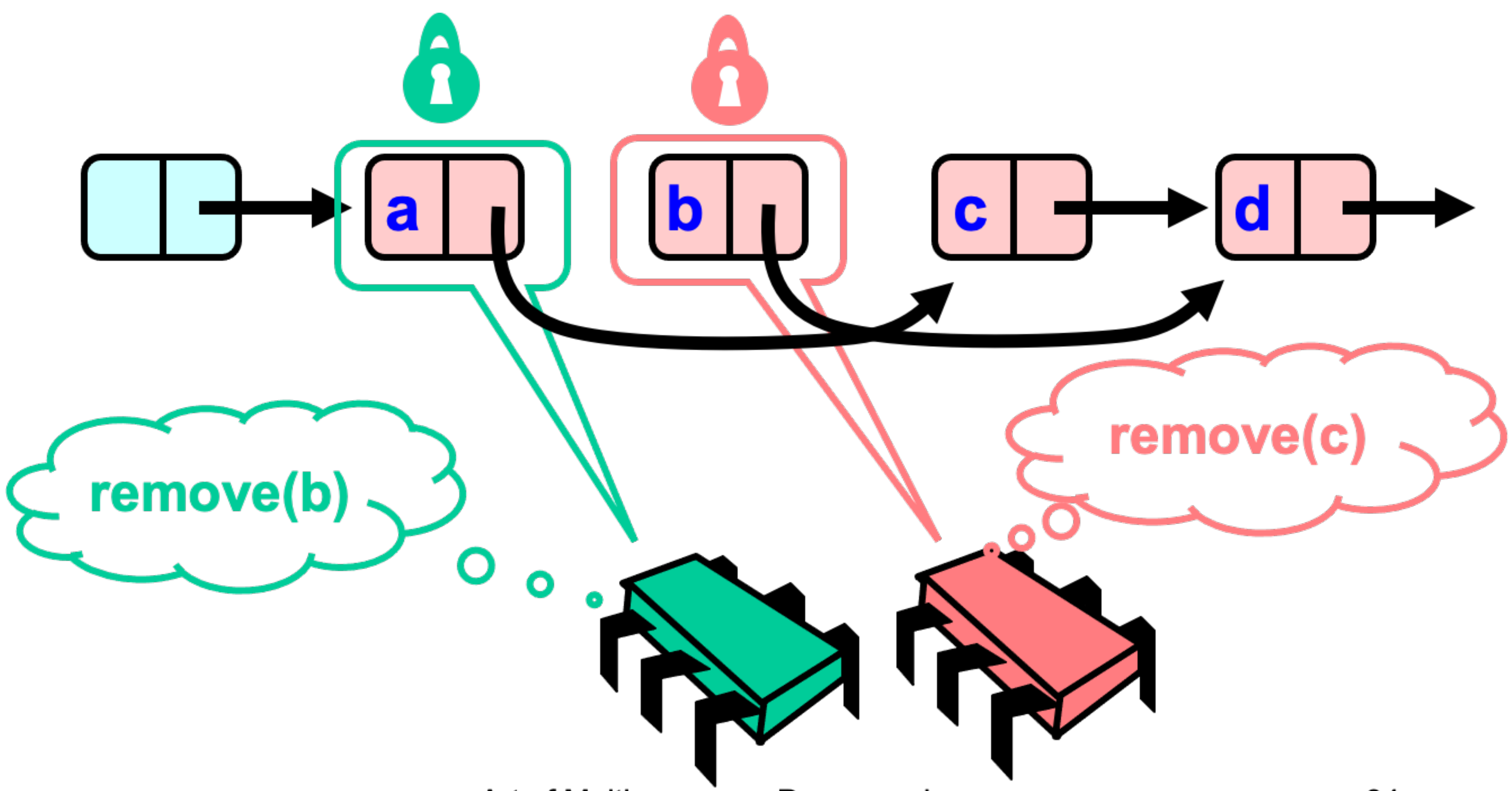
Fine Grained Locking

- Concurrent Removal



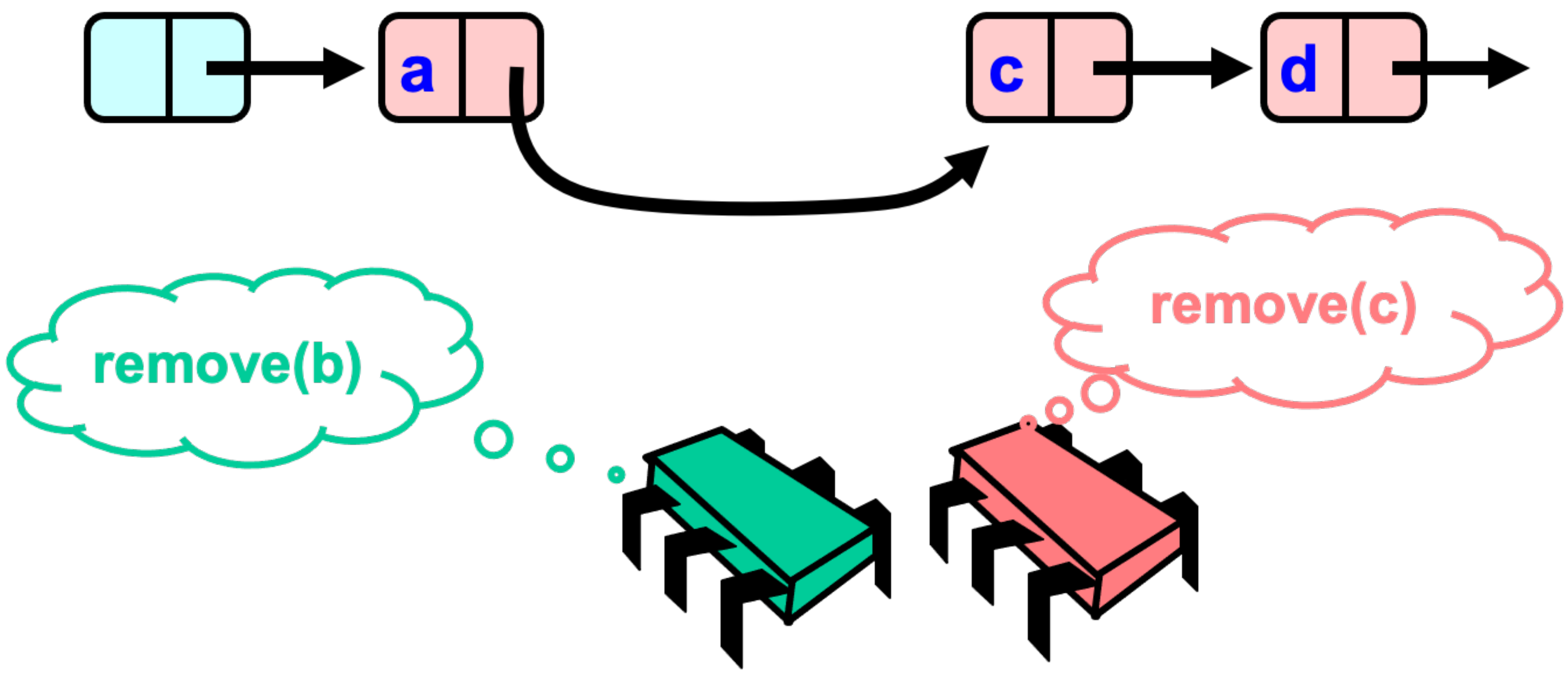
Fine Grained Locking

- Concurrent Removal



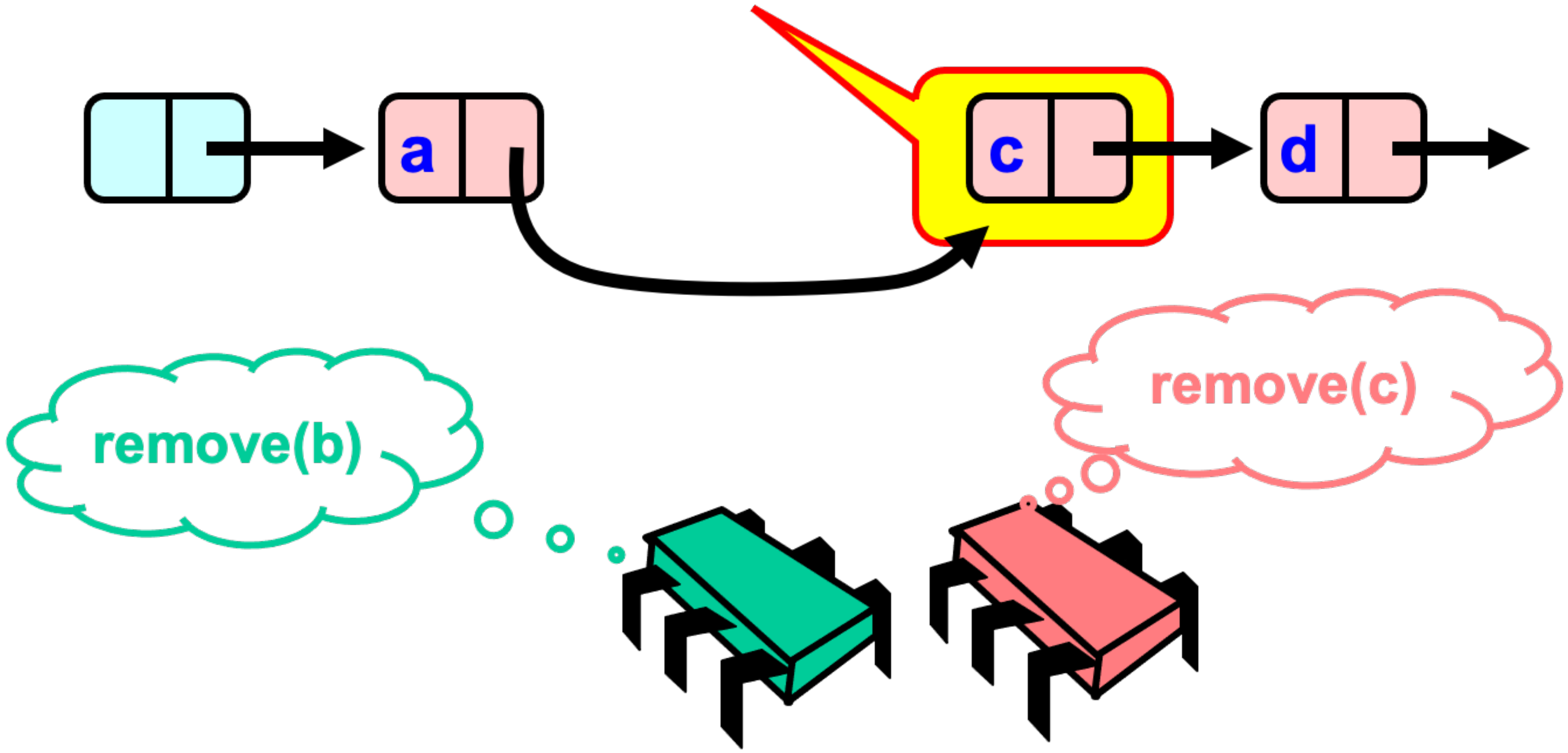
Fine Grained Locking

- Concurrent removal undoes one threads work



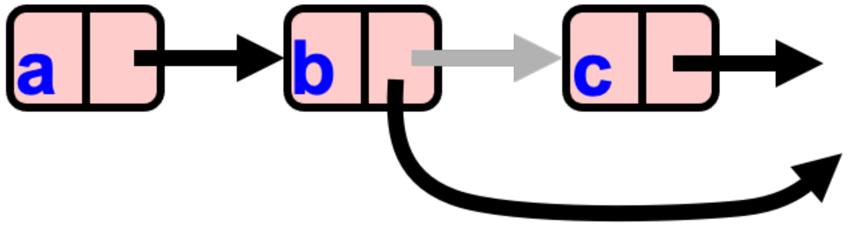
Fine Grained Locking

- Node c has not been removed

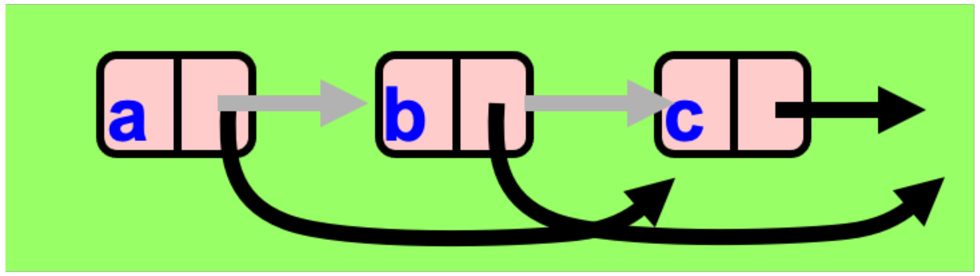


Fine Grained Locking

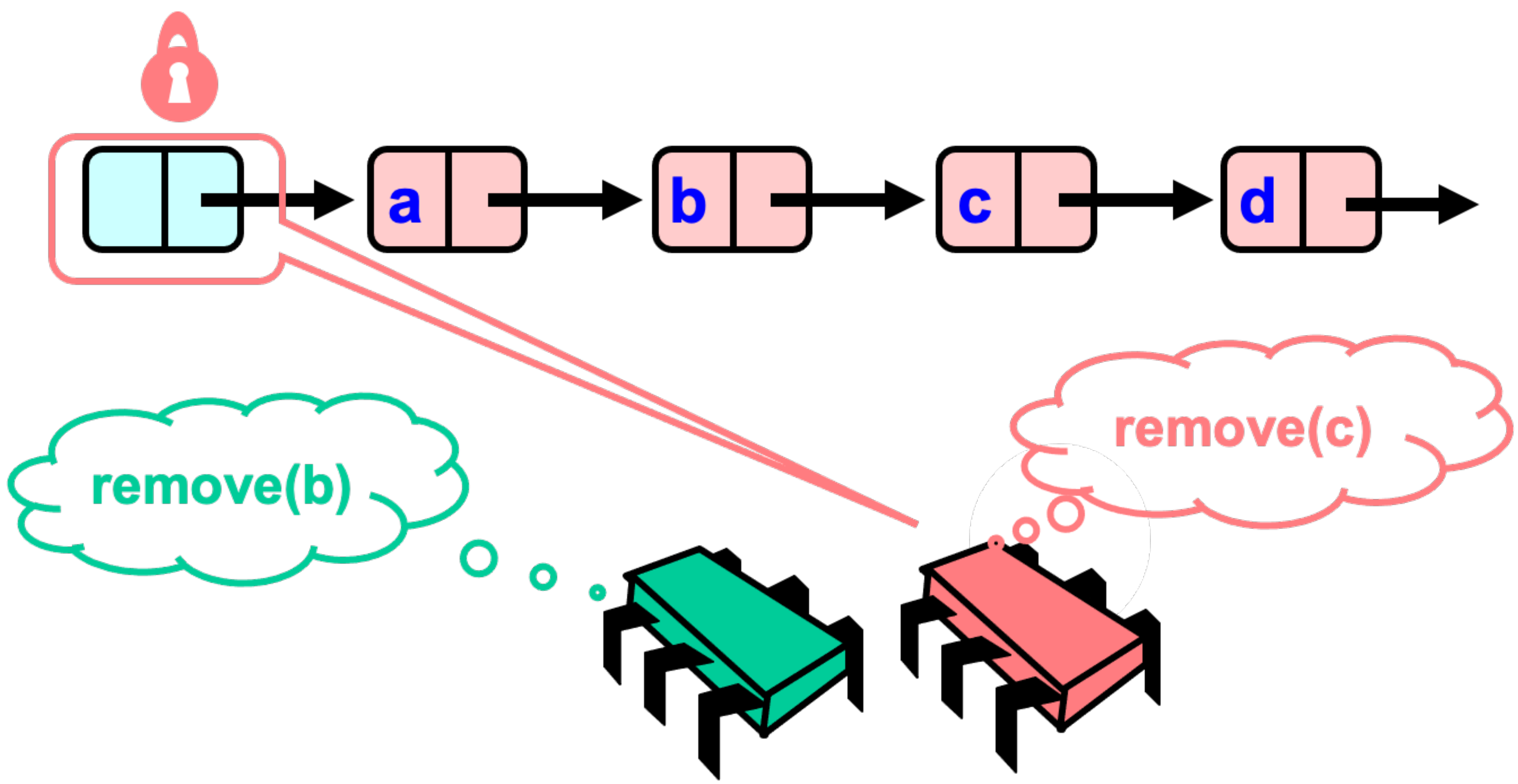
- Problem
  - To delete node C, we swing its predecessor's next-field to its successor



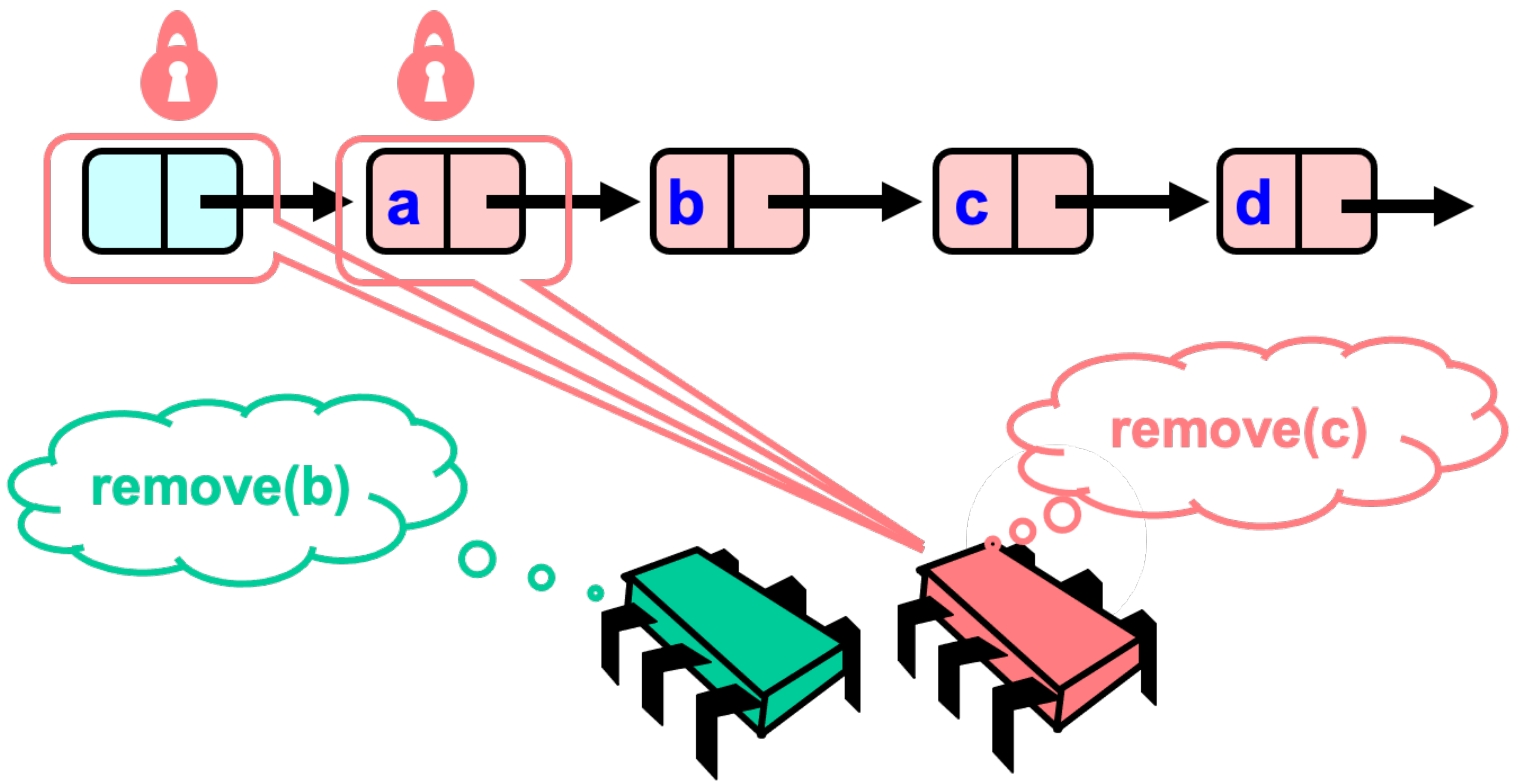
  - But someone could create another pointer to C



