Data Structures

Algorithms

Types of Data Structures

- Organize data to make access / processing fast
  - Speed depends on the internal organization
  - Internal organization allows different types of accesses
- Problems:
  - Large data is nowadays distributed over several data centers
  - Need to take advantage of storage devices

Types of Data Structures

- Internal Memory
  - DRAM: fast access, byte addressable
- Storage
  - Hard Disk Drives
    - Data in blocks
    - Decent for streaming (consecutive blocks)
    - Bad for random access (~10 msec per access)
  - Solid State Disks
    - Data in blocks (called pages)
    - Decent access times (~1msec per access)

Types of Data Structures

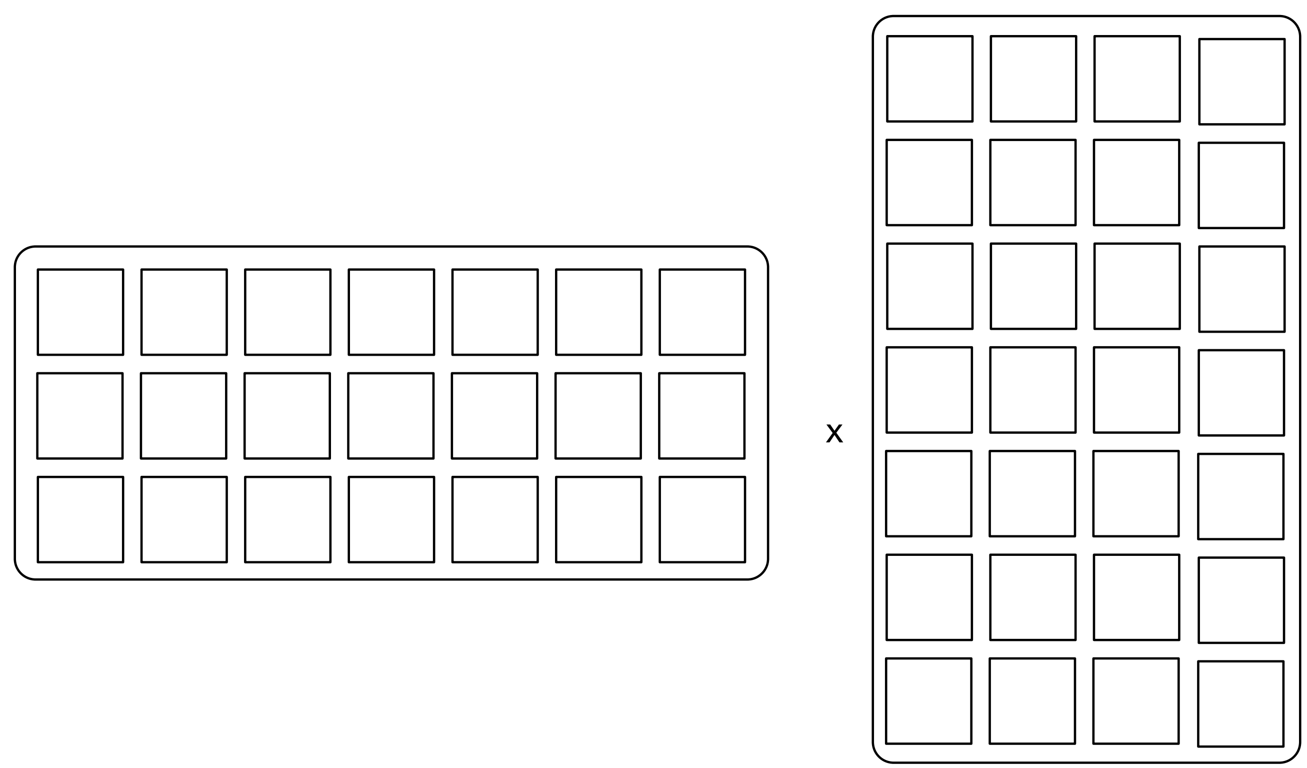
- Thread safe:
  - Several threads can safely access data structure
  - Need collaboration between threads
    - Implemented with locks
    - Implemented without locks
      - Difficult to do
      - Needs atomic instructions

Types of Data Structures

- Caches can make big performance differences
  - Cache aware algorithms
    - Get the parameter of the caches
  - Cache oblivious algorithms
    - Work well for all cache sizes
  - Dumb algorithms
    - Do not pay attention to caches at all
    - Frequent surprises with bad performance

Example

- Multiplying two big, non-dense matrices
    - Cache aware:
        - Break matrices into subsquares
        - Three subsquares fit comfortably into cache



Example

- Cache Oblivious
  - Use a Divide and Conquer Algorithm that subdivides the sub-squares repeatedly
  - Only **cold** cache misses when a new subsquare needs to be loaded into cache.

Types of Data Structure

- Dictionary — Key - Value Store
  - CRUD operations: create, read, update, delete
  - Solutions differ regarding read and write speeds

Types of Data Structure

- Range Queries (Big Table, RP)
  - CRUD and range operation

Types of Data Structure

- Priority queue:
    - Insert, retrieve minimum and delete it

Types of Data Structure

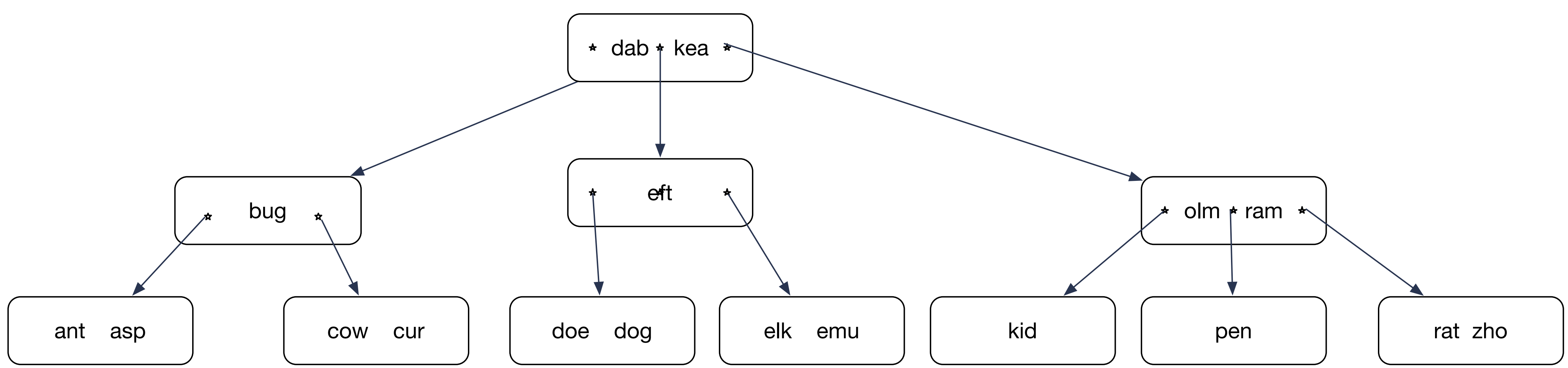
- Log:
  - Append, Read

B-Trees

- B-trees: In memory data structure for CRUD and range queries
  - Balanced Tree
  - Each node can have between *d* and 2*d* keys with the exception of the root
  - Each node consists of a sequence of node pointer, key, node pointer, key, …, key, node pointer
  - Tree is ordered.
    - All keys in a child are between the keys adjacent to the node pointer

B-Trees

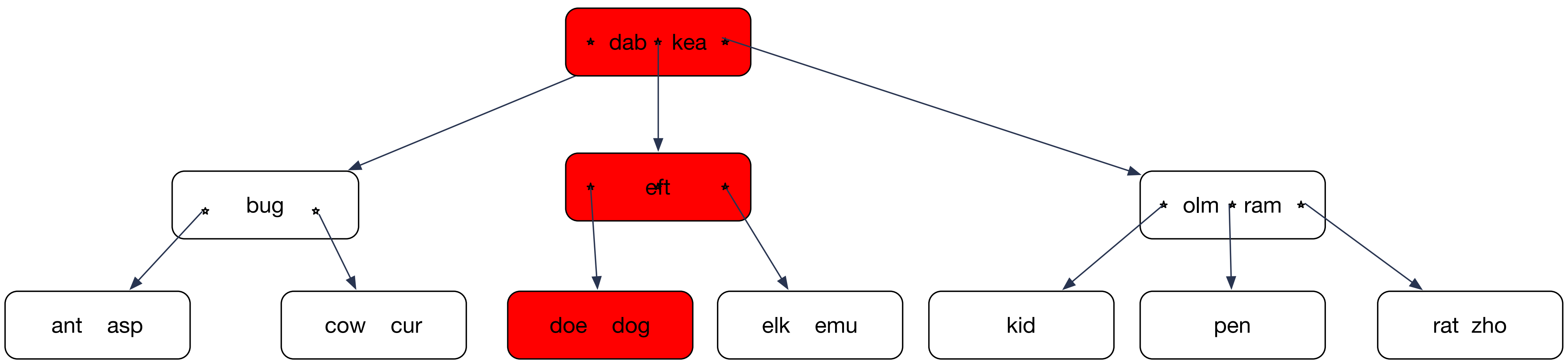
- Example: 2-3 tree: Each node has two or three children