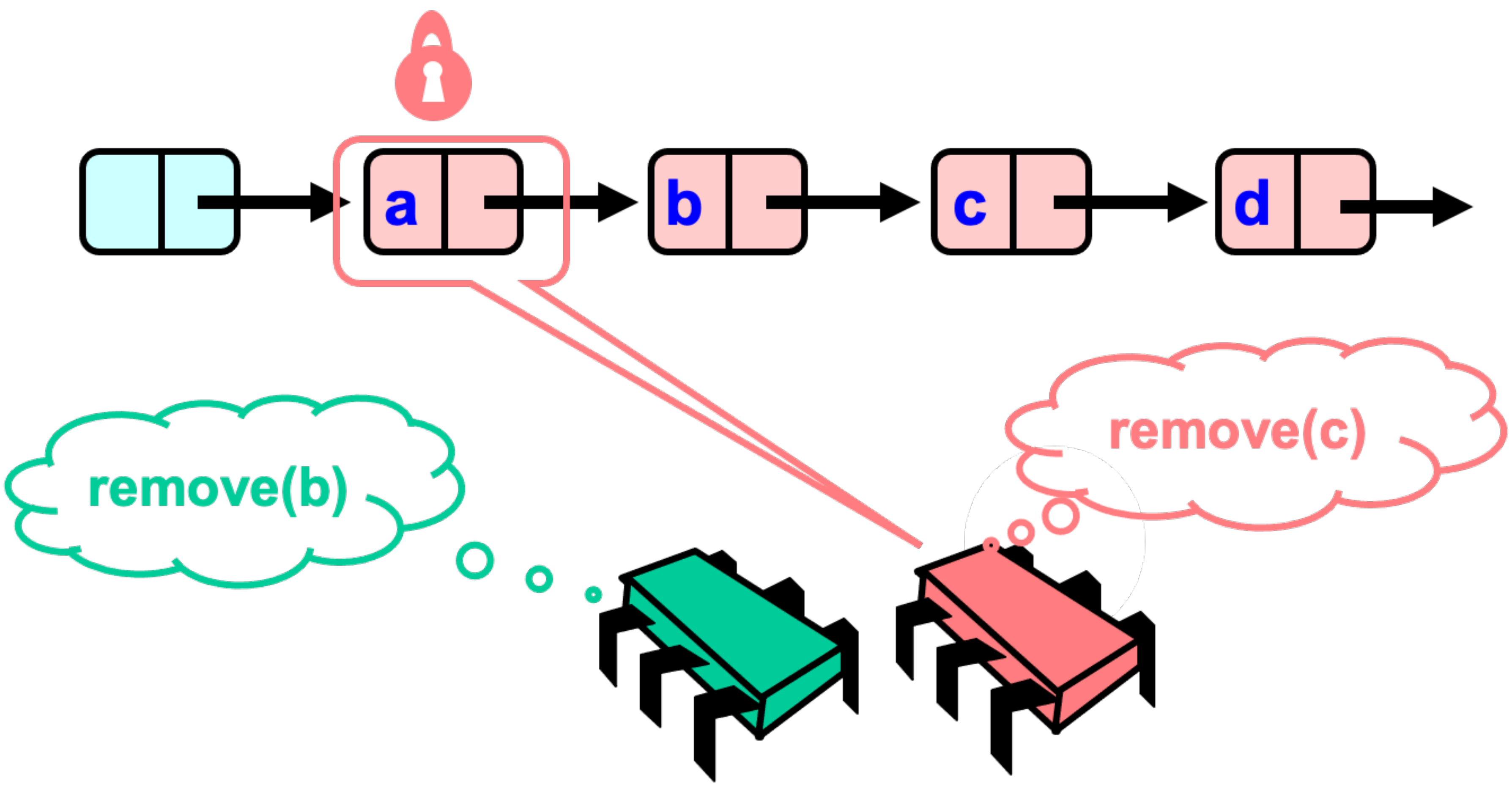
Fine Grained Locking



Fine Grained Locking



Fine Grained Locking



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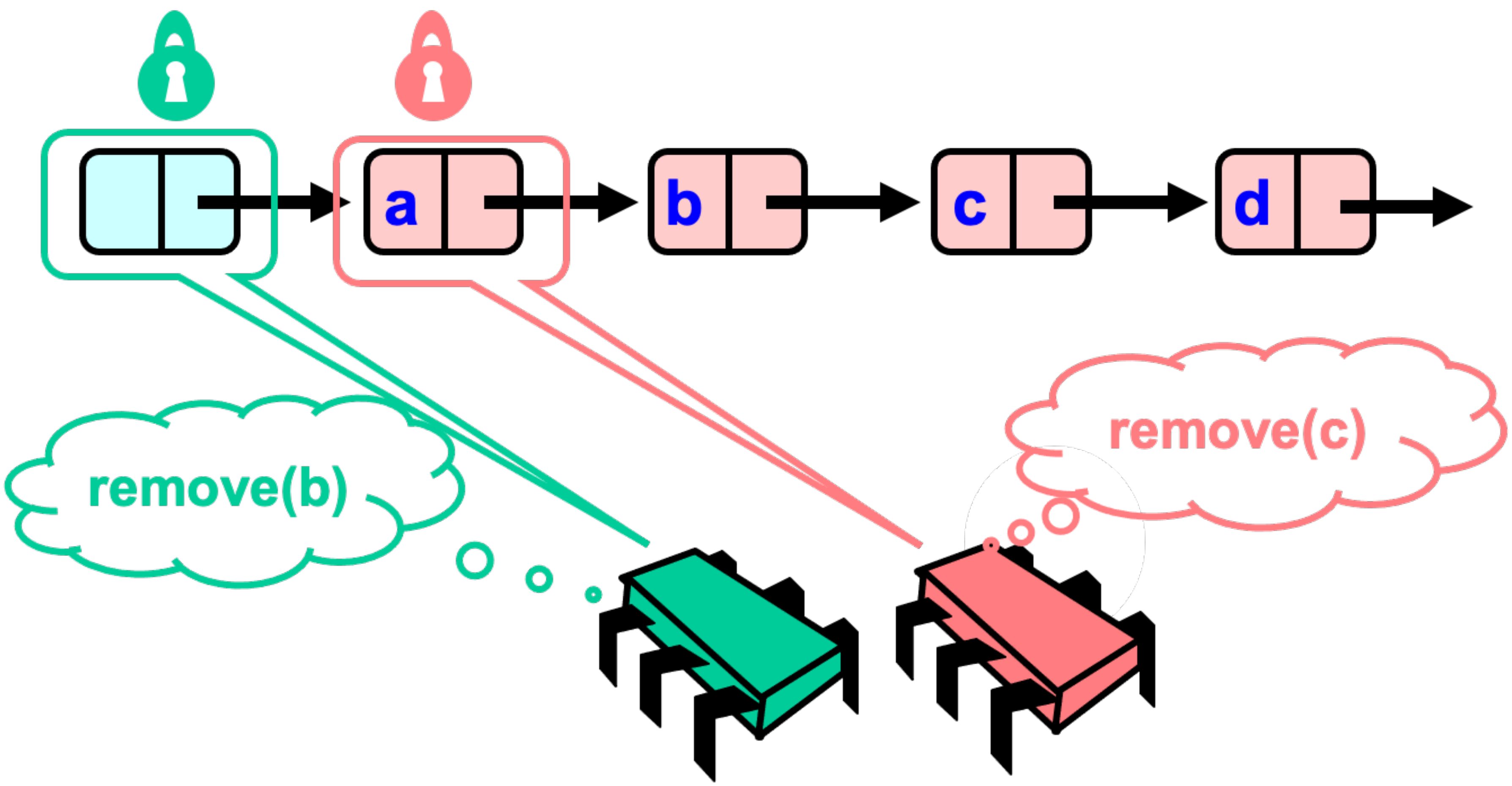

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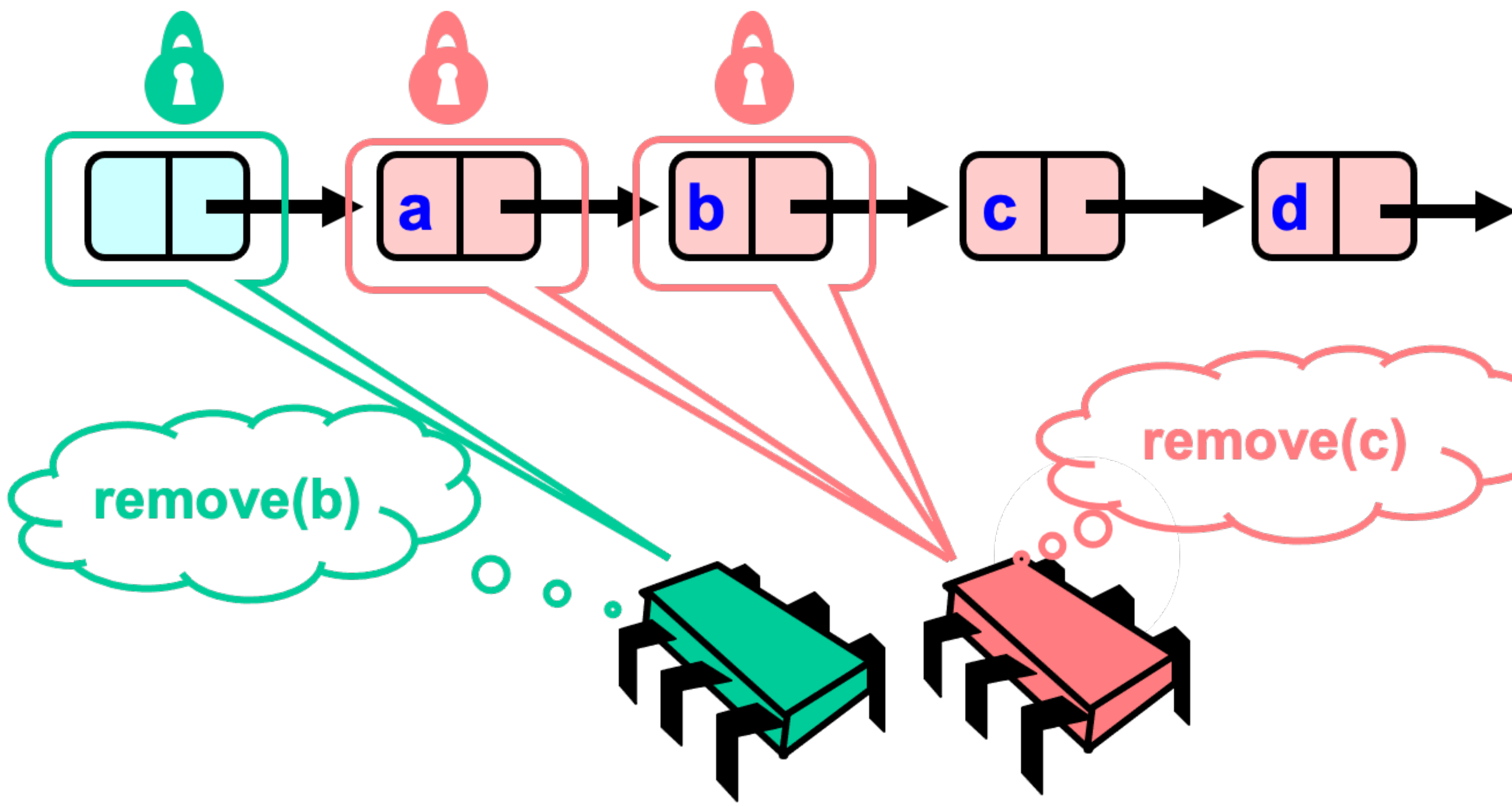

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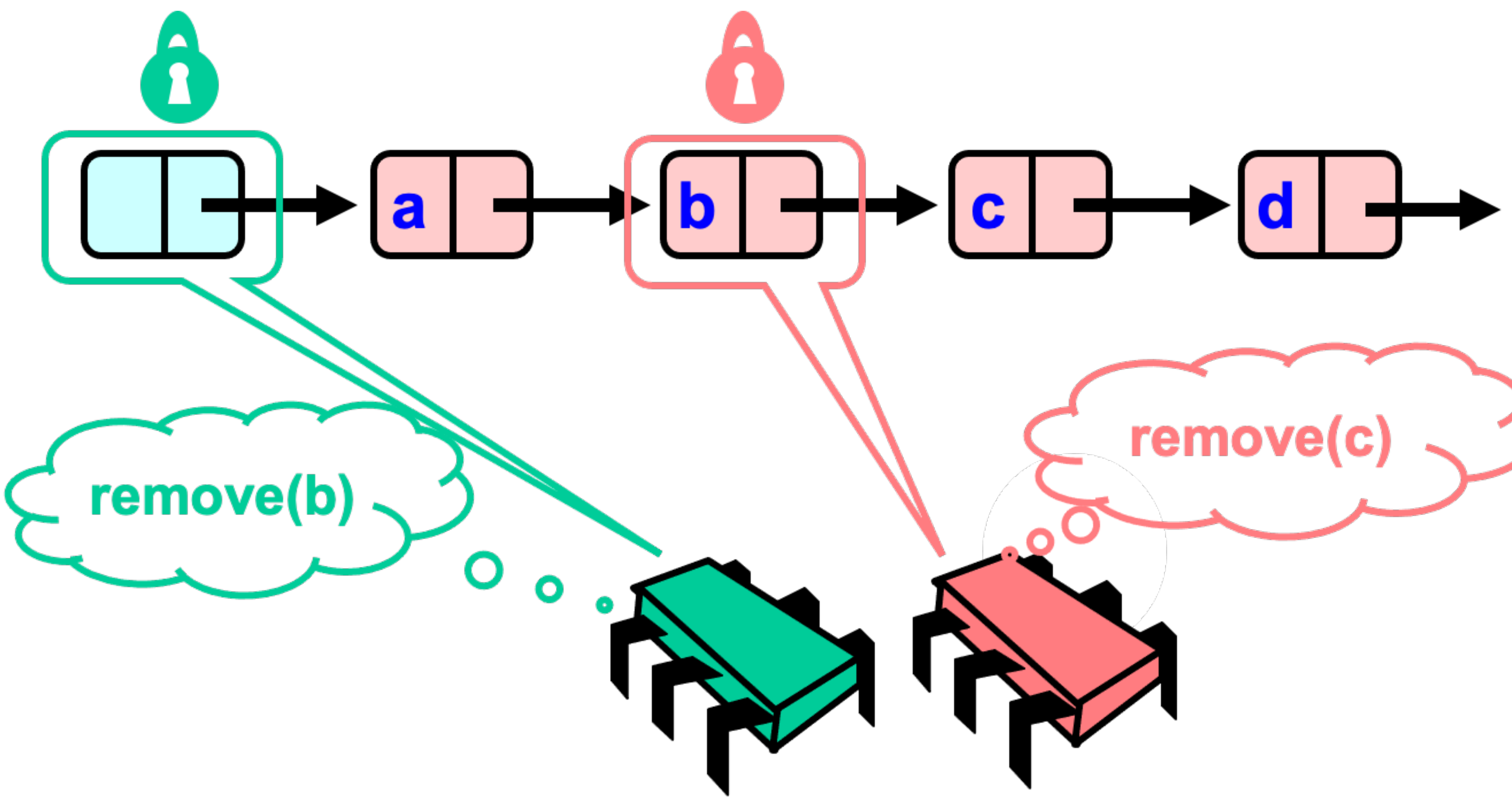

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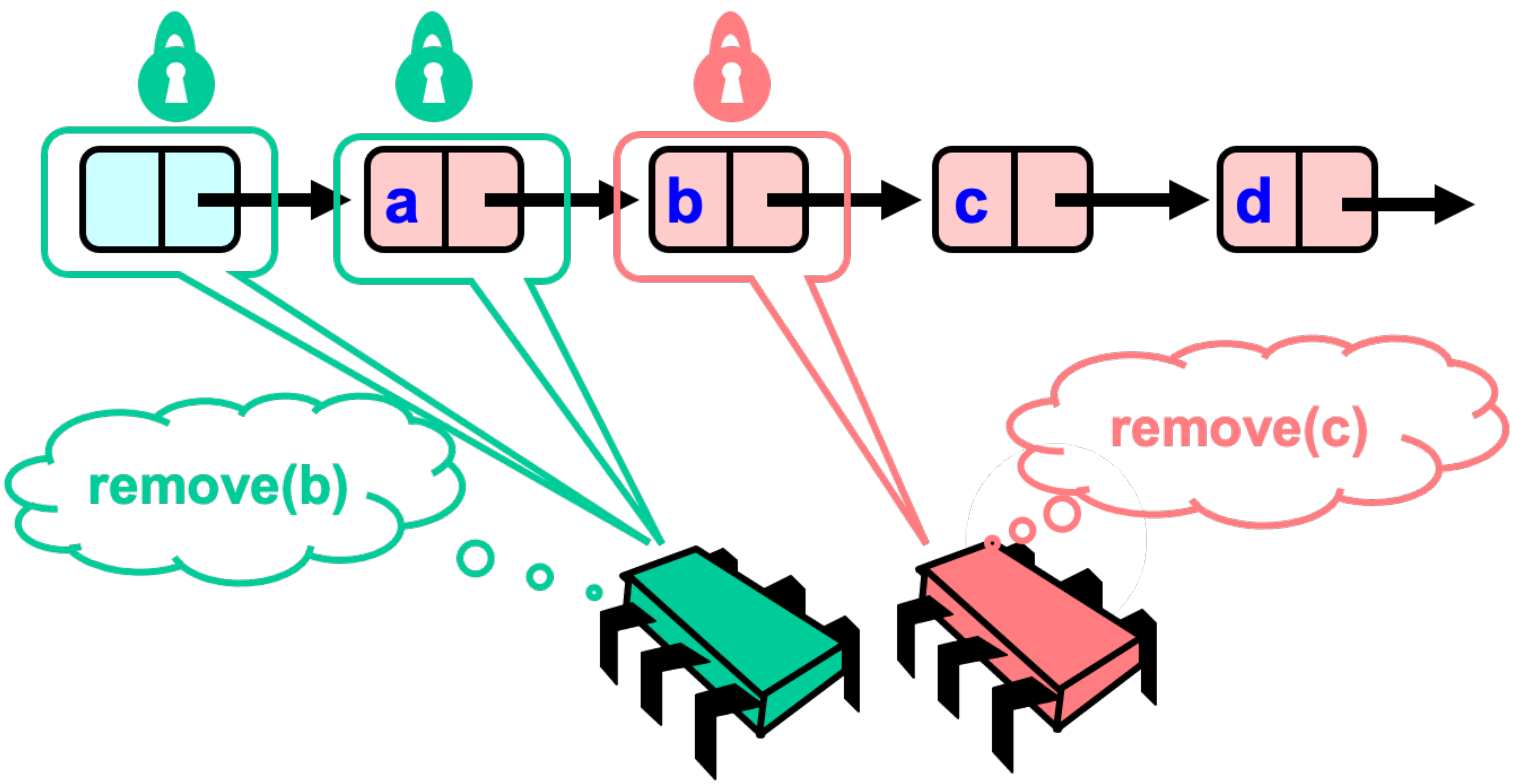

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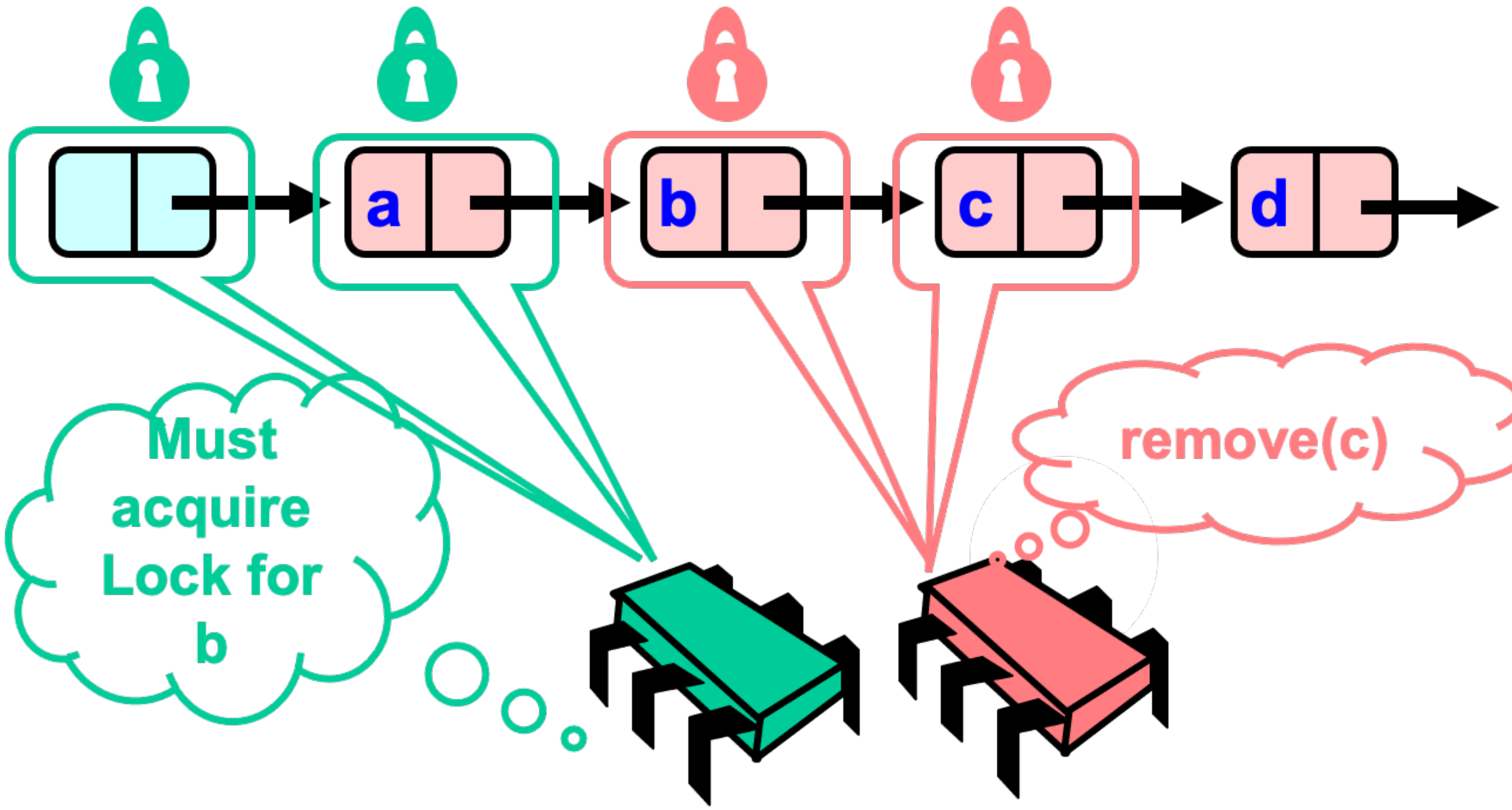
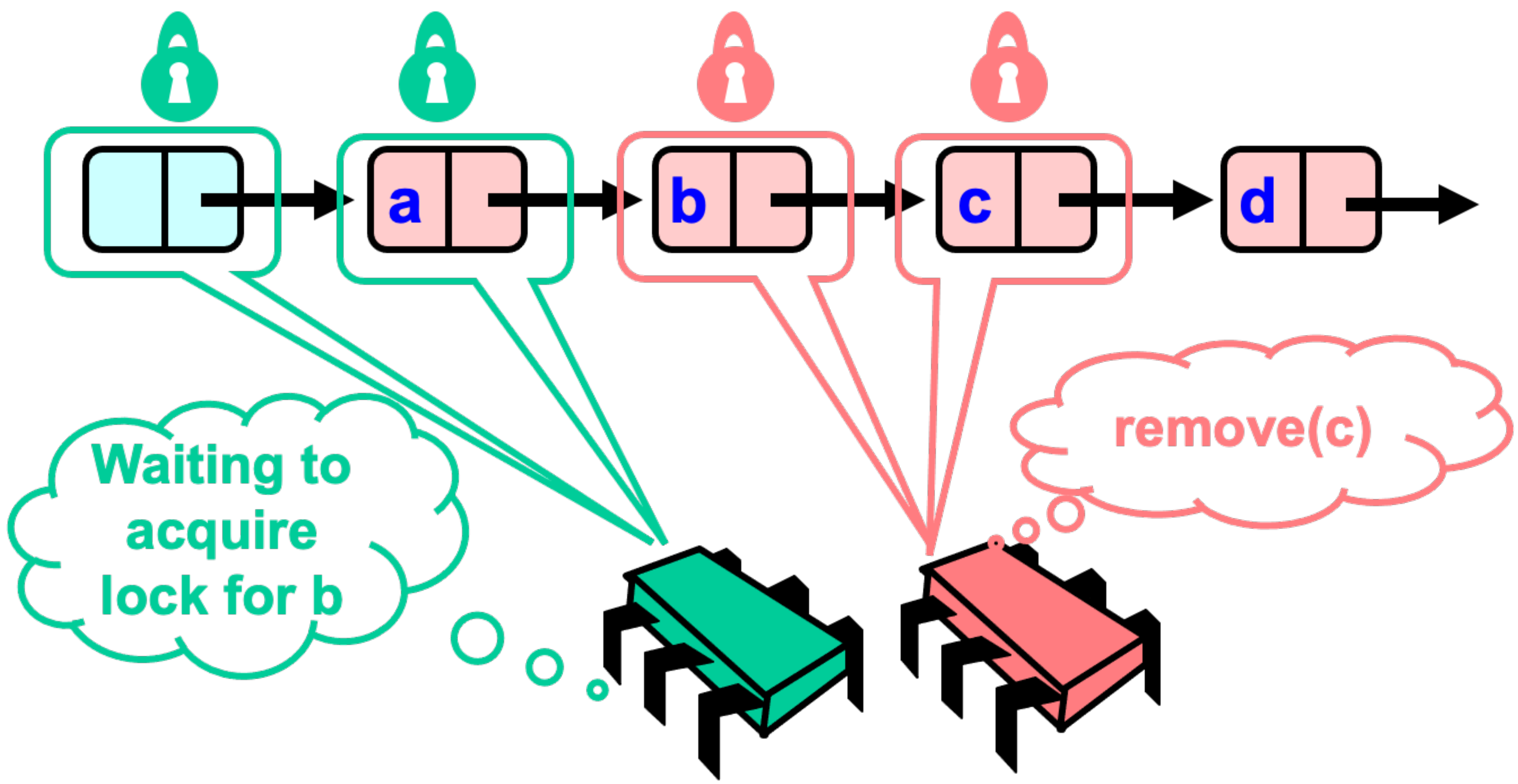
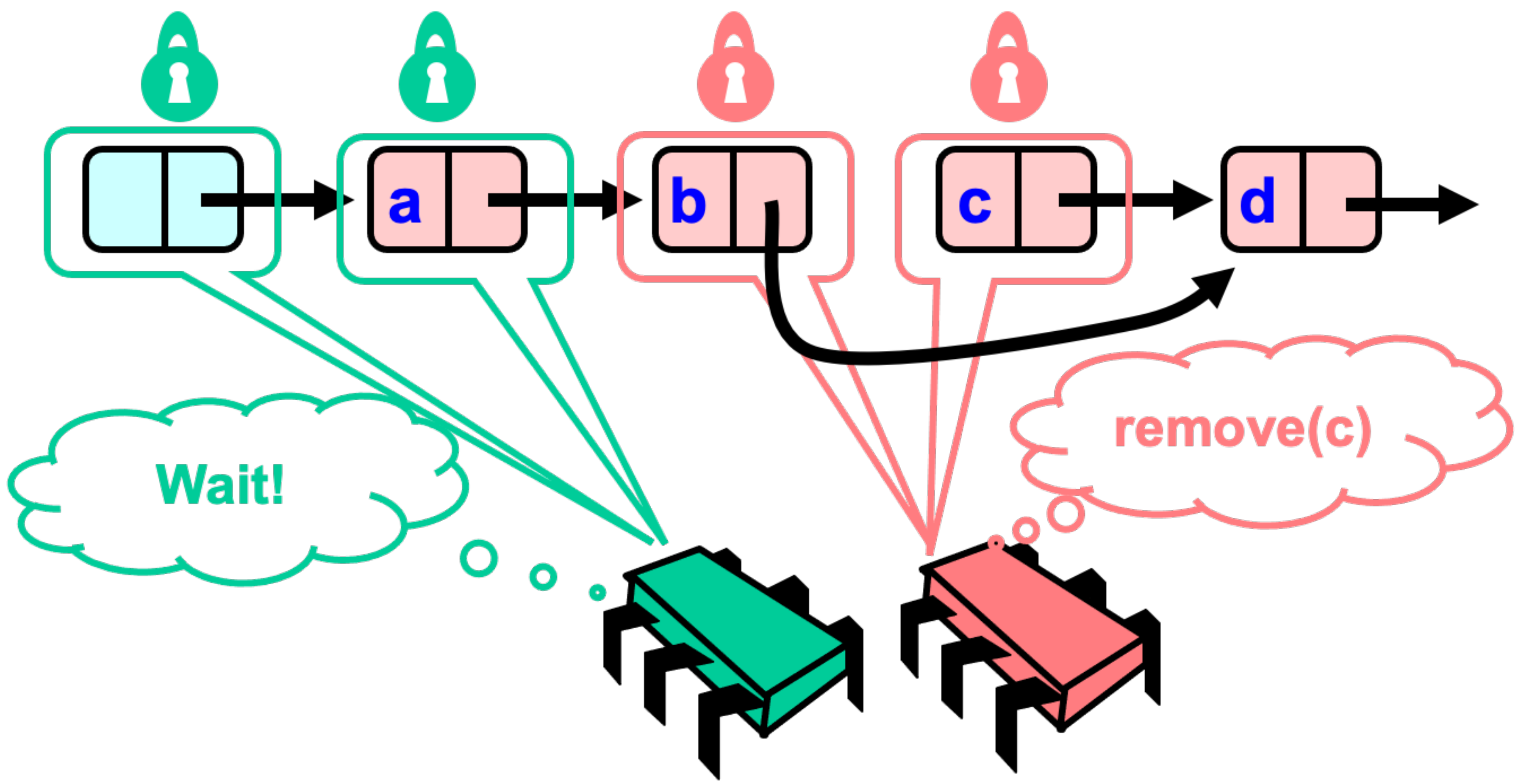