B-Trees

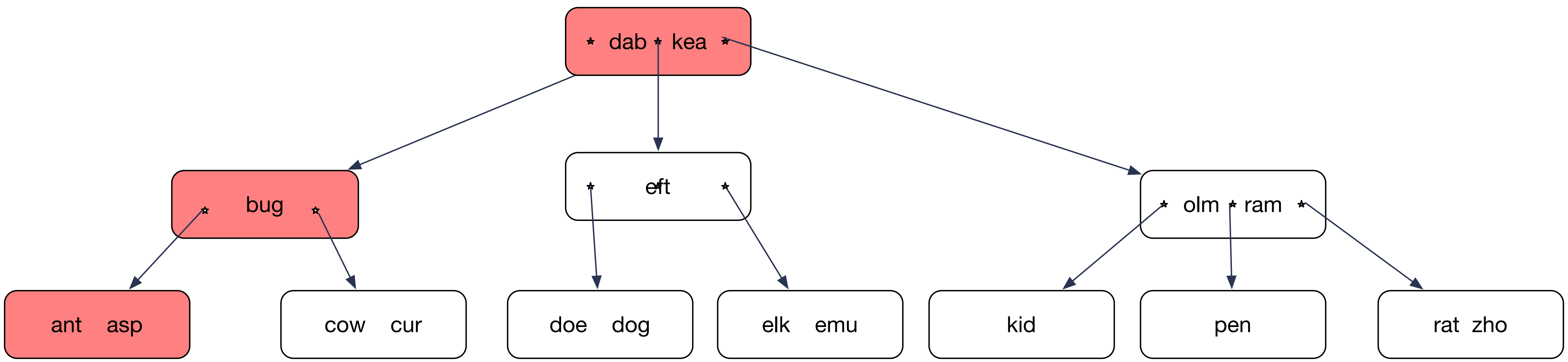
- Read dog:
  - Load root, determine location of dog in relation to the keys
  - Follow middle pointer
  - Follow pointer to the left
  - Find “dog”

B-Trees


dab kea
bug
eft
olm ram
ant asp
cow cur
doe dog
elk emu
kid
pen
rat zho

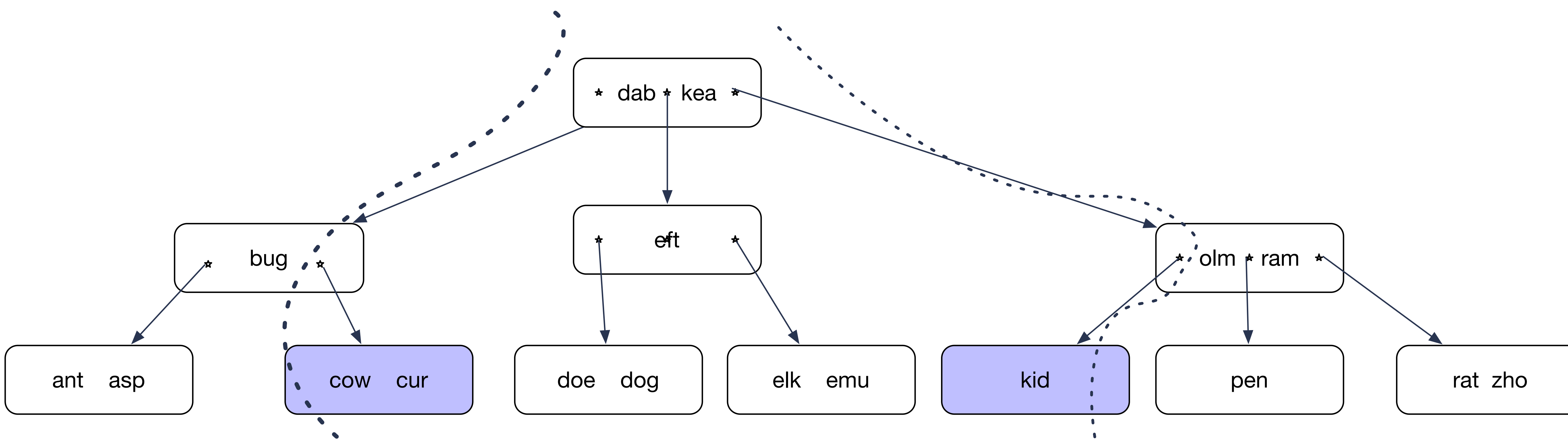
B-Trees

- Search for “auk” :



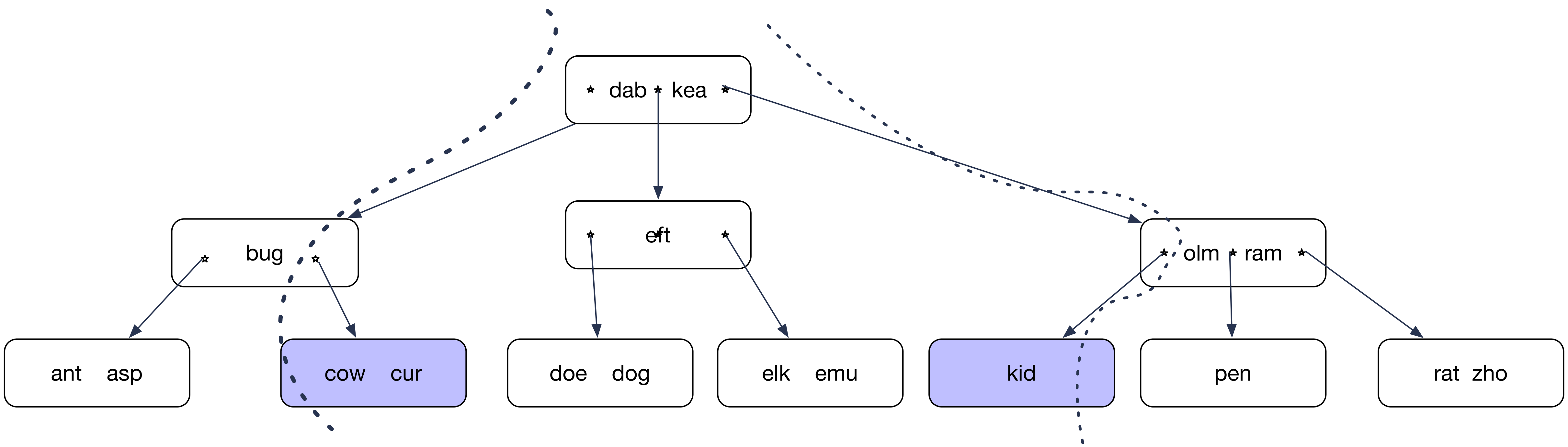
B-Trees

- Range Query c - l
  - Determine location of c and l



B-Trees

- Recursively enumerate all nodes between the lines starting with root



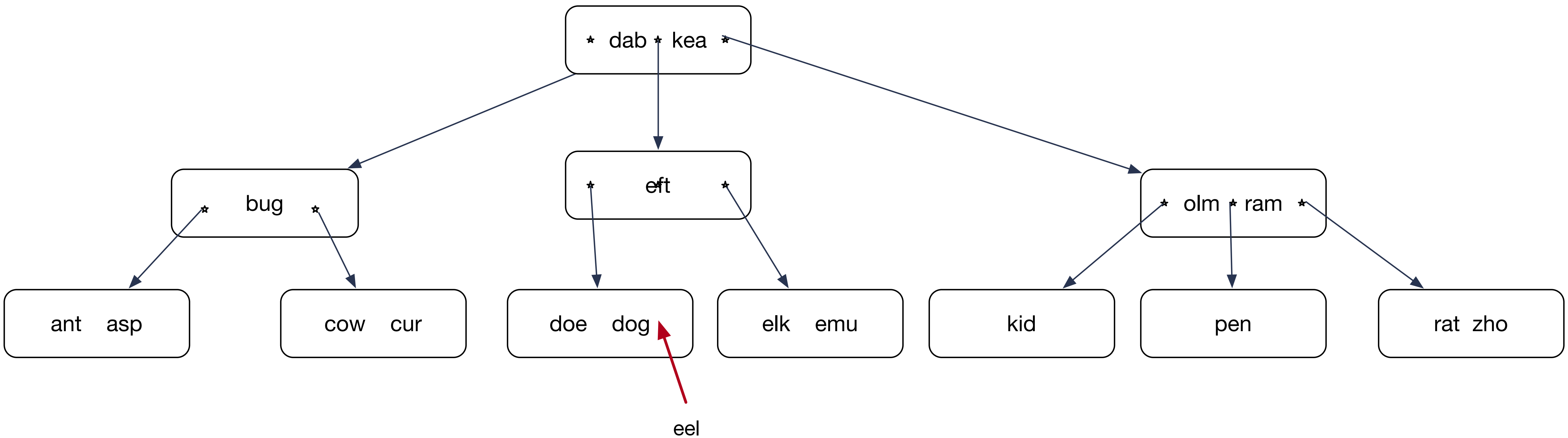
B-trees

- Capacity: With *l* levels, minimum of $1 + 2 + 2^2 + \ldots + 2^l$ nodes:
  - $1(2^{l+1} - 1)$ keys
- Maximum of $1 + 3 + 3^2 + \ldots + 3^l$ nodes
  - $\frac{2}{2}(3^{l+1} - 1)$ keys