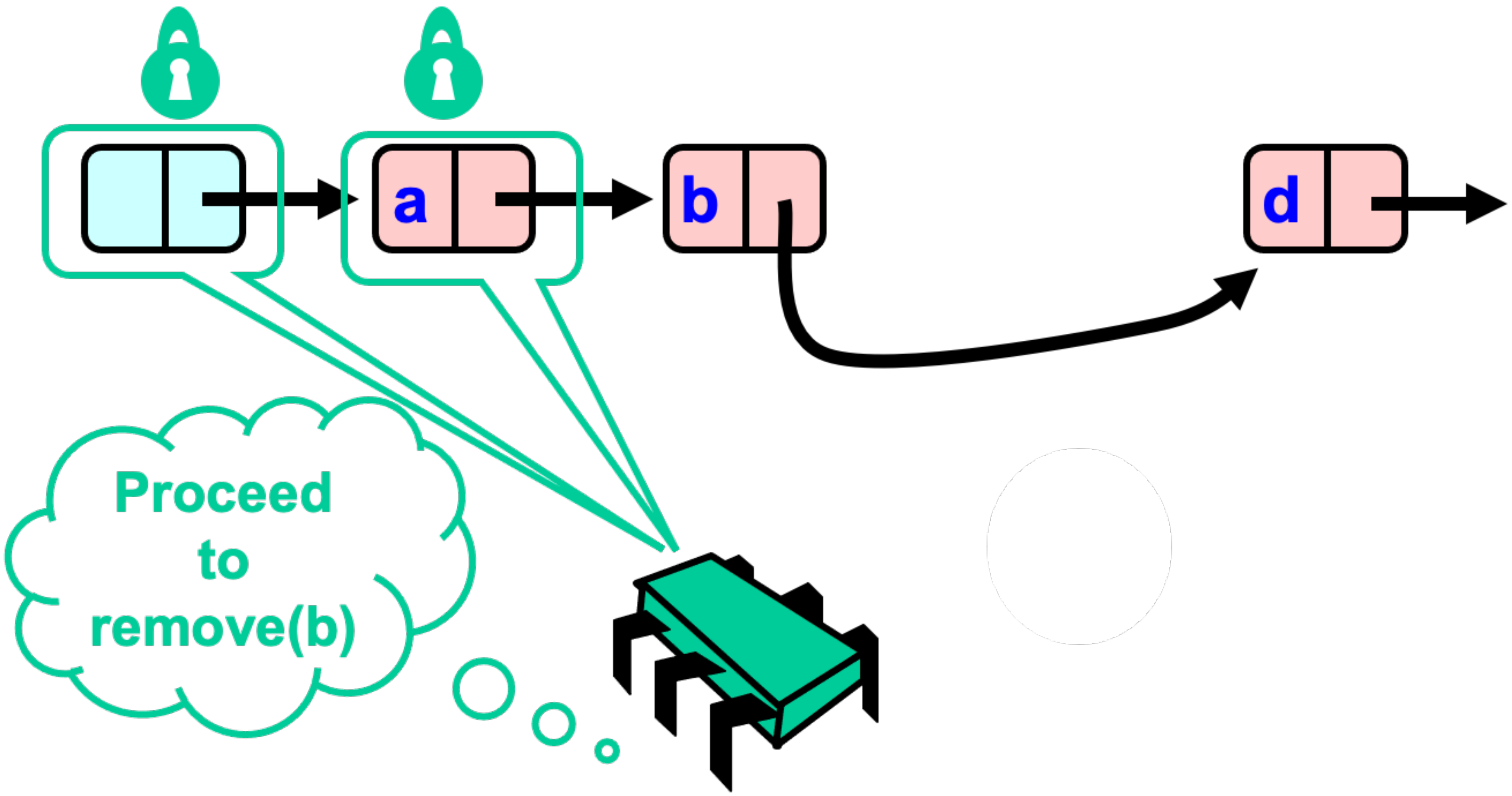
Fine Grained Locking



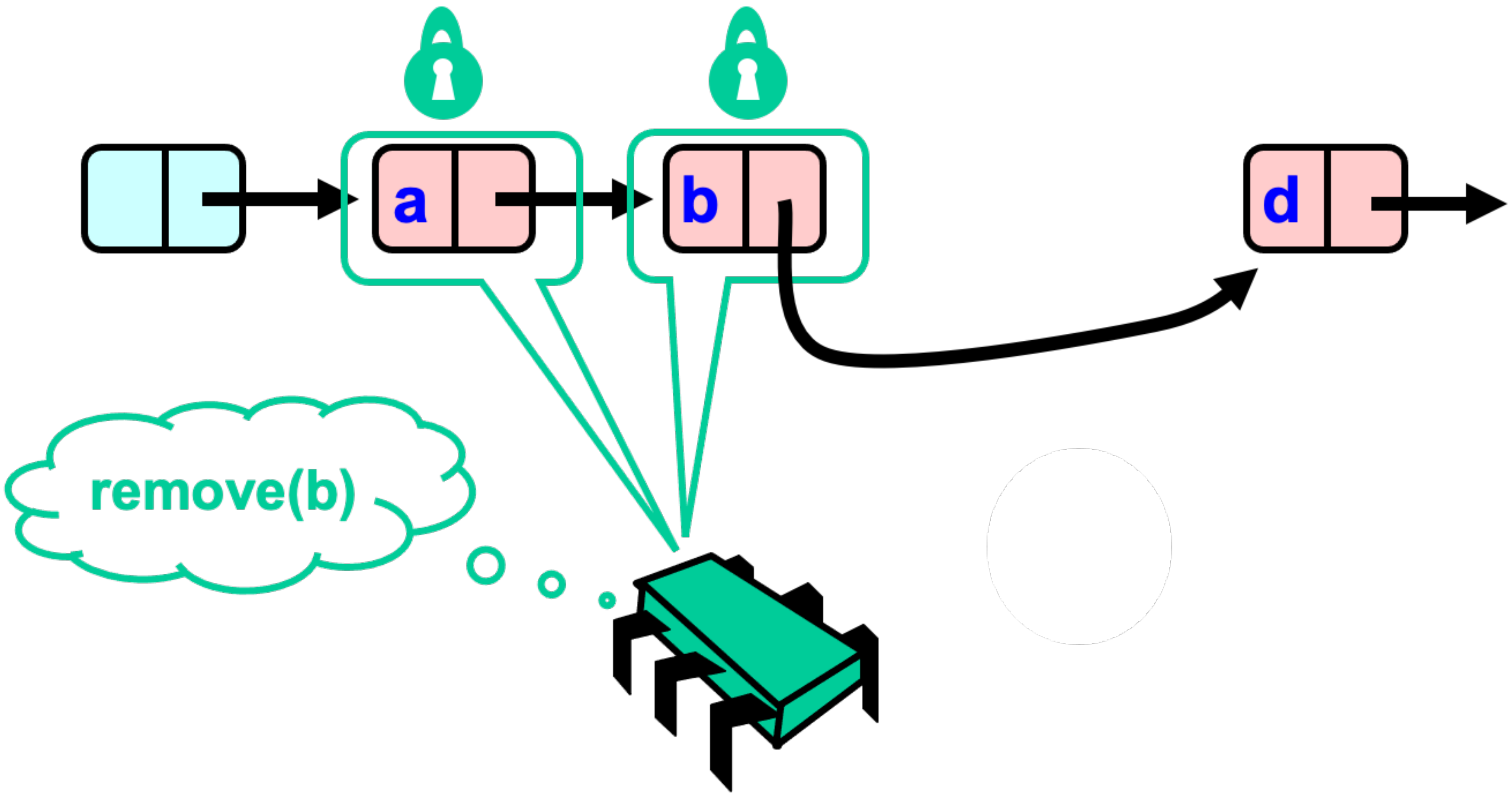
Fine Grained Locking



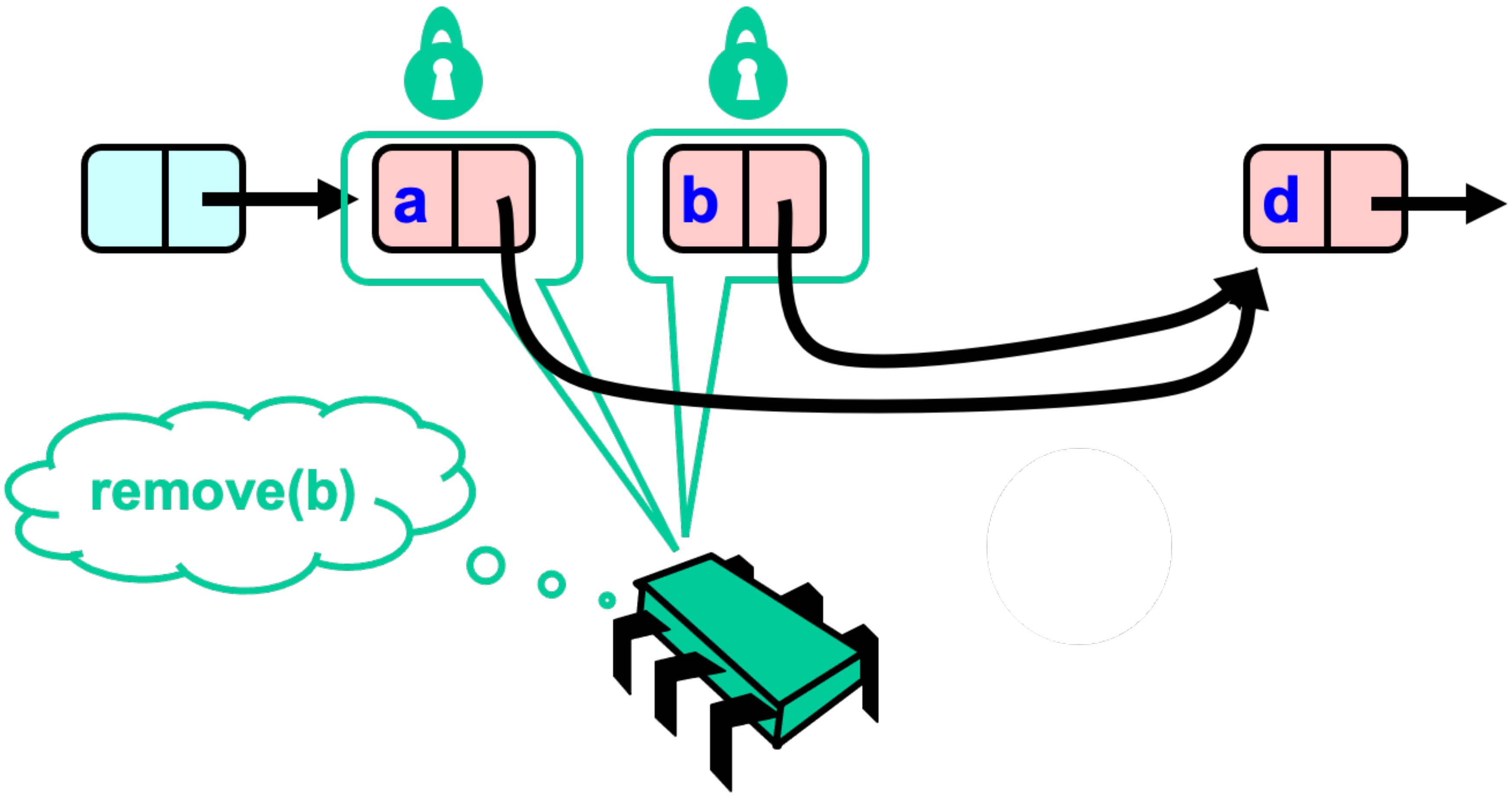
Fine Grained Locking



Fine Grained Locking



Fine Grained Locking



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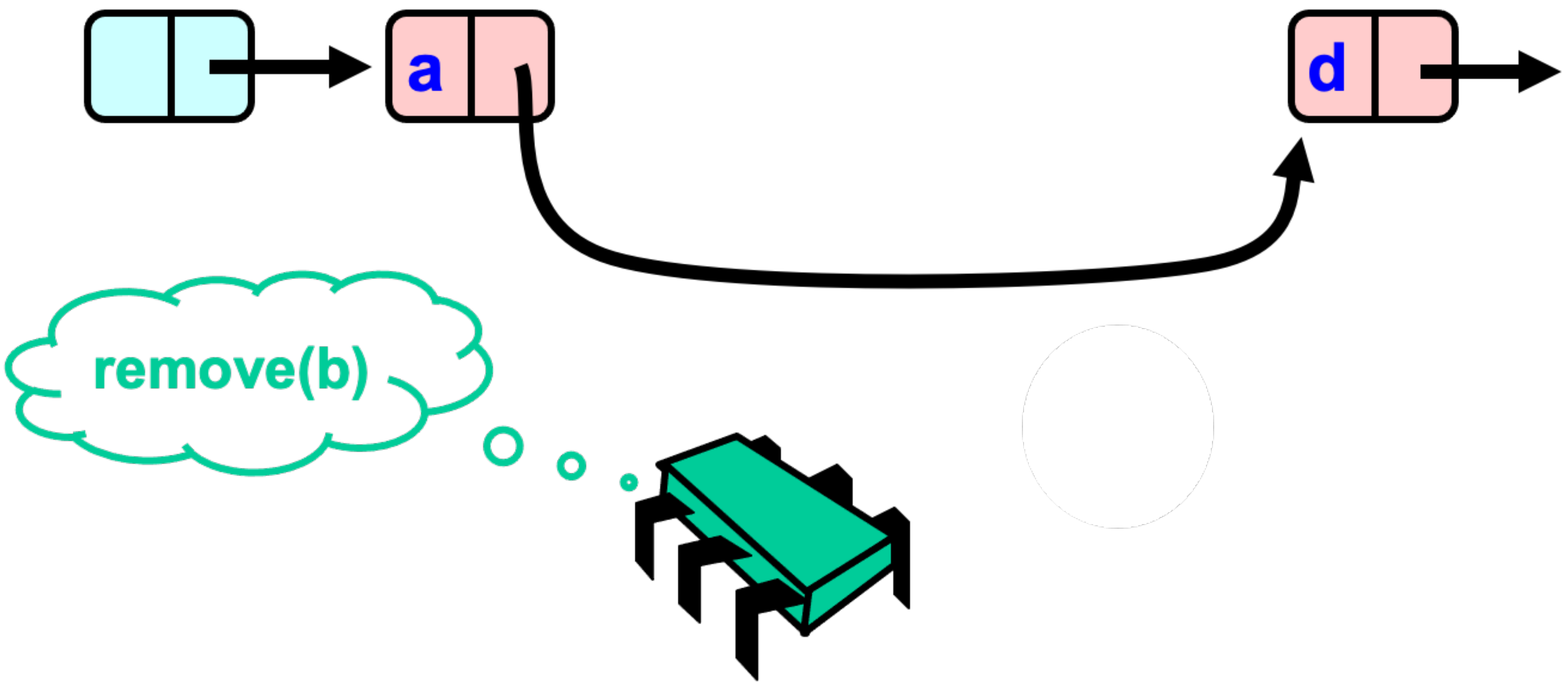

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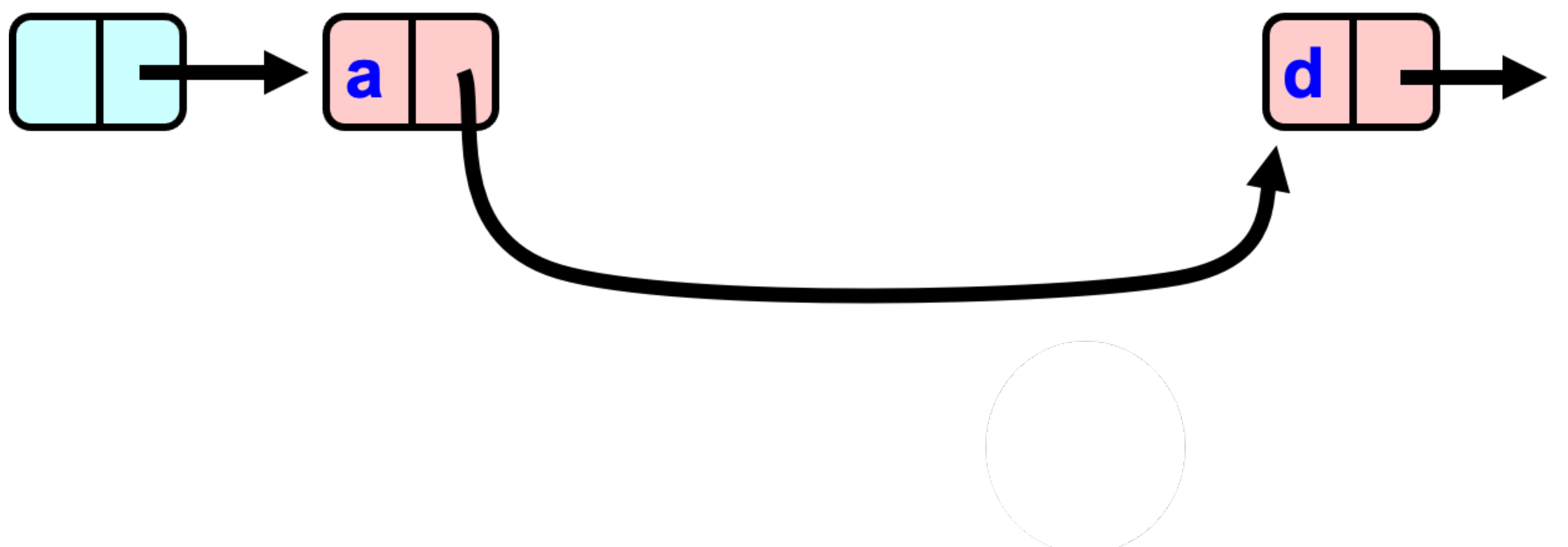


Fine Grained Locking

```
public boolean remove(T item) {
 int key = item.hashCode();
 Node pred, curr;
 try {
   …
 } finally {
  curr.unlock();
  pred.unlock();
 }}
```