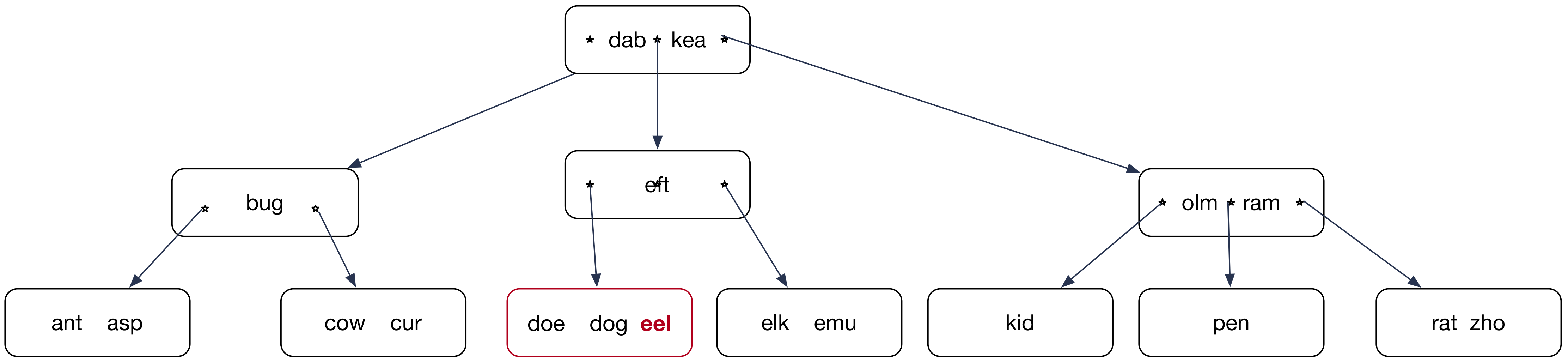
B-trees

- Inserts:
  - Determine where the key should be located in a leaf
  - Insert into leaf node
  - Leaf node can now have too many nodes
  - Take middle node and elevate it to the next higher level
  - Which can cause more “splits”

B-trees

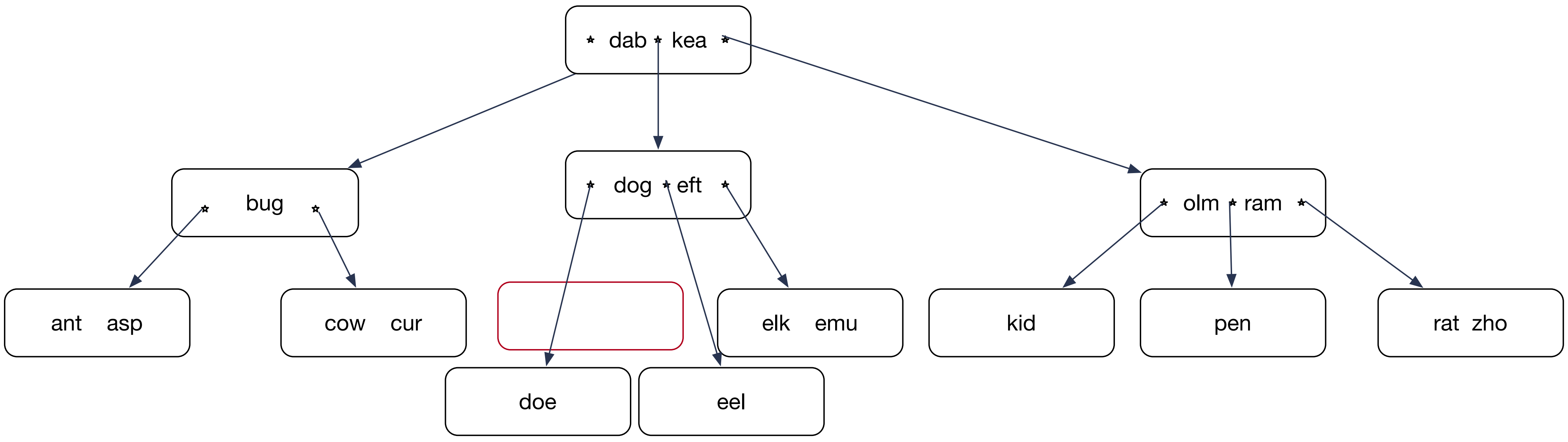

dab kea
bug
eft
olm ram
ant asp
cow cur
doe dog
elk emu
kid
pen
rat zho
eel

B-trees



* dab * kea *
* bug *
* eft *
* olm * ram *
ant asp
cow cur
doe dog **eel**
elk emu
kid
pen
rat zho

B-trees



dab kea
bug
dog eft
olm ram
ant asp
cow cur
elk emu
kid
pen
rat zho
doe
eel

B-trees

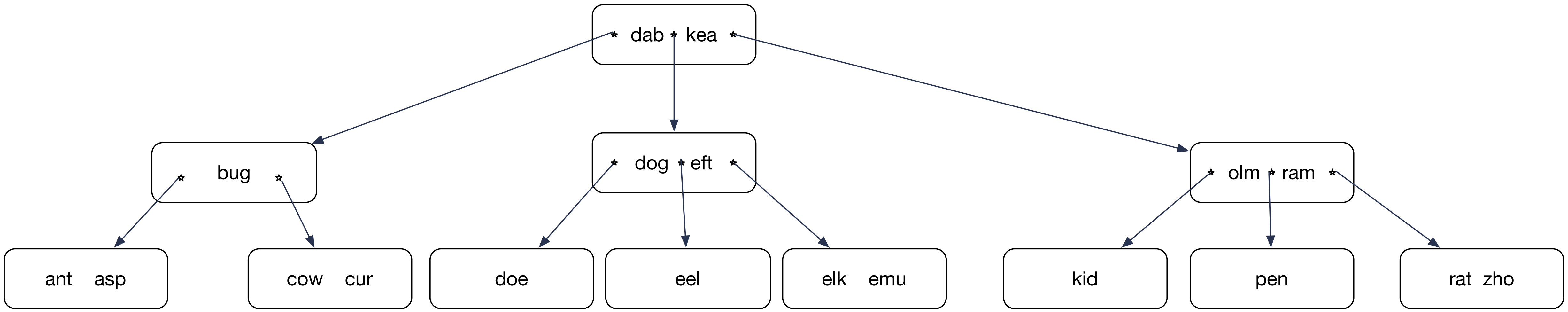
- Insert: Lock all nodes from root on down so that only one process can operate on the nodes
- Tree only grows a new level by splitting the root

B-Trees

- Using only splits leads to skinny trees
  - Better to make use of potential room in adjacent nodes
  - Insert “ewe”.
    - Node elk-emu only has one true neighbor.
      - Node kid does not count, it is a cousin, not a sibling