Key used to order nodes

Fine Grained Locking

```
public boolean remove(T item) {
 int key = item.hashCode();
 Node pred, curr;
 try {
   …
 } finally {
  curr.unlock();
  pred.unlock();
 }}
```

Precursor and current node

Fine Grained Locking

```
public boolean remove(T item) {
 int key = item.hashCode();
 Node pred, curr;
 try {
   …
 } finally {
  curr.unlock();
  pred.unlock();
 }}
```

Make sure locks are freed

Fine Grained Locking

```
public boolean remove(T item) {
 int key = item.hashCode();
 Node pred, curr;
 try {
   …
 } finally {
  curr.unlock();
  pred.unlock();
 }}
```

Everything else

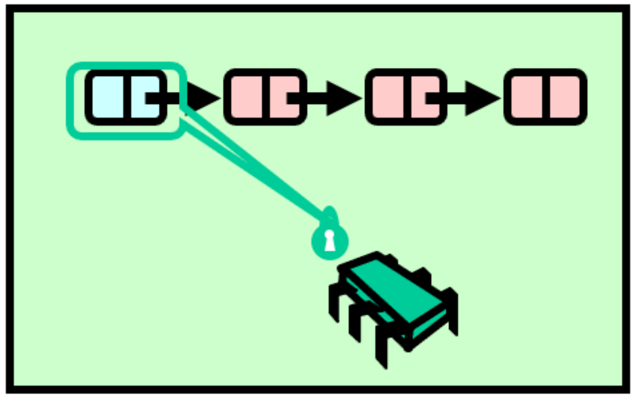
Fine Grained Locking

- Remove

```
try {
 pred = head;
 pred.lock();
 curr = pred.next;
 curr.lock();
 …
} finally { … }
```

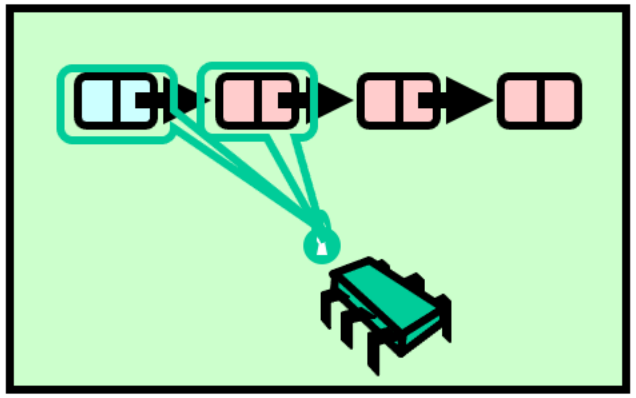
Fine Grained Locking

```
try {
 pred = head;
 pred.lock();
 curr = pred.next;
 curr.lock();
 …
} finally { … }
```



Fine Grained Locking

```
try {
 pred = head;
 pred.lock();
 curr = pred.next;
 curr.lock();
 …
} finally { … }
```



Fine Grained Locking

```
while (curr.key <= key) {
  if (item == curr.item) {
   pred.next = curr.next;
   return true;
  }
  pred.unlock();
  pred = curr;
  curr = curr.next;
  curr.lock();
 }
 return false;
```

Searching

Fine Grained Locking

```
while (curr.key <= key) {
  if (item == curr.item) {
   pred.next = curr.next;
   return true;
  }
  pred.unlock();
  pred = curr;
  curr = curr.next;
  curr.lock();
 }
 return false;
```

Loop Invariant: At start of while, pred and curr are locked

Fine Grained Locking

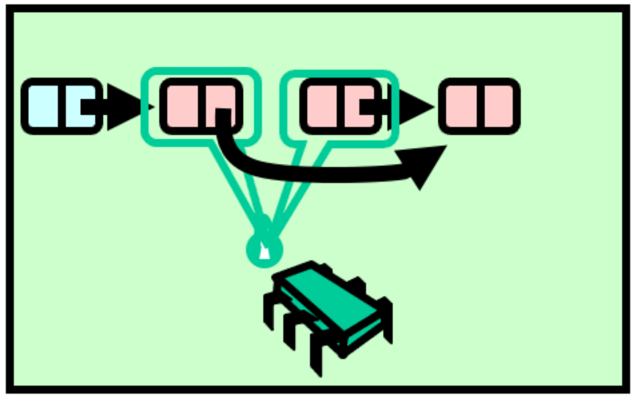
```
while (curr.key <= key) {
  if (item == curr.item) {
   pred.next = curr.next;
   return true;
  }
  pred.unlock();
  pred = curr;
  curr = curr.next;
  curr.lock();
 }
 return false;
```

Loop Invariant:
At start of while,
pred and curr are
locked

Fine Grained Locking

```
while (curr.key <= key) {
  if (item == curr.item) {
   pred.next = curr.next;
   return true;
  }
  pred.unlock();
  pred = curr;
  curr = curr.next;
  curr.lock();
 }
 return false;
```

If item found, delete node



Fine Grained Locking

```
while (curr.key <= key) {
  if (item == curr.item) {
   pred.next = curr.next;
   return true;
  }
  pred.unlock();
  pred = curr;
  curr = curr.next;
  curr.lock();
 }
 return false;
```

Unlock predecessor

Fine Grained Locking

```
while (curr.key <= key) {
  if (item == curr.item) {
   pred.next = curr.next;
   return true;
  }
  pred.unlock();
  pred = curr;
  curr = curr.next;
  curr.lock();
 }
 return false;
```

Move right

Fine Grained Locking

```
while (curr.key <= key) {
  if (item == curr.item) {
   pred.next = curr.next;
   return true;
  }
  pred.unlock();
  pred = curr;
  curr = curr.next;
  curr.lock();
 }
 return false;
```

Acquire next node

Fine Grained Locking

```
while (curr.key <= key) {
  if (item == curr.item) {
   pred.next = curr.next;
   return true;
  }
  pred.unlock();
  pred = curr;
  curr = curr.next;
  curr.lock();
 }
 return false;
```

Acquire next node

Fine Grained Locking

```
while (curr.key <= key) {
  if (item == curr.item) {
   pred.next = curr.next;
   return true;
  }
  pred.unlock();
  pred = curr;
  curr = curr.next;
  curr.lock();
 }
 return false;
```

Lock next node

Fine Grained Locking

```
while (curr.key <= key) {
  if (item == curr.item) {
   pred.next = curr.next;
   return true;
  }
  pred.unlock();
  pred = curr;
  curr = curr.next;
  curr.lock();
 }
 return false;
```

Loop invariant restored

Fine Grained Locking

```
while (curr.key <= key) {
  if (item == curr.item) {
   pred.next = curr.next;
   return true;
  }
  pred.unlock();
  pred = curr;
  curr = curr.next;
  curr.lock();
 }
 return false;
```

Otherwise, return false

Fine Grained Locking

- Execution history is **Linearizable**:
  - Equivalent to a sequential history
- To argue something is linearizable:
  - Can find "linearization points"

Fine Grained Locking

- Invariants:
  - All items in the set are in nodes reachable from head
  - All nodes are arranged in order
- We show that invariants are maintained by methods

Fine Grained Locking

- Why remove is linearizable
  - Case 1: Item is in the list

```
while (curr.key <= key) {
  if (item == curr.item) {
   pred.next = curr.next;
   return true;
  }
  pred.unlock();
  pred = curr;
  curr = curr.next;
  curr.lock();
 }
 return false;
```

Fine Grained Locking

- Why remove is linearizable
  - Case 1: Item is in the list
    - Then pred.next = curr.next is a linearization point
- Invariants:
  - pred is reachable from head
  - curr is pred.next
  - curr is in the set
- No other thread can access either pred or curr during assignment

```
while (curr.key <= key) {
  if (item == curr.item) {
   pred.next = curr.next;
   return true;
  }
  pred.unlock();
  pred = curr;
  curr = curr.next;
  curr.lock();
 }
return false;
```

Fine Grained Locking

- Why remove is linearizable
  - After removal:
    - curr is no longer reachable: item is removed
    - pred is reachable from head
    - old curr.next is reachable
    - for all other nodes, reachability has not changed

```
while (curr.key <= key) {
  if (item == curr.item) {
   pred.next = curr.next;
   return true;
  }
  pred.unlock();
  pred = curr;
  curr = curr.next;
  curr.lock();
 }
return false;
```

Fine Grained Locking

- Why remove is linearizable
  - Case 2: Item is not in the list
    -

```
while (curr.key <= key) {
  if (item == curr.item) {
   pred.next = curr.next;
   return true;
  }
  pred.unlock();
  pred = curr;
  curr = curr.next;
  curr.lock();
 }
 return false;
```

Fine Grained Locking

- Why remove is linearizable
  - Case 2: Item is not in the list
    - return false is linearization point

```
while (curr.key <= key) {
  if (item == curr.item) {
   pred.next = curr.next;
   return true;
  }
  pred.unlock();
  pred = curr;
  curr = curr.next;
  curr.lock();
 }
 return false;
```

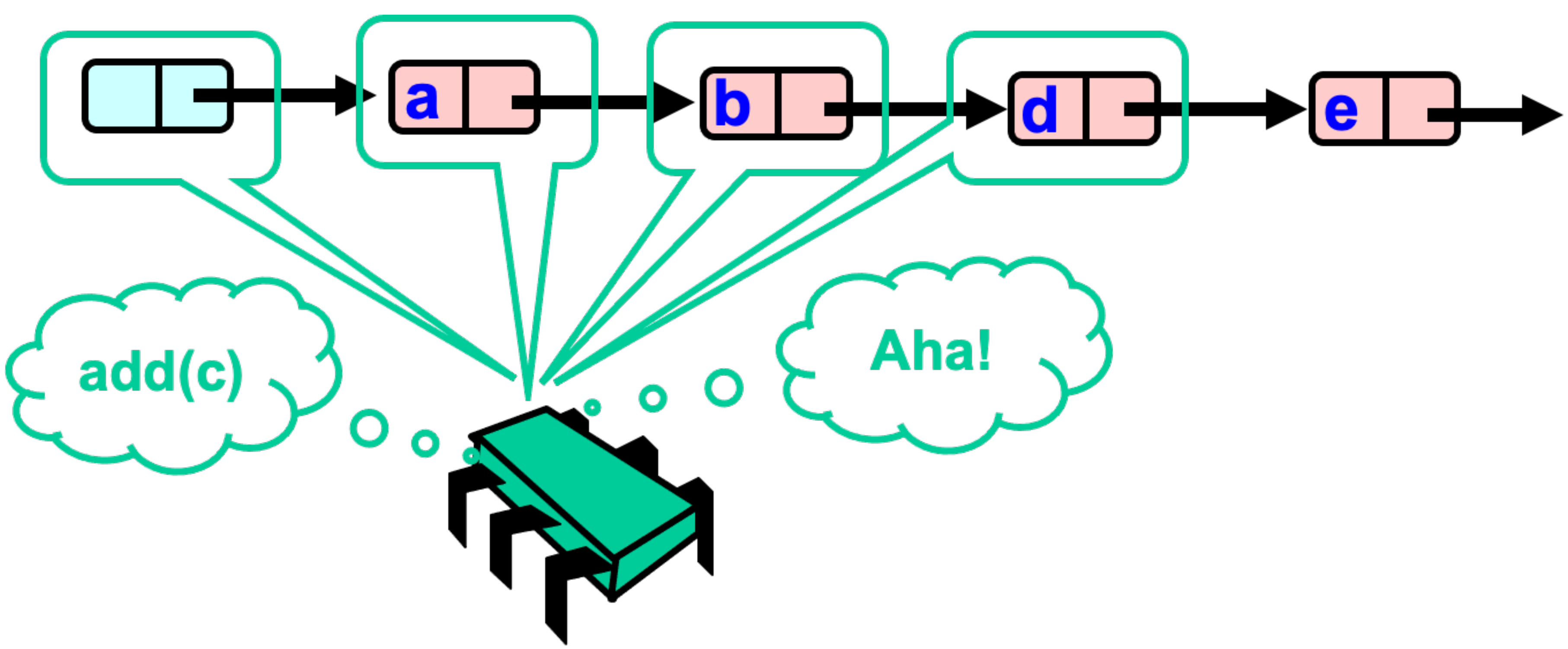
Fine Grained Locking

- Why remove is linearizable
  - Invariants are not changed
  - Need to show correctness:
    - Use induction to argue that item is not in the set

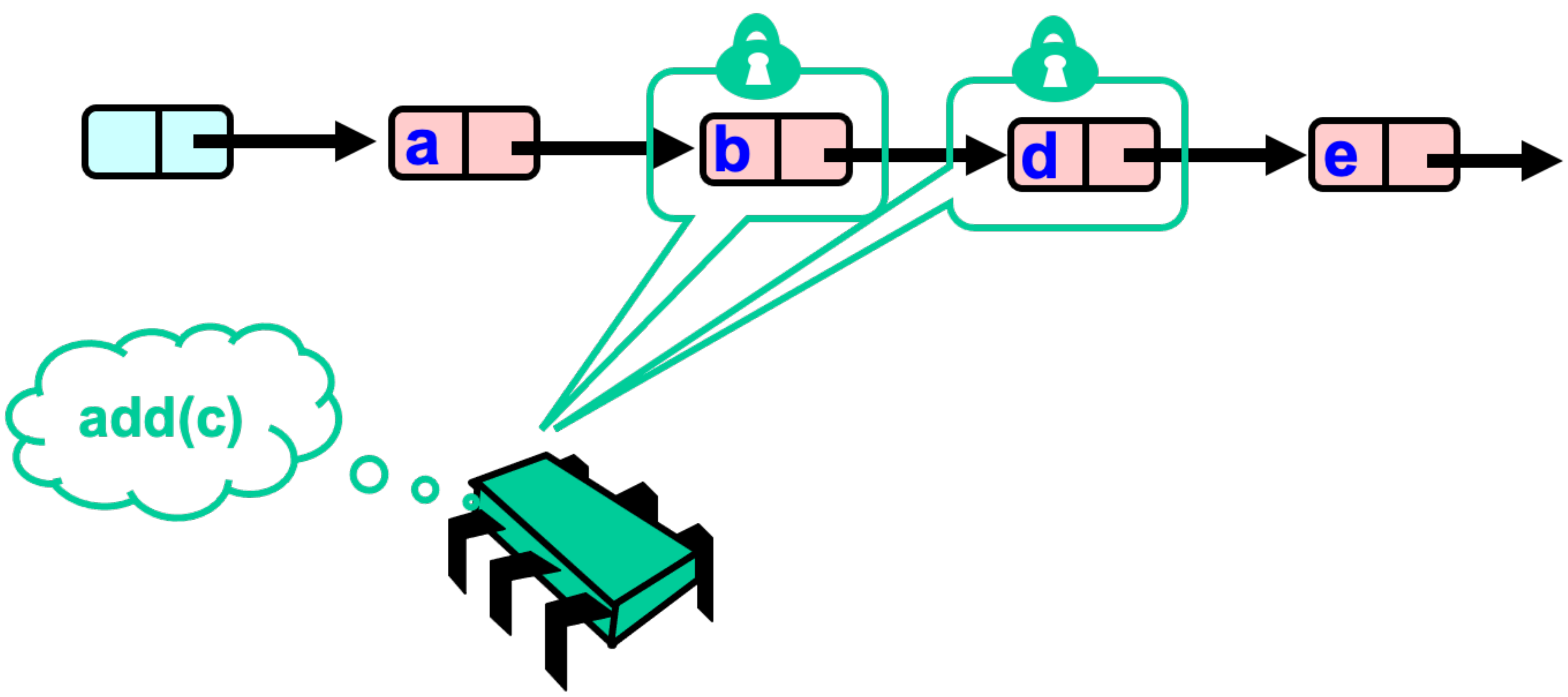
```
while (curr.key <= key) {
  if (item == curr.item) {
   pred.next = curr.next;
   return true;
  }
  pred.unlock();
  pred = curr;
  curr = curr.next;
  curr.lock();
 }
 return false;
```

Optimistic Locking

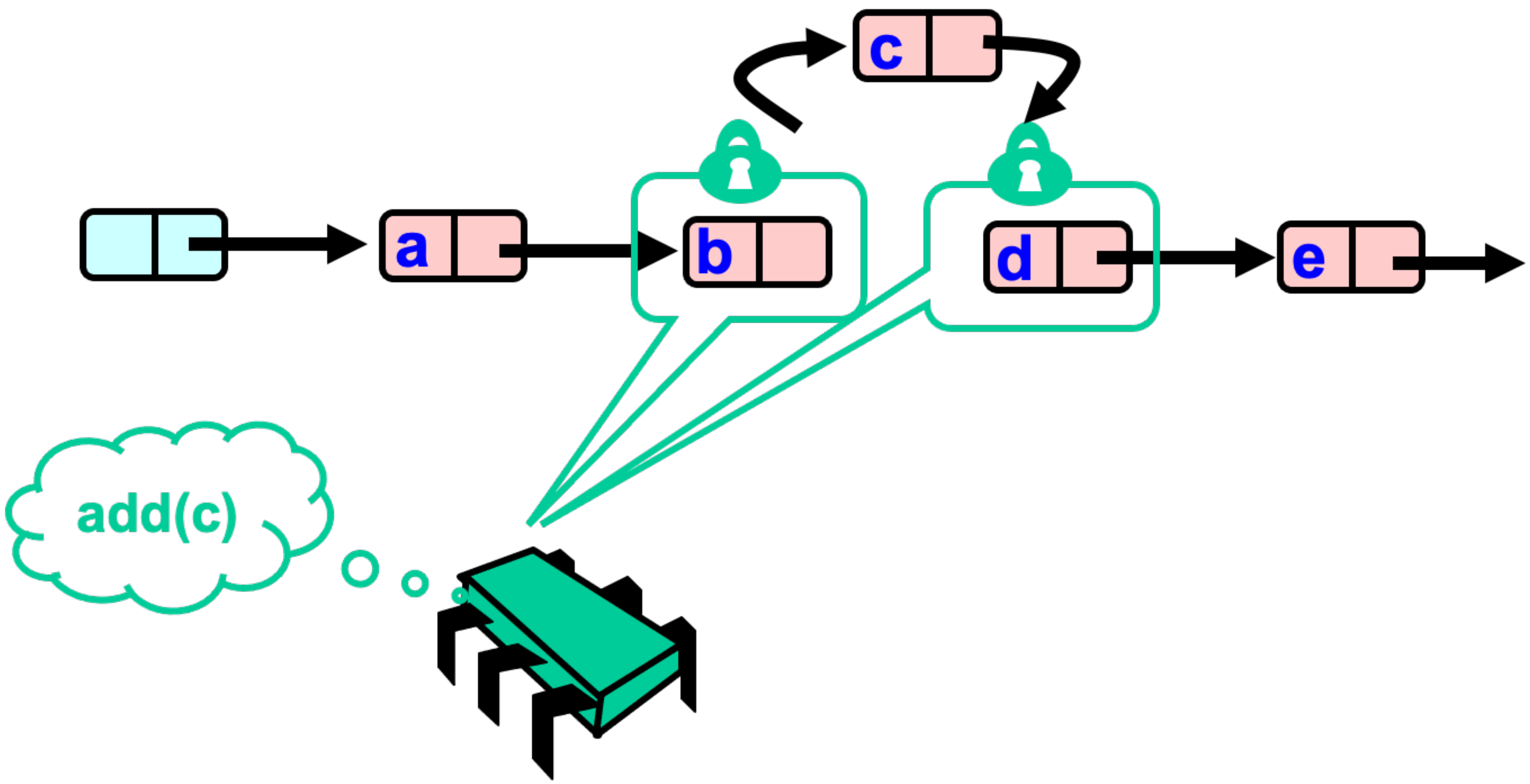
- Only lock when you are ready
  - Traverse list to find insertion / removal point
  - Then lock needed nodes after validation!