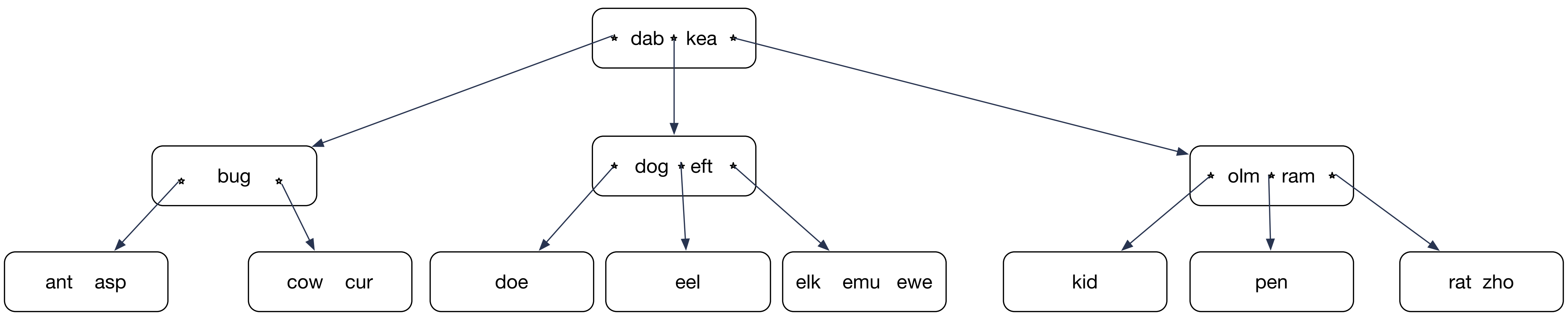
B-tree

- Insert ewe into



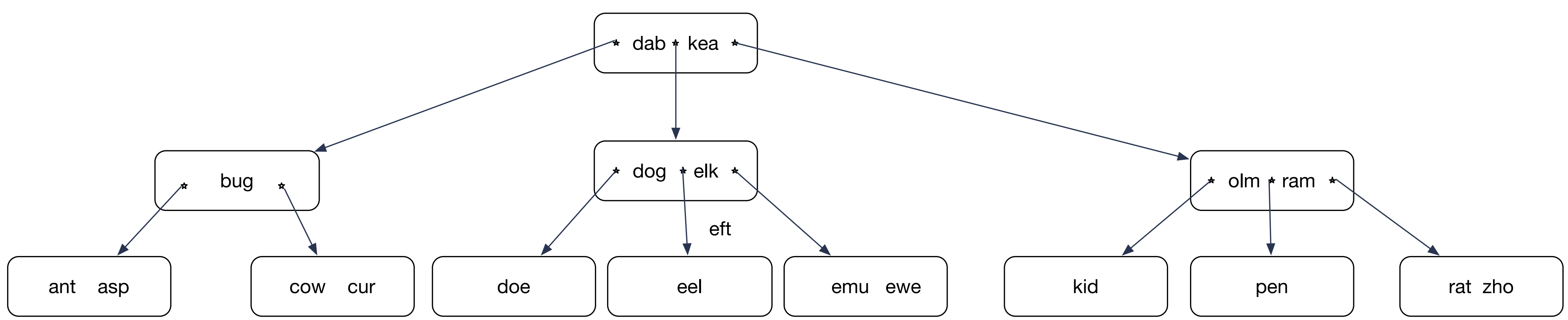
B-tree

- Insert ewe



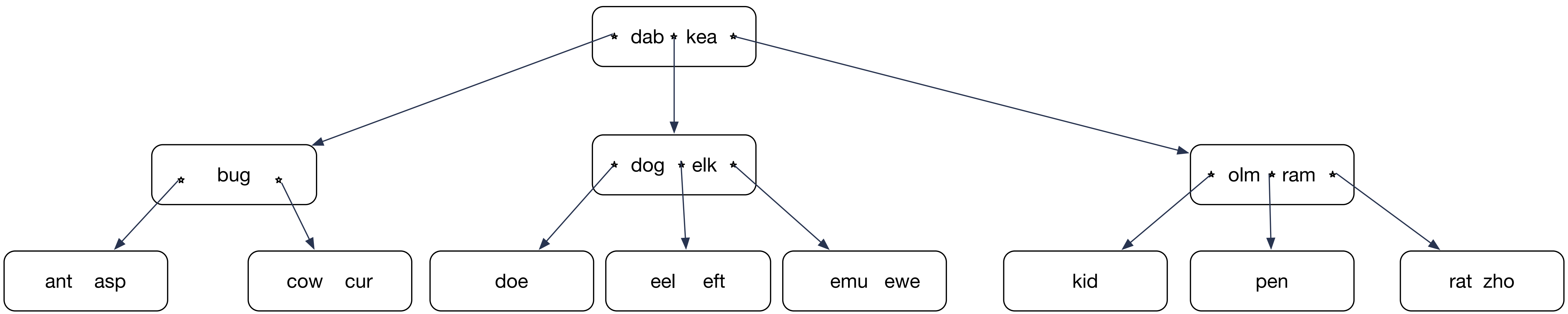
B-tree

- Promote elk. elk is guaranteed to come right after eft.
- Demote eft