Optimistic Locking

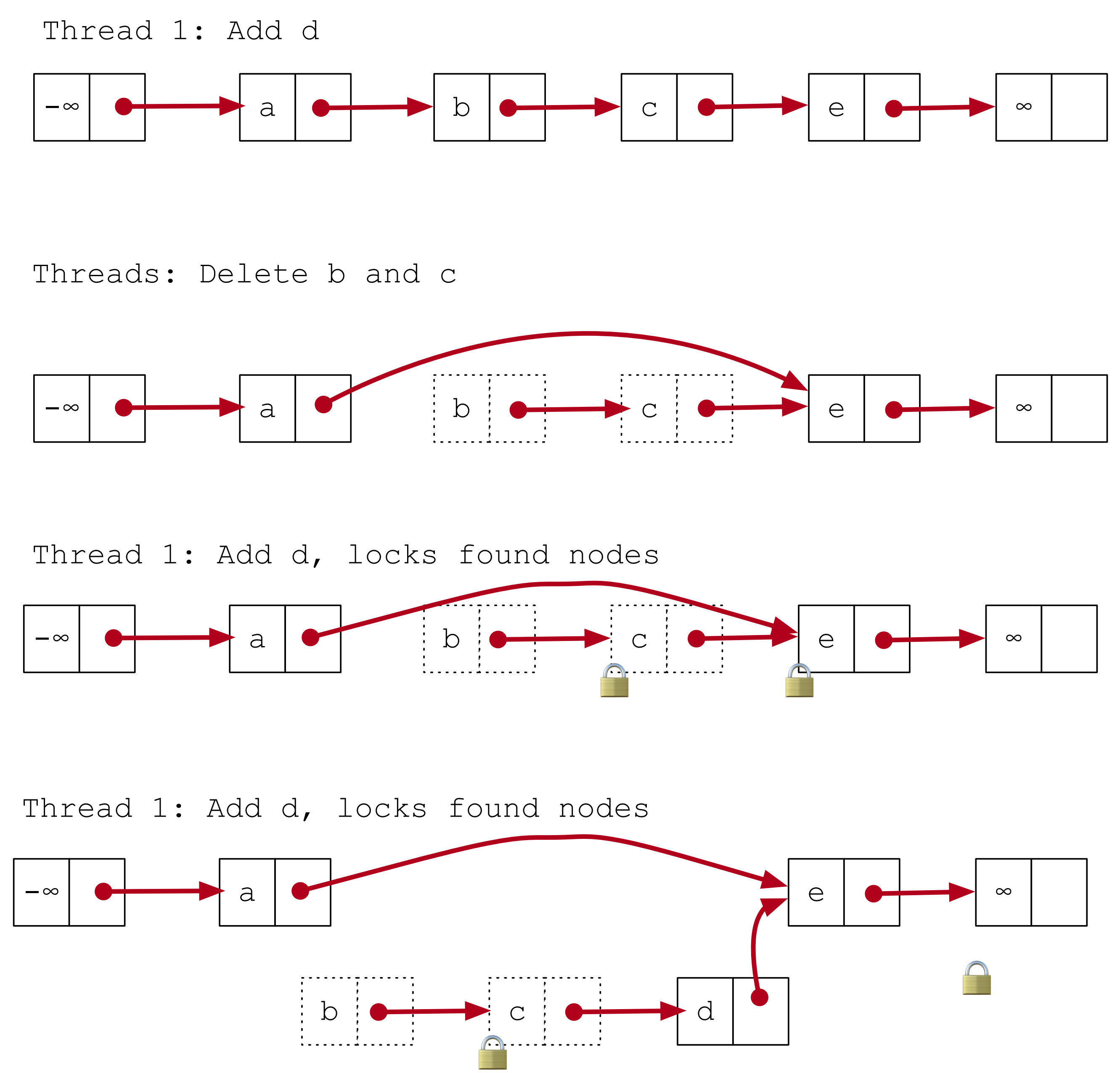


Optimistic Locking



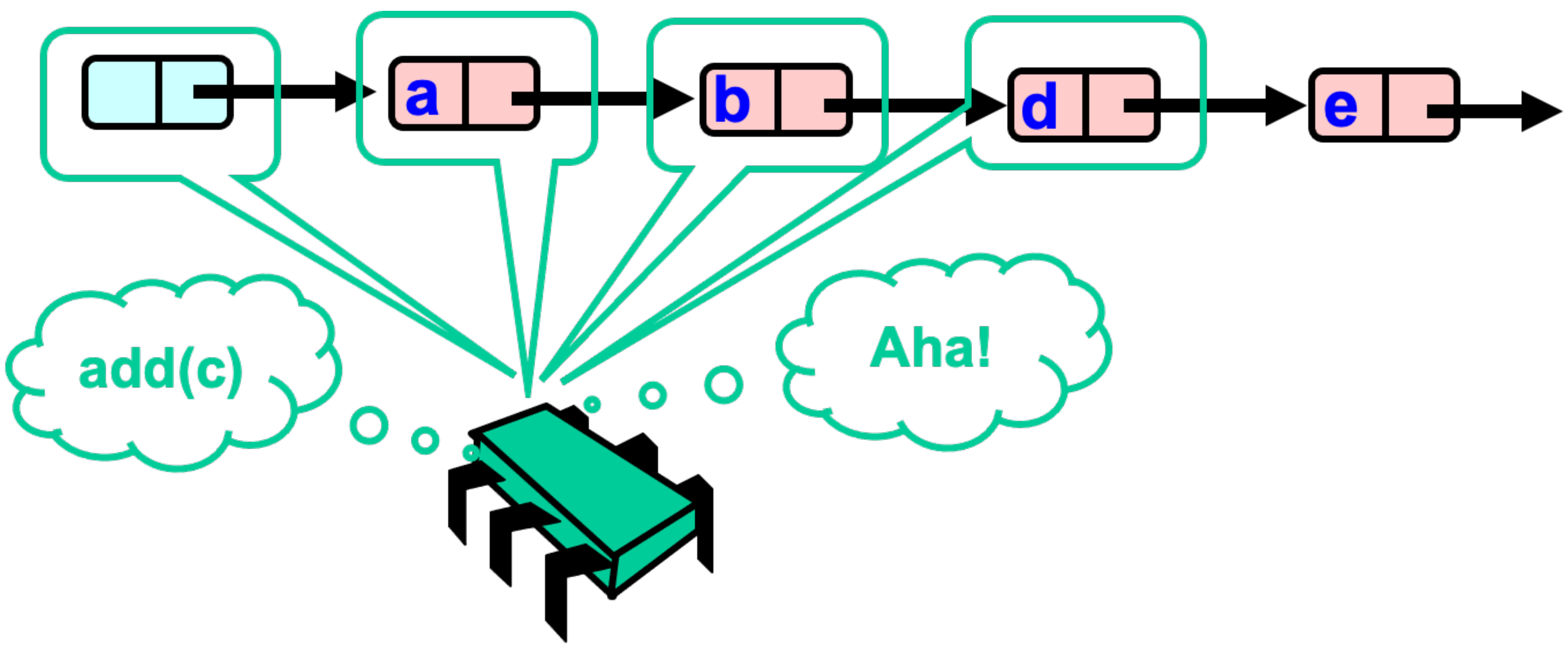
Optimistic Locking

- Why we need validation

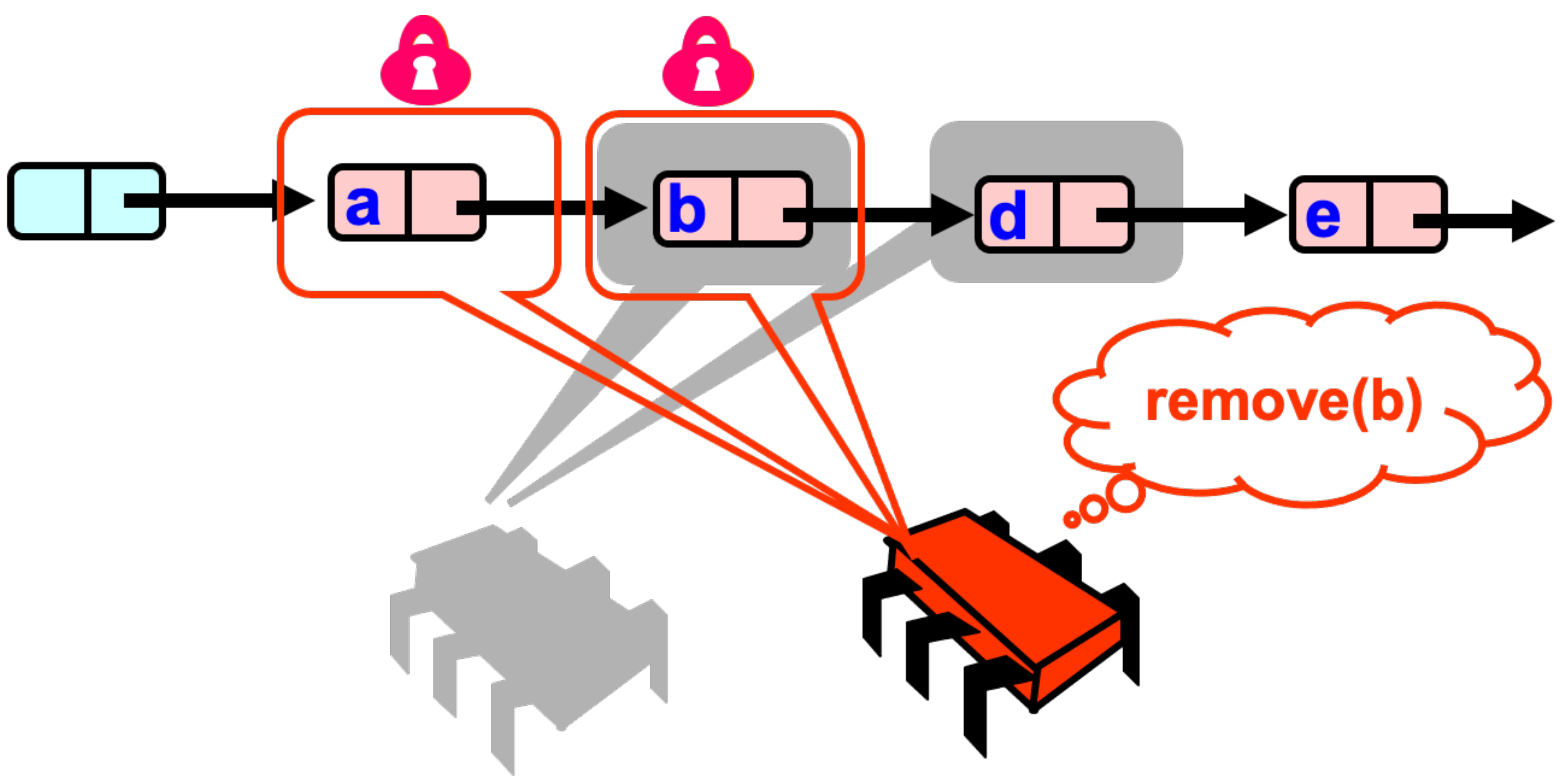


Optimistic Locking

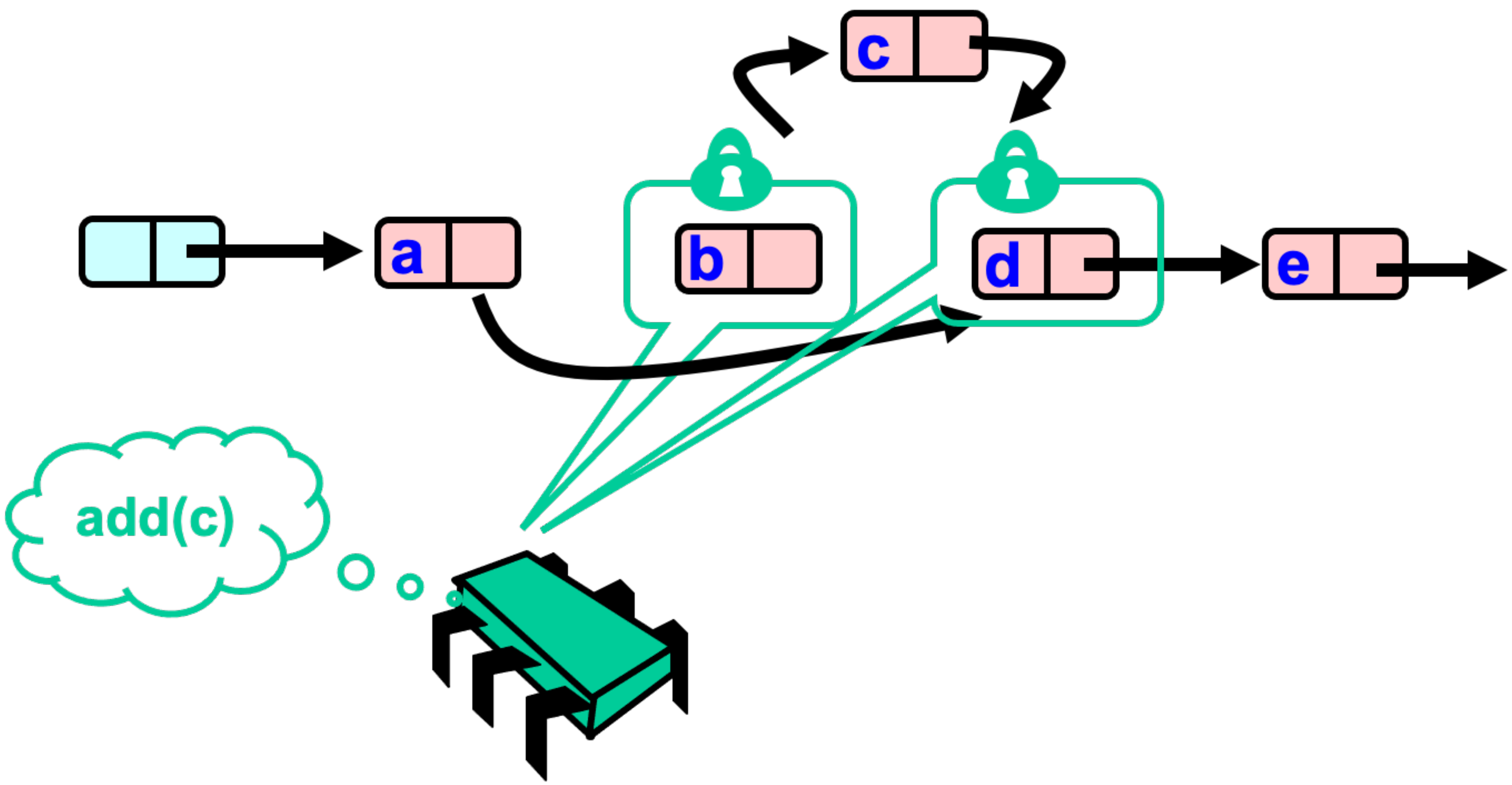
- What can go wrong?
  - Nodes might no longer be there



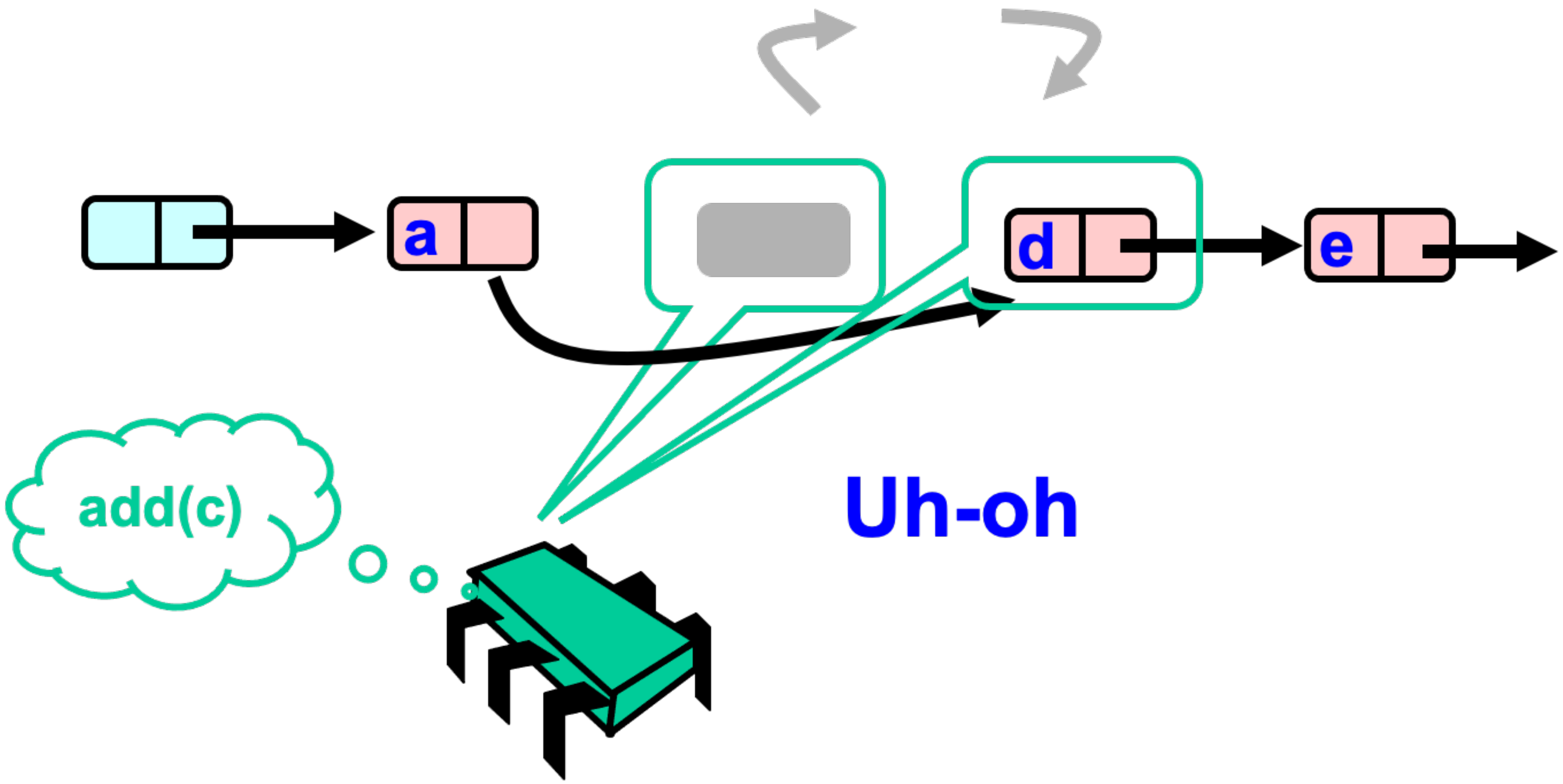
Optimistic Locking



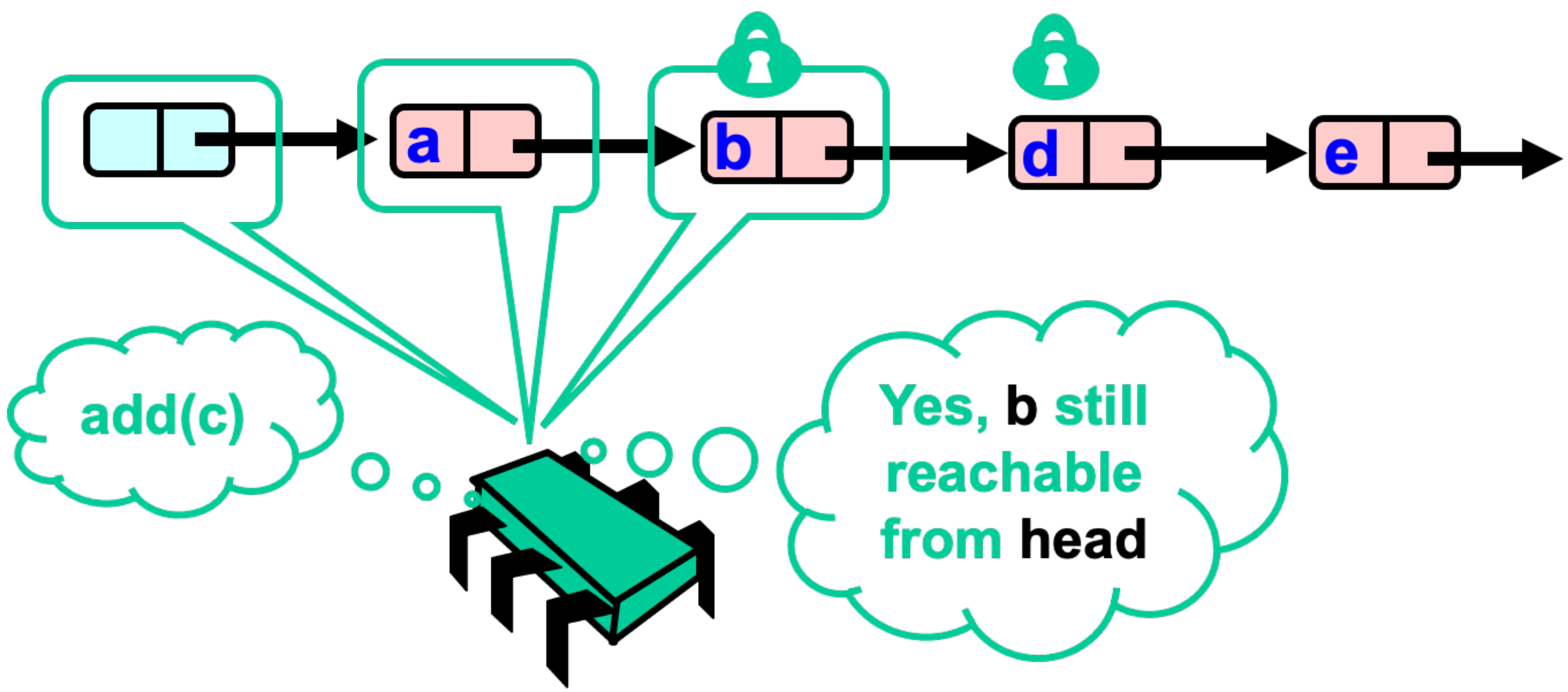
Optimistic Locking



Optimistic Locking



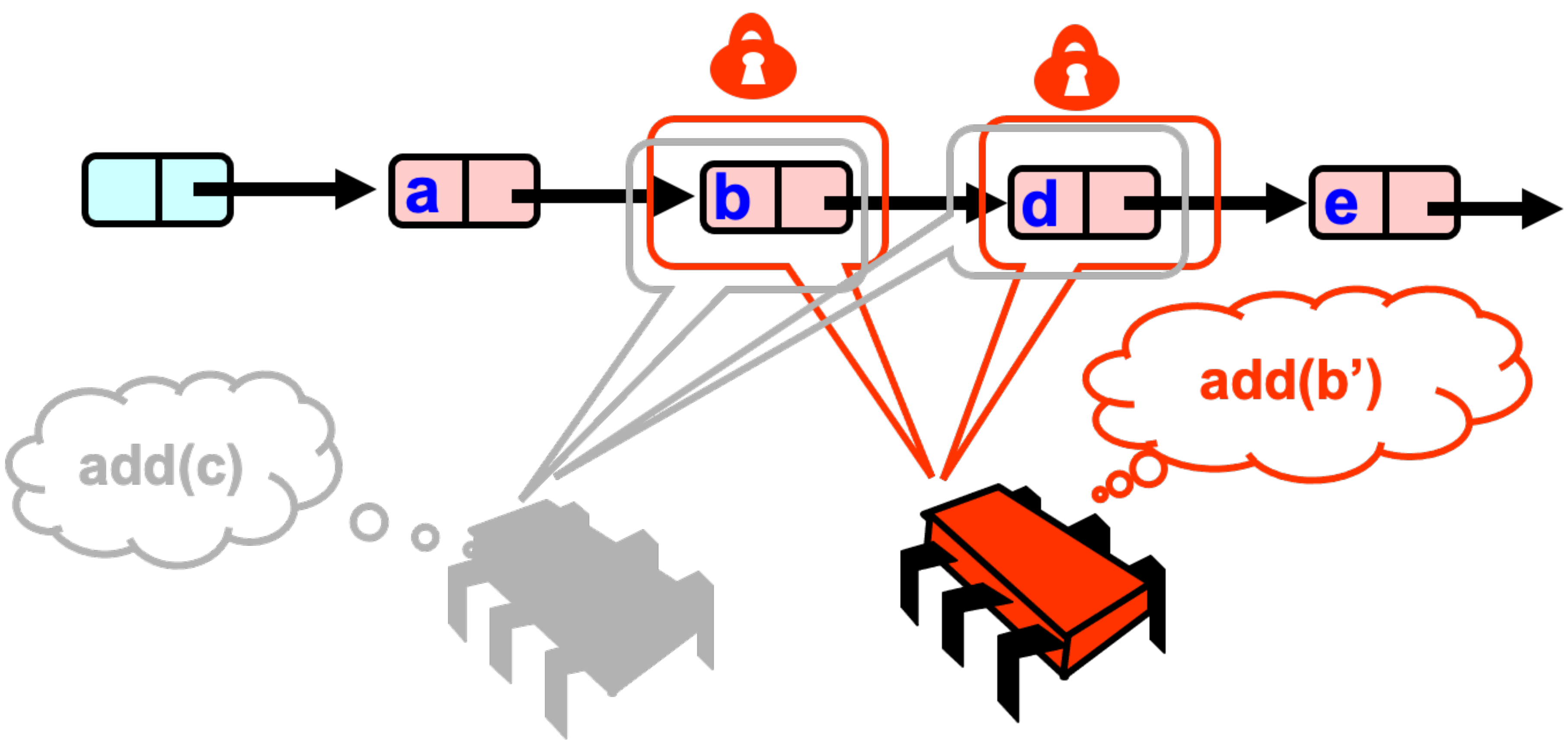
Optimistic Locking



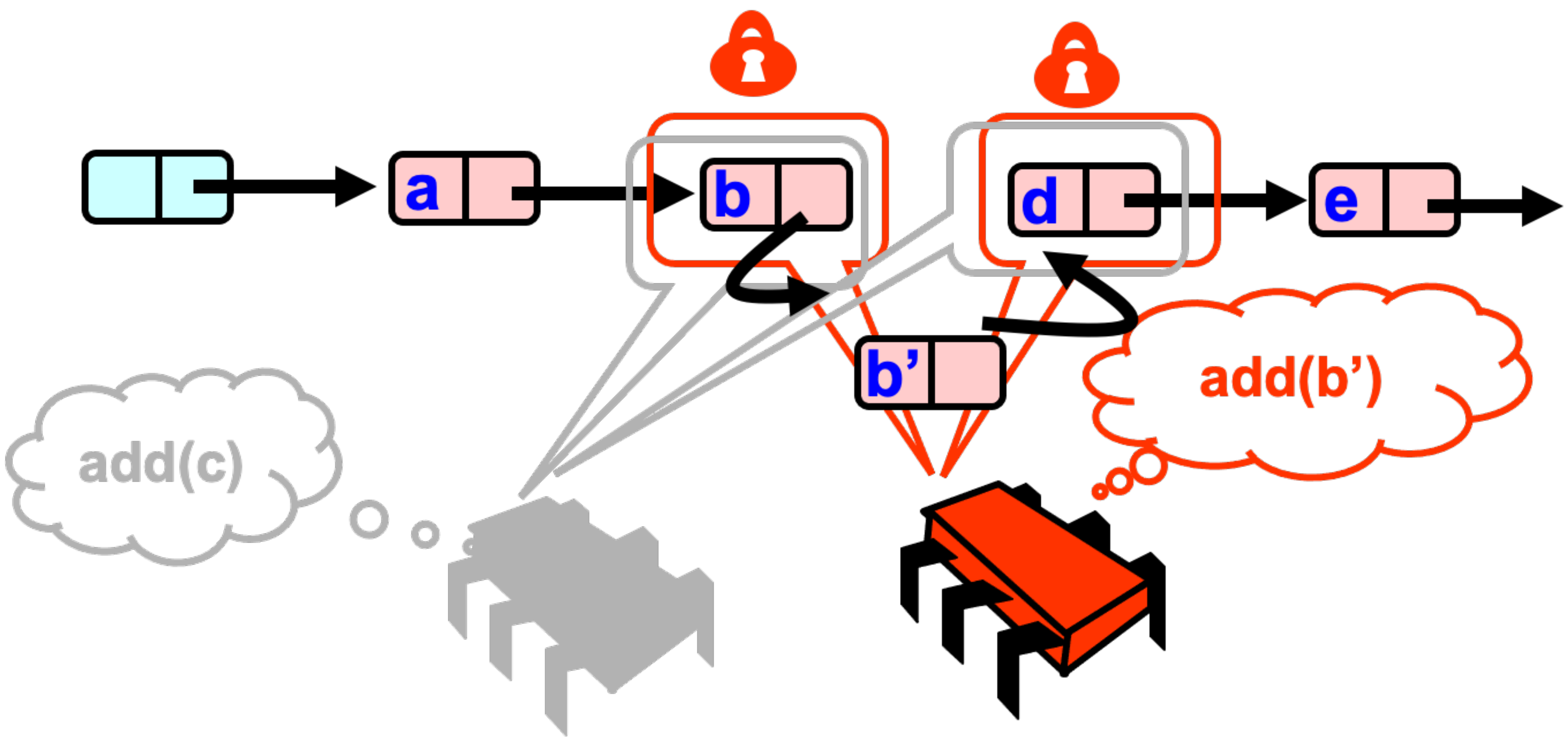
Need to validate

Optimistic Locking

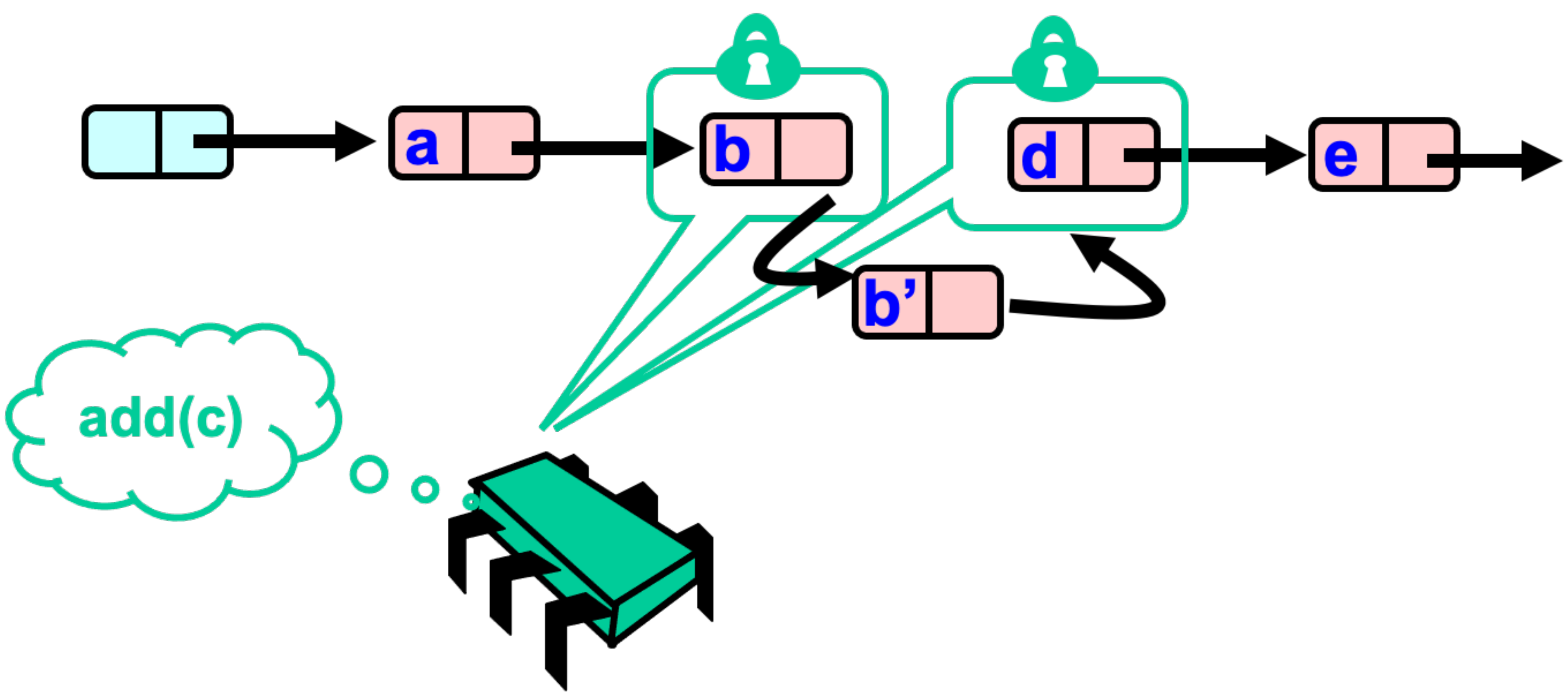
- What else can go wrong?



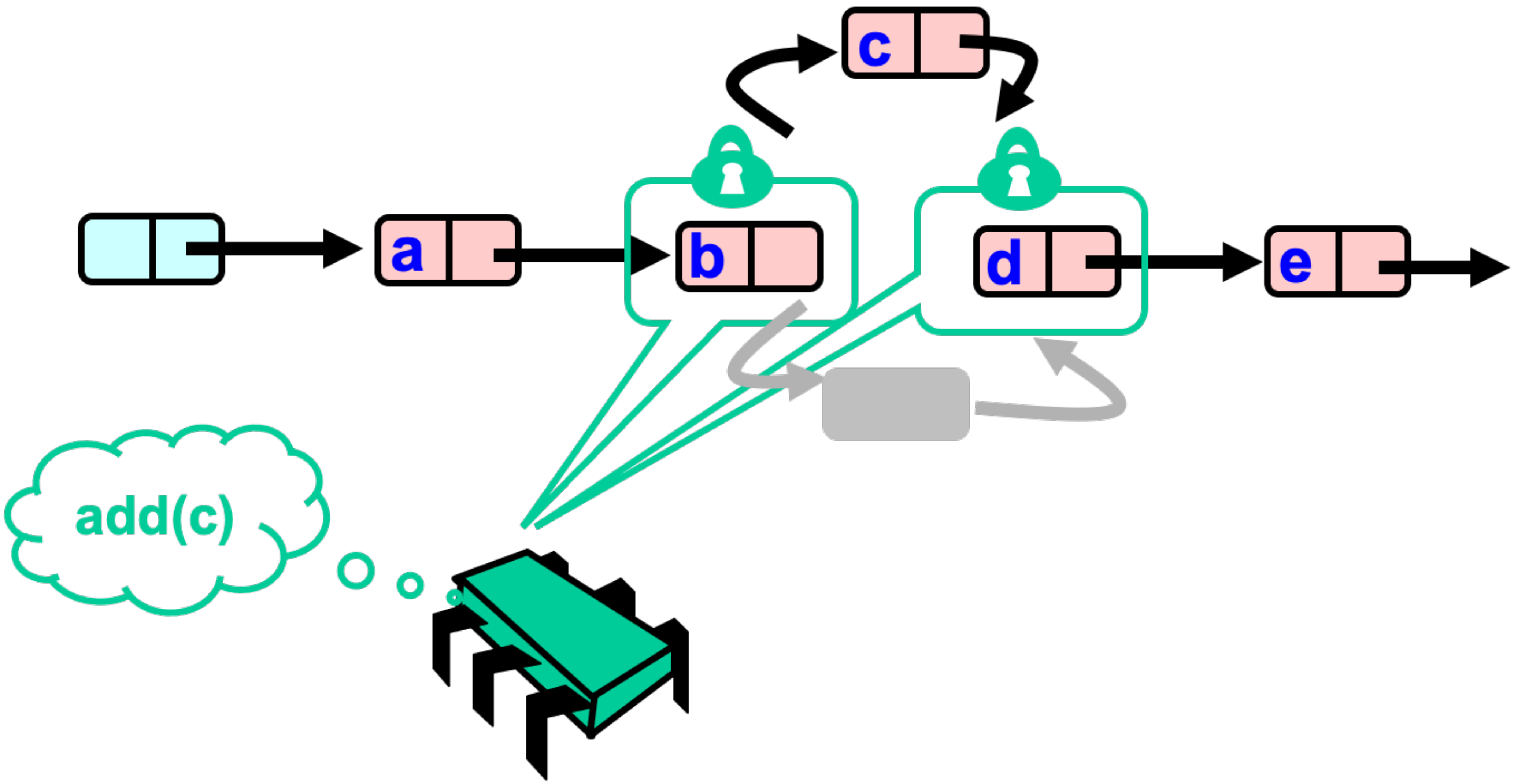
Optimistic Locking



Optimistic Locking

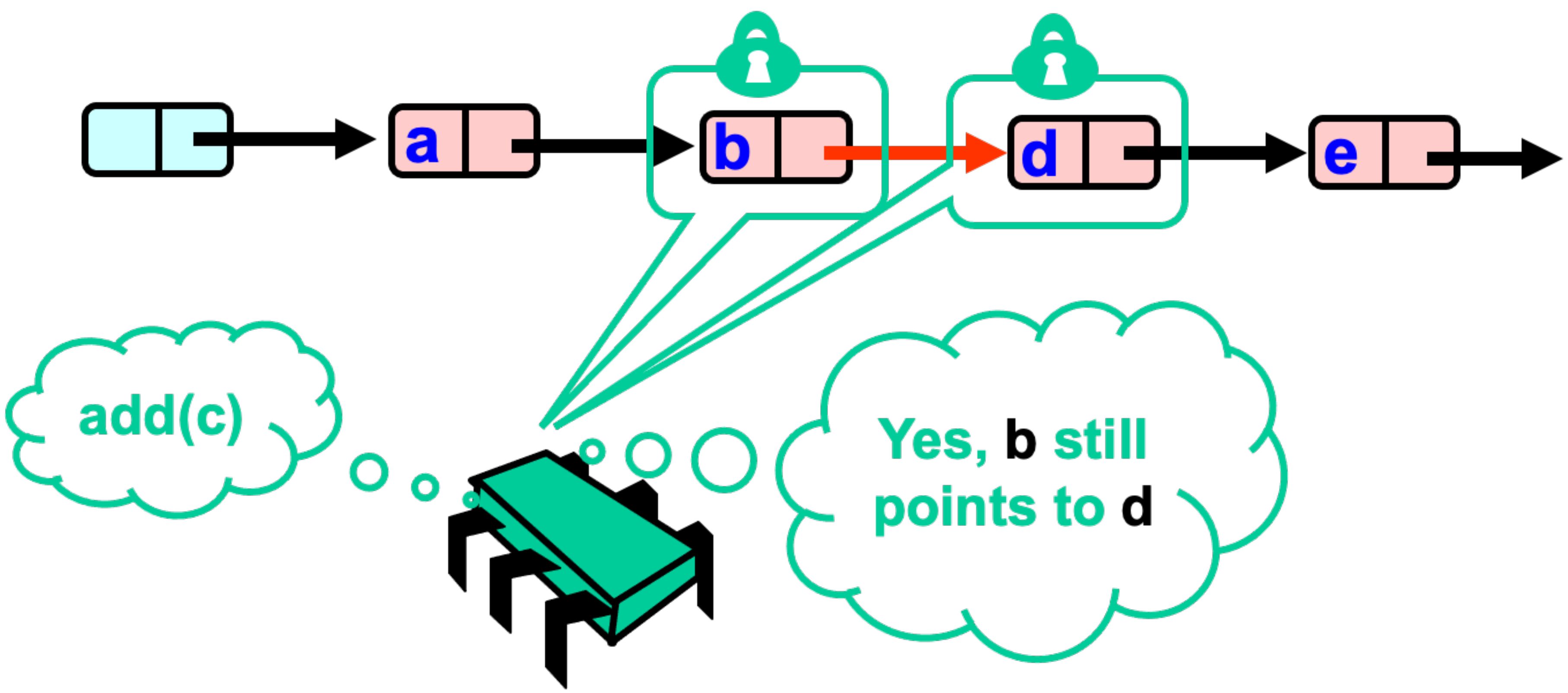


Optimistic Locking



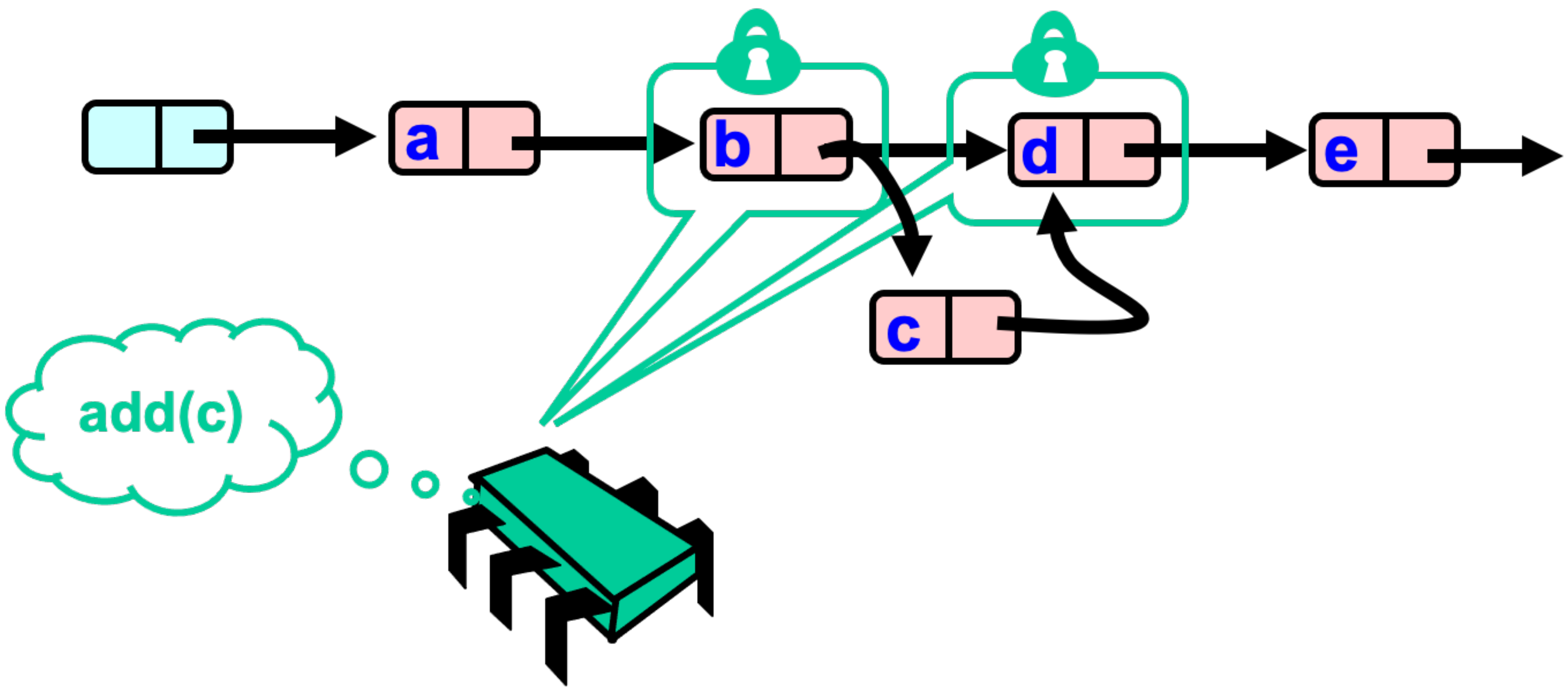
Optimistic Locking

- Need to validate **while holding locks**



Optimistic Locking

- Need to validate **while holding locks**
- Linearization point



Optimistic Locking

- Optimistic locking:
  - Search without acquiring locks
  - Lock the nodes found
  - Confirm that locked nodes are correct
    - For inserting a node between Node A and Node B:
      - Node A is reachable from head
      - Node B is still the successor of Node A

Optimistic Locking

- Validation:
  - Reachability of Node A
    - No operation changes reachability with exception of the Node being removed
      - Verify that!
    - Therefore: we do not need locks to verify reachability

Optimistic Locking

```
private boolean
 validate(Node pred,
          Node curry) {
 Node node = head;
 while (node.key <= pred.key) {
  if (node == pred)
   return pred.next == curr;
  node = node.next;
 }
 return false;
}
```

Optimistic Locking

- Addition: Phase 1: searching

```
public boolean add(T item) {
    int key = item.hashCode();
    while (true) {
        Node pred = head;
        Node curr = pred.next;
        while (curr.key <= key) {
            pred = curr; curr = curr.next;
```

Optimistic Locking

- Addition: Phase 2: Locking

```
pred.lock();
curr.lock();
```

Optimistic Locking

- Addition: Phase 3: Validation and Update

```
try {
   if (validate(pred, curr)) {
      if (curr.key == key) {
          return false;
      } else {
            Node node = new Node(item);
            node.next = curr;
            pred.next = node;
            return true;
      }
   }
} finally {
     pred.unlock();
     curr.unlock();
}
```

Optimistic Locking

- Remove

```
public boolean remove(T item) {
   int key = item.hashCode();
    while( true ){
        Node pred = head;
        Node curr = pred.next;
        while (curr.key < key)  {
            pred = curr;
            curr = curr.next;
        }
```

Optimistic Locking

- Remove: Lock phase

```
pred.lock();
curr.lock();
```

Optimistic Locking

- Remove: Validation and deletion phase

```
try {
   if (validate(pred, curr)) {
      if (curr.key == key) {
         pred.next = curr.next;
         return true;
      } else {
         return false;
   }
}
} finally {
           pred.unlock(); curr.unlock();
         }
}}
```

Optimistic Locking

- On exit from loop **and in the absence of synchronization problems:**
  - If item is present:
    - curr holds item
    - pred just before curr
  - If item is absent:
    - curr has higher key
    - pred just before curr

```
public boolean remove(T item) {
    int key = item.hashCode();
     while( true ){
         Node pred = head;
         Node curr = pred.next;
         while (curr.key < key)  {
             pred = curr;
             curr = curr.next;
         }
```

Optimistic Locking

- Remove: Validation and deletion phase

```
try {
    if (validate(pred, curr)) {
        if (curr.key == key) {
            pred.next = curr.next;
            return true;
        } else {
            return false;
    }
}
} finally {
            pred.unlock(); curr.unlock();
        }
}}
```

Check for synchronization problems

Optimistic Locking

- Limited hot-spots:
  - Targets of add, remove, contains
- No contention on traversals
- Traversals are wait-free

Lazy Locking

- Optimistic locking:
  - Traverses list twice
  - Contains locks
- Lazy locking:
  - Make validation simpler
    - By marking deleted nodes

Lazy Locking

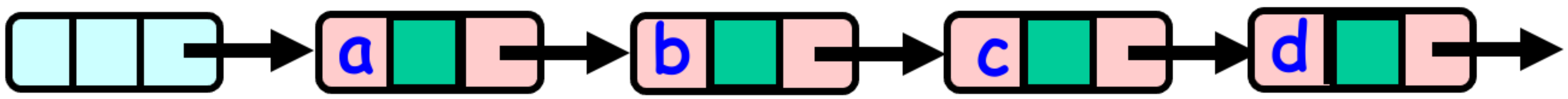
- Add to each node a Boolean `marked` field
- Traversals no longer need to validate that a node is reachable:
  - New invariant:
    - Every unmarked node is reachable

Lazy Locking

- Contains:
  - Just traverse the list, including nodes marked deleted
  - If the item is in the list and the node is not marked deleted, then it is in the set

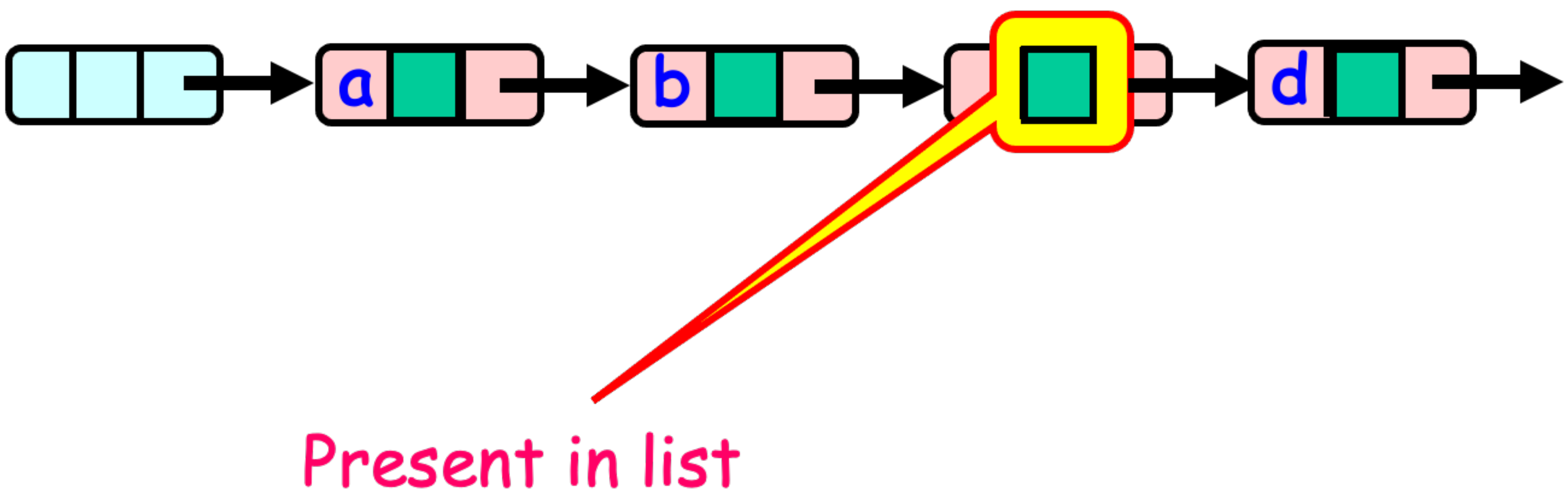
Lazy Locking

- Lazy removal



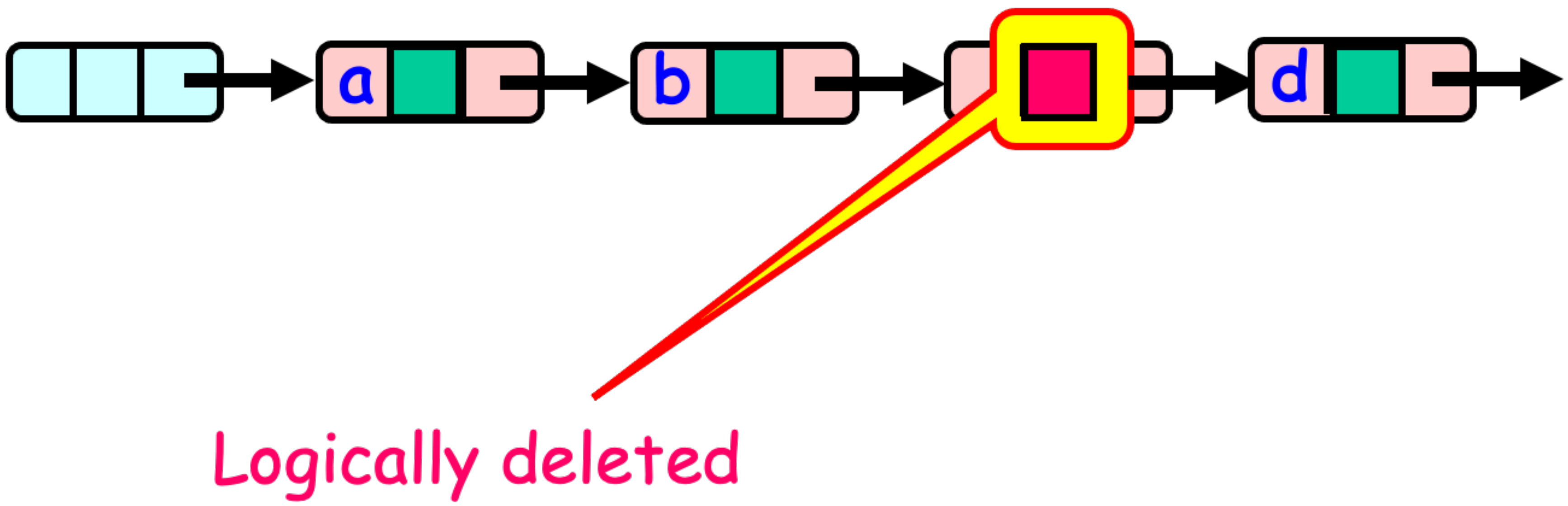
Lazy Locking

- Lazy removal

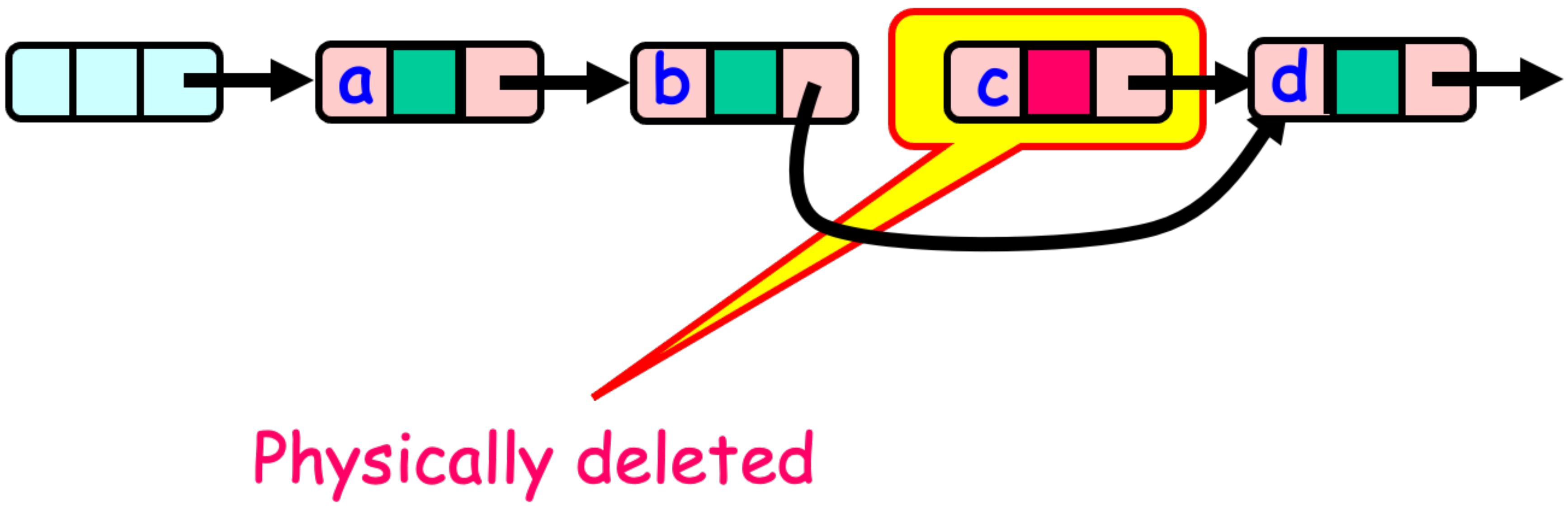


Lazy Locking

- Lazy removal

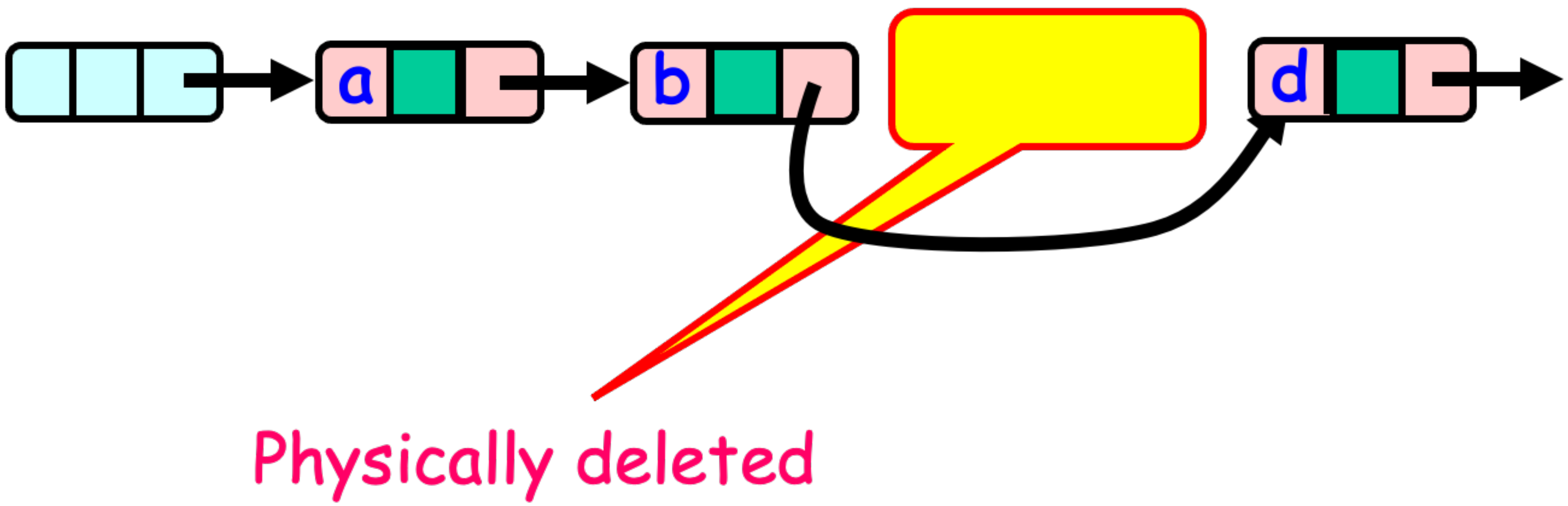


Lazy Locking



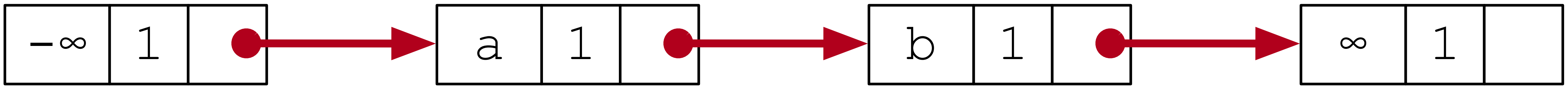
Lazy Locking

- Lazy removal



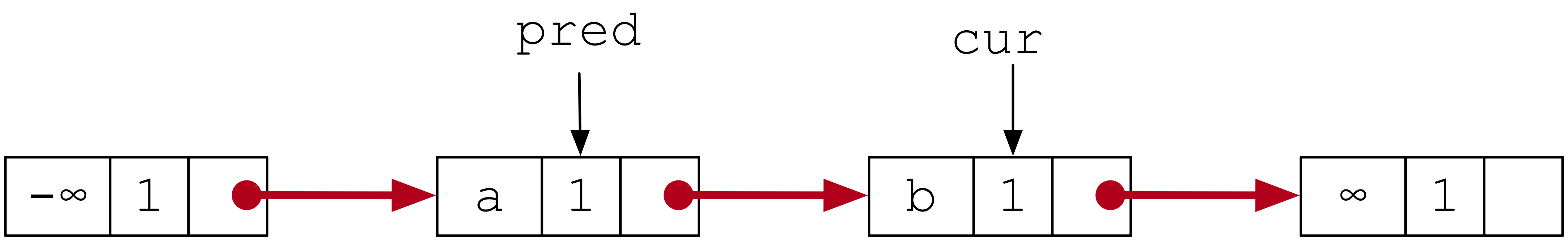
Lazy Locking

- Why do we need to validate?
  - Thread I removes b



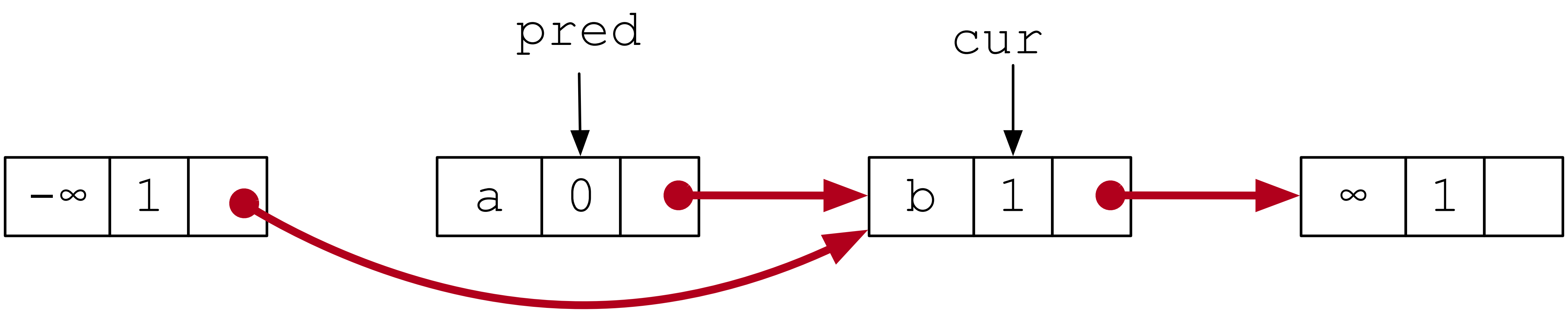
Lazy Locking

- Thread 1 finds b



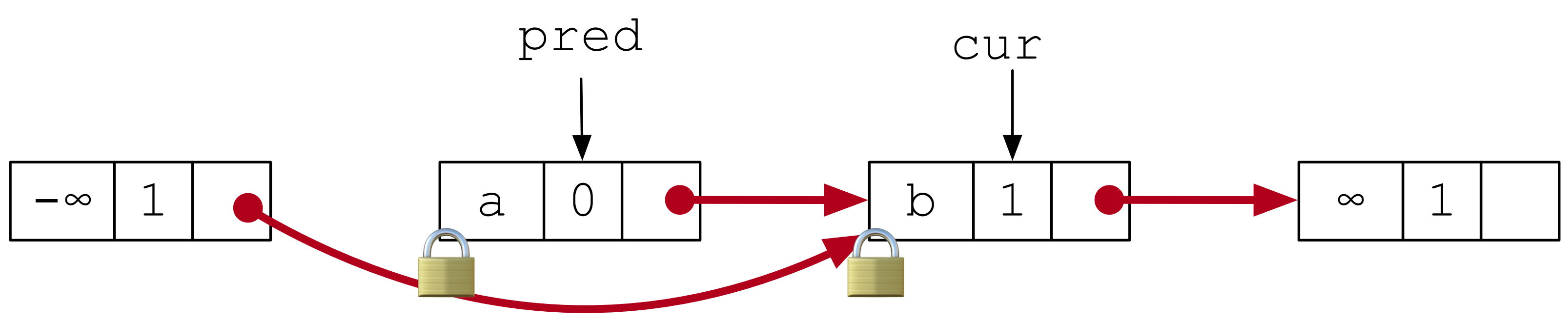
Lazy Locking

- Before Thread 1 acquires the lock, another thread logically and physically removes the predecessor



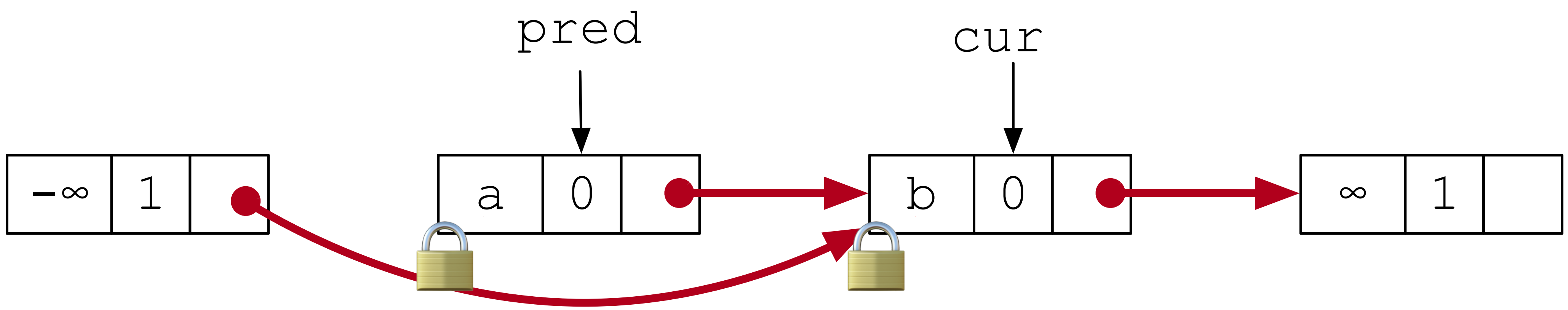
Lazy Locking

- Thread 1 now acquires the lock



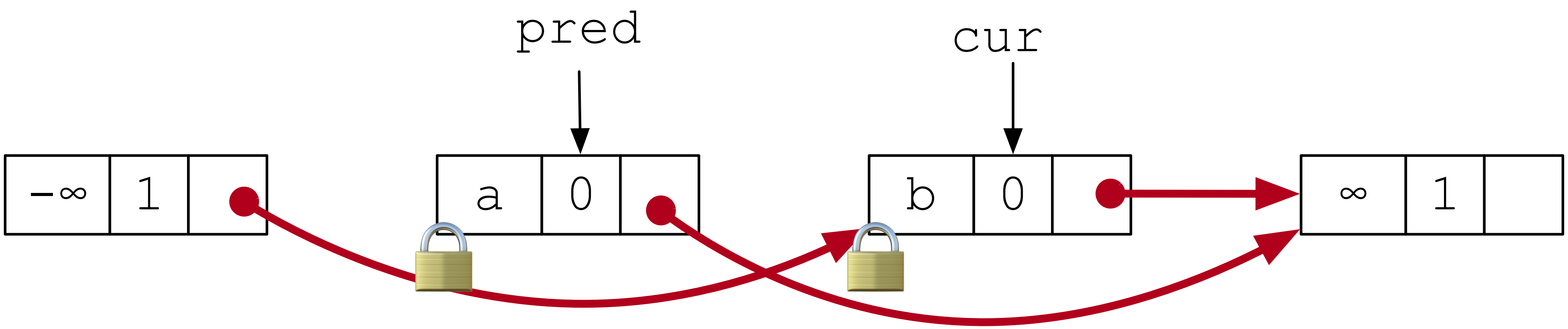
Lazy Locking

- Thread I marks b as deleted



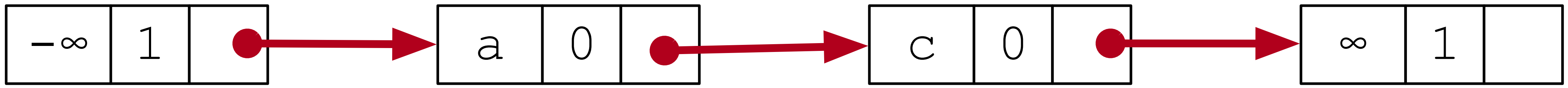
Lazy Locking

- And then removes it physically



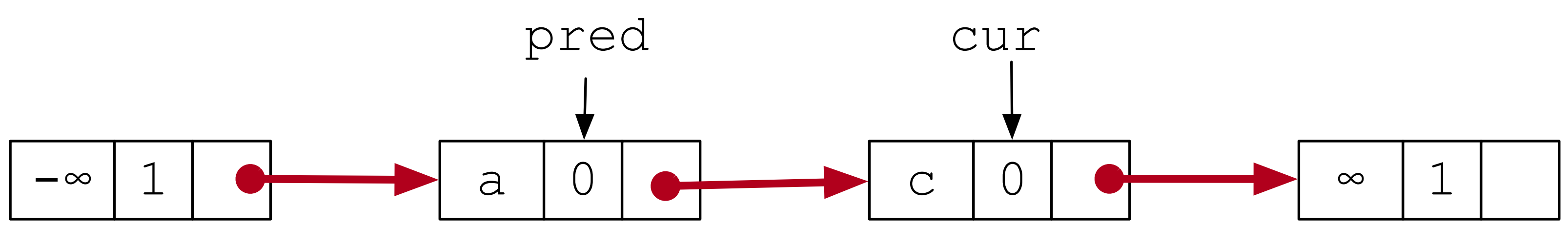
Lazy Locking

- Another scenario:
    - Thread I tries to remove c



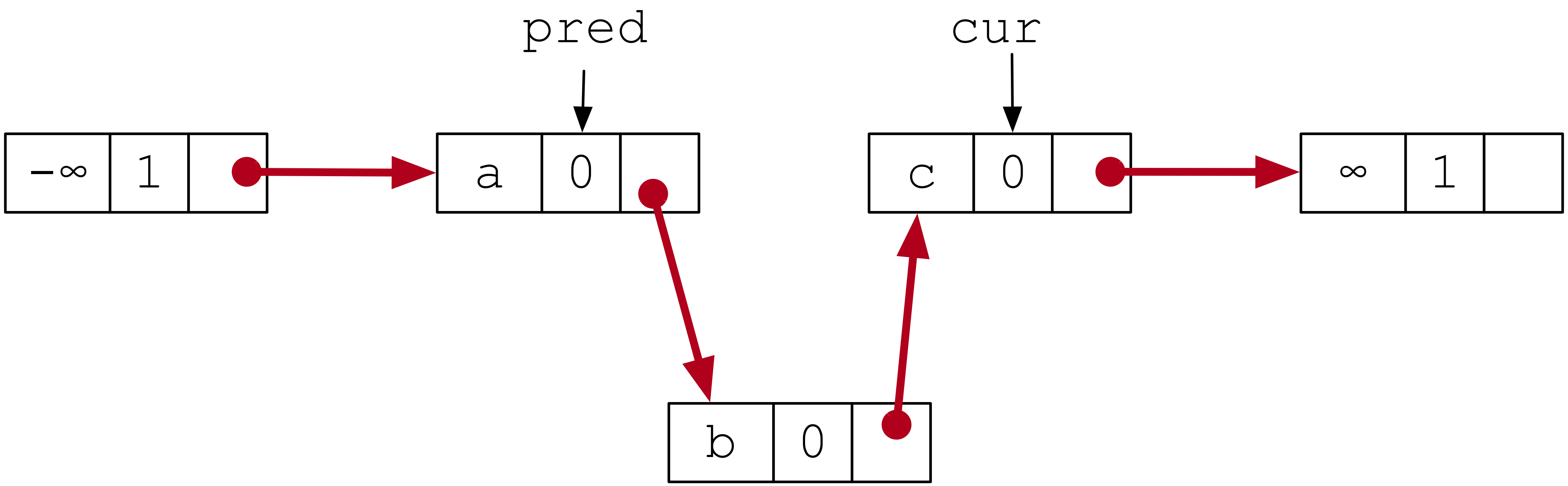
Lazy Locking

- Thread I finds them



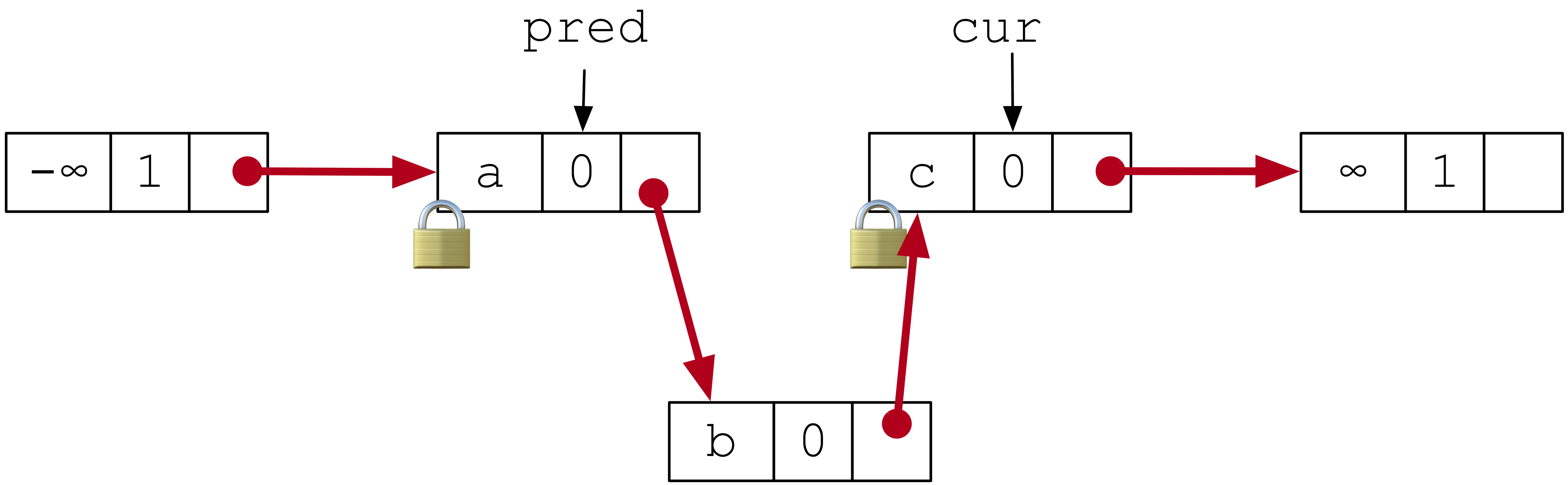
Lazy Locking

- But before locking, another thread adds a node b



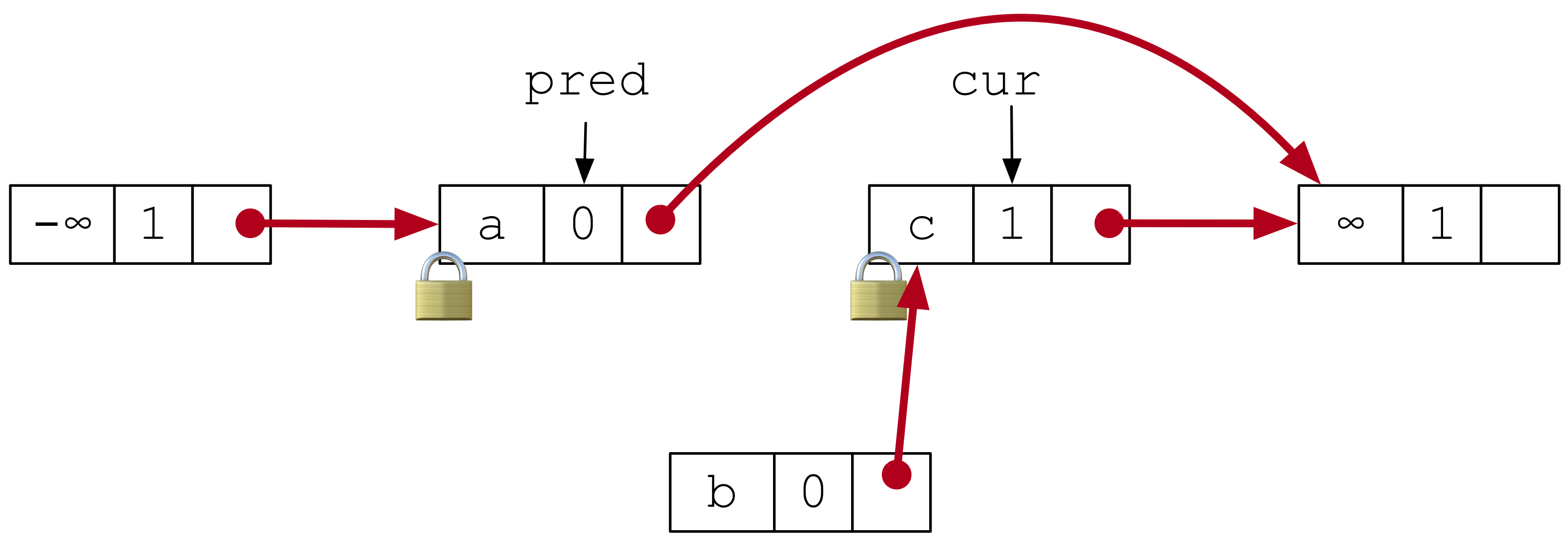
Lazy Locking

- Thread I now locks



Lazy Locking

- And virtually and physically removes node c



Lazy Locking

- Validation:
    - Check that pred is not marked
    - Check that curr is not marked
    - Check that pred.next == curr

Lazy Locking

- Validation

```
private boolean
  validate(Node pred, Node curr) {
 return
    !pred.marked &&
    !curr.marked &&
    pred.next == curr);
  }
```

Lazy Locking

- Validation

```
private boolean
  validate(Node pred, Node curr) {
 return
    !pred.marked &&
    !curr.marked &&
    pred.next == curr);
  }
```

predecessor not logically deleted

Lazy Locking

- Validation

```
private boolean
   validate(Node pred, Node curr) {
  return
     !pred.marked &&
     !curr.marked &&
     pred.next == curr);
   }
```

current node not logically deleted

Lazy Locking

- Validation

```
private boolean
  validate(Node pred, Node curr) {
 return
    !pred.marked &&
    !curr.marked &&
    pred.next == curr);
  }
```

predecessor still predecessor

Lazy Locking

- Removal

```
try {
  pred.lock(); curr.lock();
  if (validate(pred,curr) {
   if (curr.key == key) {
    curr.marked = true;
    pred.next = curr.next;
    return true;
   } else {
    return false;
   }}} finally {
  pred.unlock();
  curr.unlock();
   }}}
```

Lazy Locking

- Removal

```
try {
  pred.lock(); curr.lock();
  if (validate(pred,curr) {
   if (curr.key == key) {
    curr.marked = true;
    pred.next = curr.next;
    return true;
   } else {
    return false;
   }}} finally {
  pred.unlock();
  curr.unlock();
   }}}
```

lock both nodes

Lazy Locking

- Removal

```
try {
  pred.lock(); curr.lock();
  if (validate(pred,curr) {
   if (curr.key == key) {
    curr.marked = true;
    pred.next = curr.next;
    return true;
   } else {
    return false;
   }}} finally {
  pred.unlock();
  curr.unlock();
   }}}
```

validate

Lazy Locking

- Removal

```
try {
  pred.lock(); curr.lock();
  if (validate(pred,curr) {
   if (curr.key == key) {
    curr.marked = true;
    pred.next = curr.next;
    return true;
   } else {
    return false;
   }}} finally {
  pred.unlock();
  curr.unlock();
   }}}
```

key found

Lazy Locking

- Removal

```
try {
  pred.lock(); curr.lock();
  if (validate(pred,curr) {
   if (curr.key == key) {
    curr.marked = true;
    pred.next = curr.next;
    return true;
   } else {
    return false;
   }}} finally {
  pred.unlock();
  curr.unlock();
   }}}
```

logic delete

Lazy Locking

- Removal

```
try {
  pred.lock(); curr.lock();
  if (validate(pred,curr) {
   if (curr.key == key) {
    curr.marked = true;
    pred.next = curr.next;
    return true;
   } else {
    return false;
   }}} finally {
  pred.unlock();
  curr.unlock();
   }}}
```

logic delete

Lazy Locking

- Removal

```
try {
  pred.lock(); curr.lock();
  if (validate(pred,curr) {
   if (curr.key == key) {
    curr.marked = true;
    pred.next = curr.next;
    return true;
   } else {
    return false;
   }}} finally {
  pred.unlock();
  curr.unlock();
   }}}
```

physical delete

Lazy Locking

- Removal

```
try {
  pred.lock(); curr.lock();
  if (validate(pred,curr) {
   if (curr.key == key) {
    curr.marked = true;
    pred.next = curr.next;
    return true;
   } else {
    return false;
   }}} finally {
  pred.unlock();
  curr.unlock();
   }}}
```

done

Lazy Locking

- Containment

```
public boolean contains(Item item) {
  int key = item.hashCode();
  Node curr = this.head;
  while (curr.key < key) {
    curr = curr.next;
  }
  return curr.key == key && !curr.marked;
}
```

Lazy Locking

- Containment

```
public boolean contains(Item item) {
  int key = item.hashCode();
  Node curr = this.head;
  while (curr.key < key) {
    curr = curr.next;
  }
  return curr.key == key && !curr.marked;
}
```

start at head

Lazy Locking

- Containment

```
public boolean contains(Item item) {
  int key = item.hashCode();
  Node curr = this.head;
  while (curr.key < key) {
    curr = curr.next;
  }
  return curr.key == key && !curr.marked;
}
```

traverse list without locking

Nodes might be deleted

Lazy Locking

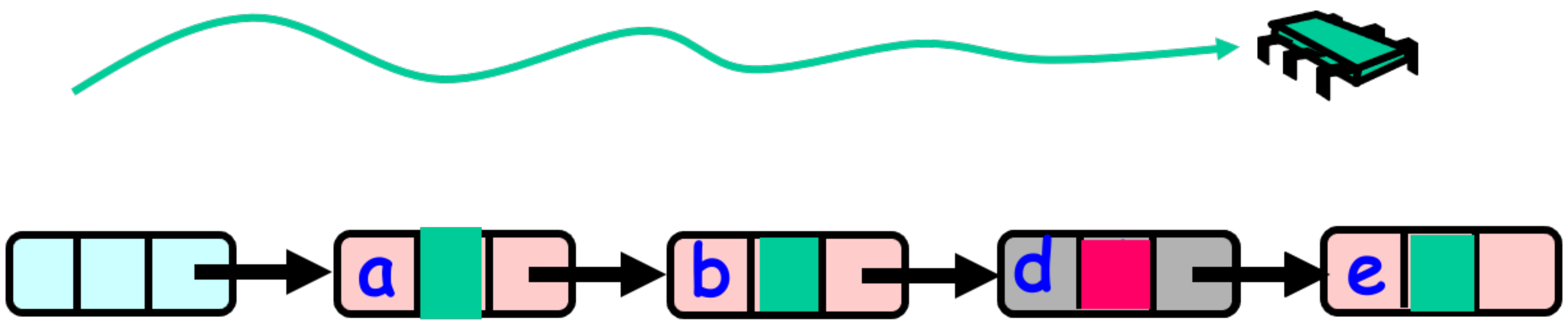
- Containment

```
public boolean contains(Item item) {
  int key = item.hashCode();
  Node curr = this.head;
  while (curr.key < key) {
    curr = curr.next;
  }
  return curr.key == key && !curr.marked;
}
```

Present and undeleted?

Lazy Locking

- Summary



- Combine mark bit and list ordering

Lazy Locking

- Lazy adds and removes
- Wait-free contains

Lazy Locking

- Good:
  - Contains is wait-free
  - Uncontended calls do not re-traverse
- Bad:
  - Contended add / removes require re-traversion

CAS

- CAS instruction: Compare And Set
  - `Boolean register.CAS(expected, update)`
    - Atomic operation
    - If `register` value is equal to `expected` then its value becomes `update` and returns `true`
    - If `register` value is not equal to `expected`, returns `false`, but does not change the value

CAS

- Example: Consensus protocol for *n* threads 0, …, *n*-1
- `AtomicInteger` class has a CAS method

```
class CASConsensus extends ConsensusProtocol {
    private final int FIRST = -1;
    private AtomicInteger r = new AtomicInteger(FIRST);
    public Object decide(Object value) {
        propose(value);
        int i = ThreadID.get();
        if (r.compareAndSet(FIRST, i)) // I won
             return proposed[i];
        else // I lost
             return proposed[r.get()];
    }
}
```

CAS

```
class CASConsensus extends ConsensusProtocol {
   private final int FIRST = -1;
   private AtomicInteger r = new AtomicInteger(FIRST);
   public Object decide(Object value) {
      propose(value);
      int i = ThreadID.get();
      if (r.compareAndSet(FIRST, i)) // I won
           return proposed[i];
      else // I lost
          return proposed[r.get()];
   }
}
```

Load r with First

CAS

```
class CASConsensus extends ConsensusProtocol {
    private final int FIRST = -1;
    private AtomicInteger r = new AtomicInteger(FIRST);
    public Object decide(Object value) {
        propose(value);
        int i = ThreadID.get();
        if (r.compareAndSet(FIRST, i)) // I won
            return proposed[i];
        else // I lost
            return proposed[r.get()];
    }
}
```

Each thread loads global array proposed with a value

CAS

```
class CASConsensus extends ConsensusProtocol {
   private final int FIRST = -1;
   private AtomicInteger r = new AtomicInteger(FIRST);
   public Object decide(Object value) {
      propose(value);
      int i = ThreadID.get();
      if (r.compareAndSet(FIRST, i)) // I won
           return proposed[i];
      else // I lost
          return proposed[r.get()];
   }
}
```

Try whether there is still the original value in r

CAS

```
class CASConsensus extends ConsensusProtocol {
   private final int FIRST = -1;
   private AtomicInteger r = new AtomicInteger(FIRST);
   public Object decide(Object value) {
      propose(value);
      int i = ThreadID.get();
      if (r.compareAndSet(FIRST, i)) // I won
           return proposed[i];
      else // I lost
           return proposed[r.get()];
   }
}
```

If it is, exchange with thread-number

CAS

```
class CASConsensus extends ConsensusProtocol {
   private final int FIRST = -1;
   private AtomicInteger r = new AtomicInteger(FIRST);
   public Object decide(Object value) {
      propose(value);
      int i = ThreadID.get();
      if (r.compareAndSet(FIRST, i)) // I won
           return proposed[i];
      else // I lost
          return proposed[r.get()];
   }
}
```

This happens for only one thread, who gets to update the value of r with its thread number

CAS

```
class CASConsensus extends ConsensusProtocol {
   private final int FIRST = -1;
   private AtomicInteger r = new AtomicInteger(FIRST);
   public Object decide(Object value) {
      propose(value);
      int i = ThreadID.get();
      if (r.compareAndSet(FIRST, i)) // I won
           return proposed[i];
      else // I lost
          return proposed[r.get()];
   }
}
```

All other threads will find the value different

CAS

```
class CASConsensus extends ConsensusProtocol {
   private final int FIRST = -1;
   private AtomicInteger r = new AtomicInteger(FIRST);
   public Object decide(Object value) {
      propose(value);
      int i = ThreadID.get();
      if (r.compareAndSet(FIRST, i)) // I won
           return proposed[i];
      else // I lost
          return proposed[r.get()];
   }
}
```

All other threads will find the value different:
The value is the number of the winning thread
Therefore, they return its proposed value

CAS

```
class CASConsensus extends ConsensusProtocol {
   private final int FIRST = -1;
   private AtomicInteger r = new AtomicInteger(FIRST);
   public Object decide(Object value) {
      propose(value);
      int i = ThreadID.get();
      if (r.compareAndSet(FIRST, i)) // I won
           return proposed[i];
      else // I lost
          return proposed[r.get()];
   }
}
```

The one and only thread to win will get its value as the consensus

CAS

- A register with CAS and get has an infinite consensus number

Bit-Stealing

- C++ has pointers
  - To atomically mark a pointer with a boolean value:
    - Observe that pointers to objects never have the least significant two bit set
    - In fact, alignment is usually in multiples of 16, so 4 least significant bits are zero
    - Use one of these bits as a marker
      - Can still recover the original pointer

Bit-Stealing

- In Java:
  - java.util.concurrent.atomic has an object
  - AtomicMarkableReference<T>:
    - Reference to an object of type T
    - Boolean mark field
    - Can be updated atomically together or individually

Bit Stealing

- Interface:

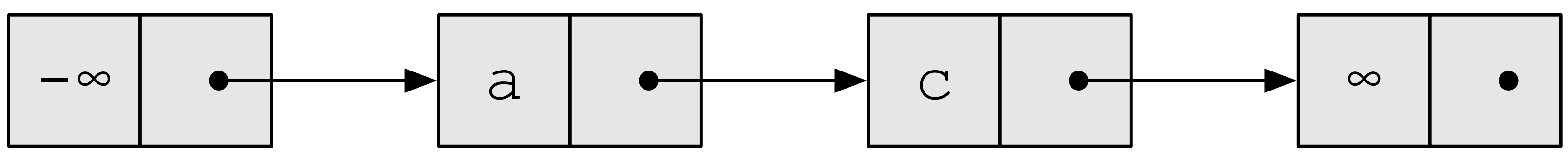
```
public boolean compareAndSet(T expectedReference,
                             T newReference,
                             boolean expectedMark,
                             boolean newMark);

public boolean attemptMark(T expectedReference,
                           boolean newMark);

public T get(boolean[] marked);
```

  - returns the encapsulated reference and stores mark at position 0 in the array

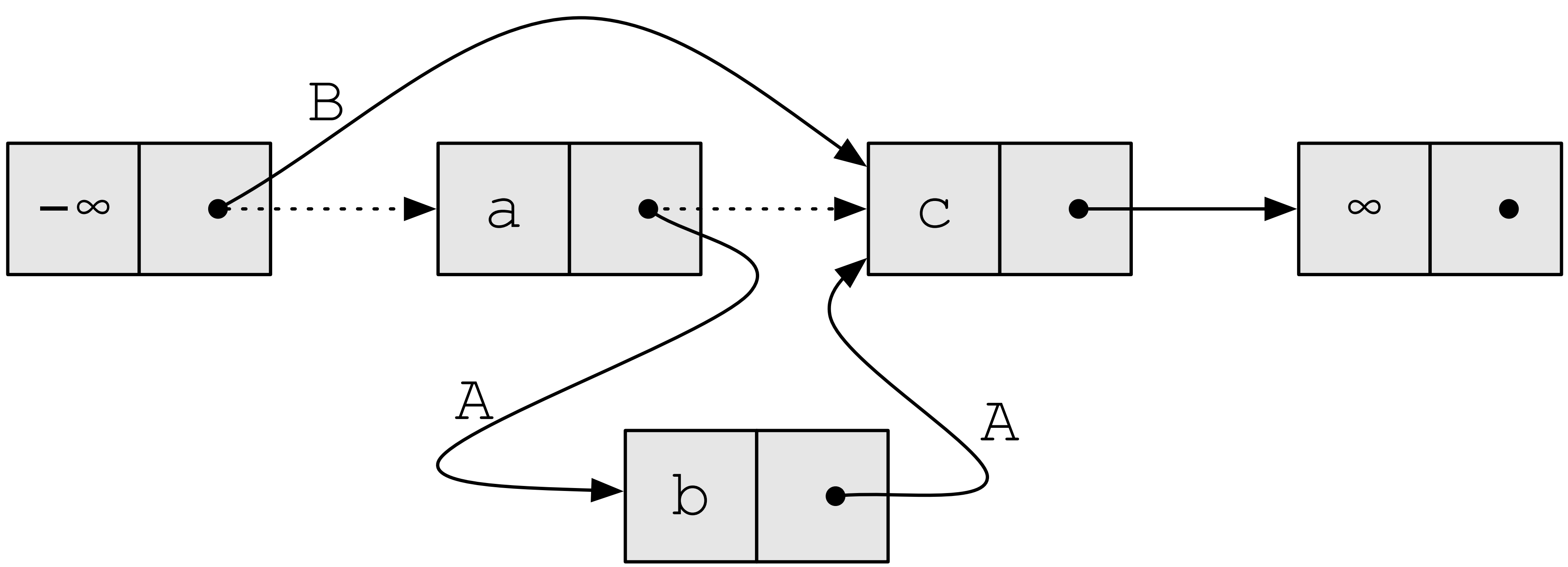
Lock-free Lists

- First attempt:
  - Use compareAndSet to change the next field
- Example:
  - 

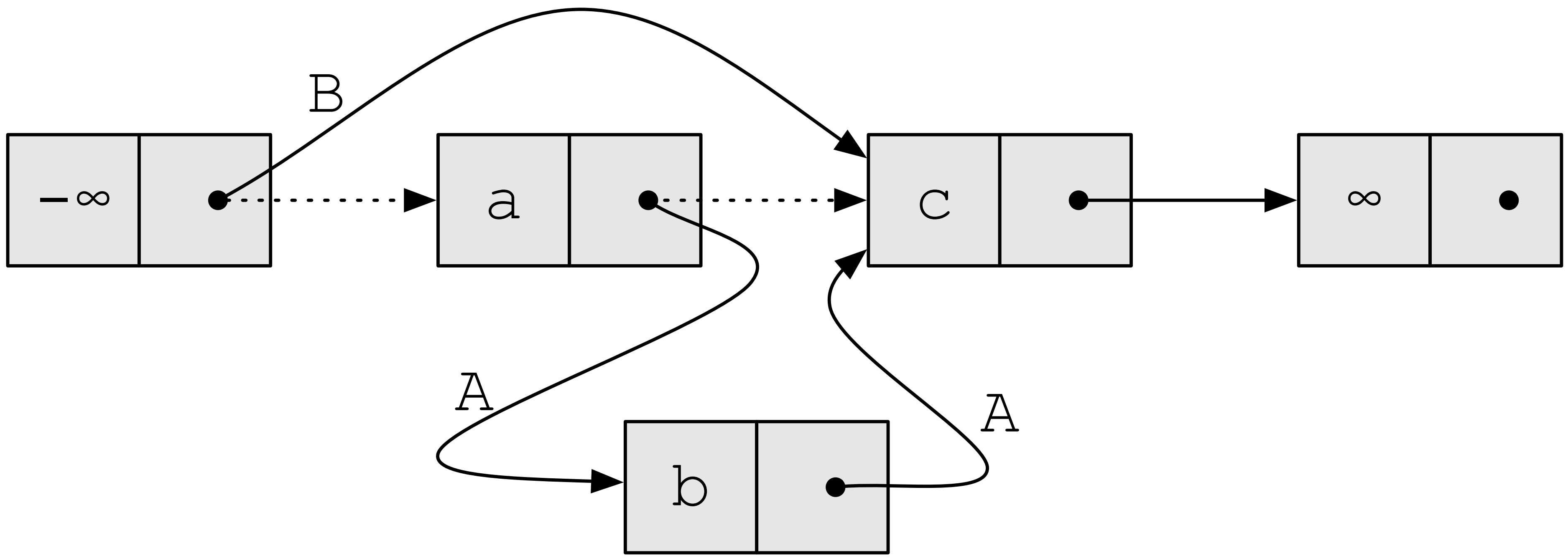
  - Thread I: add b
  - Thread II: remove a

Lock-free Lists

- Thread A applies CAS to a.next
- Thread B applies CAS to -∞.next
- Both succeed regardless of who comes first:



Lock-free Lists



- We must prevent manipulation of a removed node!

Lock-free Lists

Lock-free Lists

Lock-free Lists

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