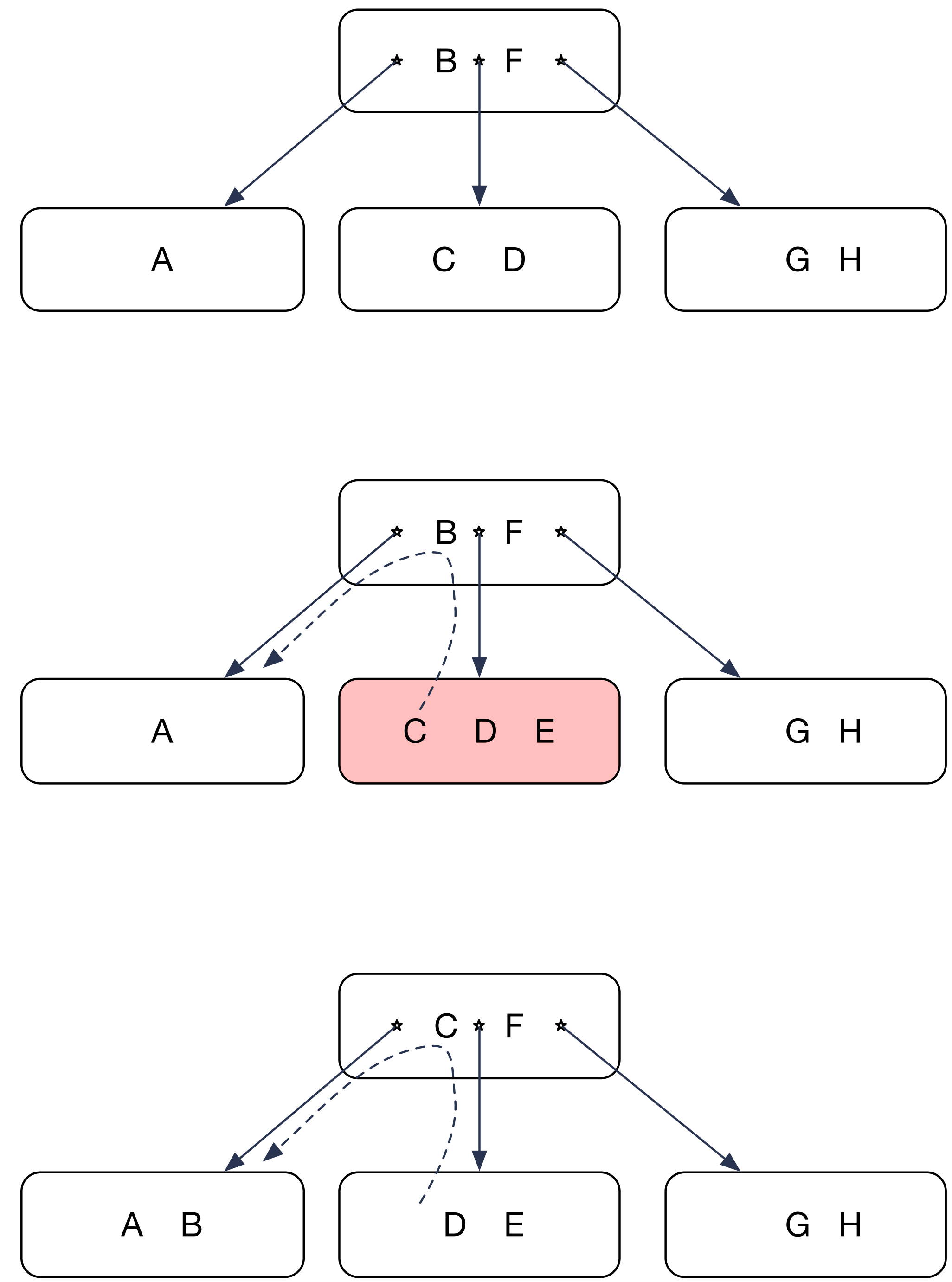
B-tree

- Insert eft into the leaf node

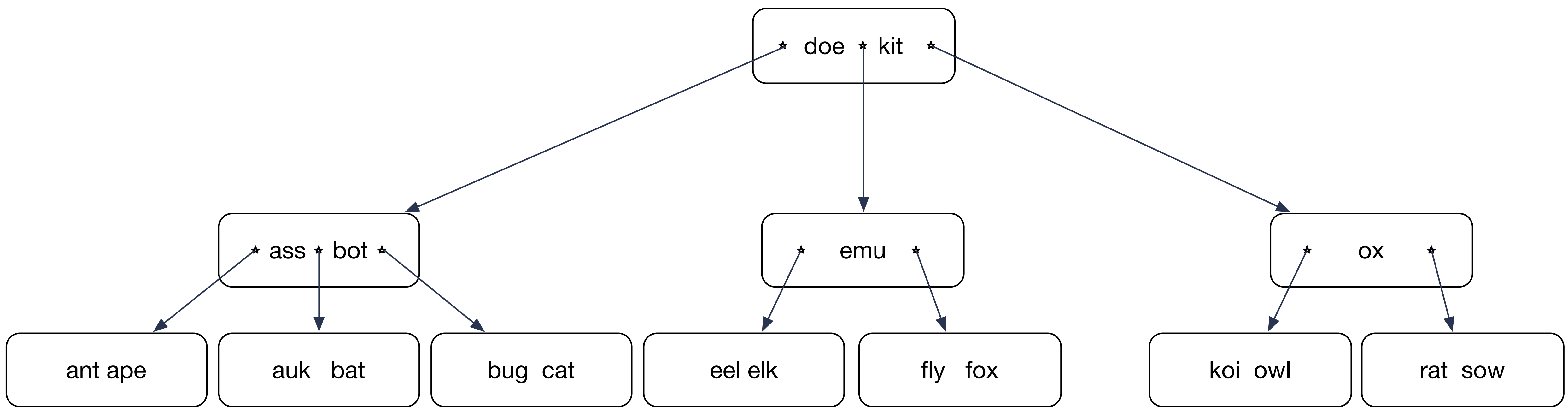


B-tree

- Left rotate
  - Overflowing node has a sibling to the left with space
  - Move left-most key up
  - Lower left-most key

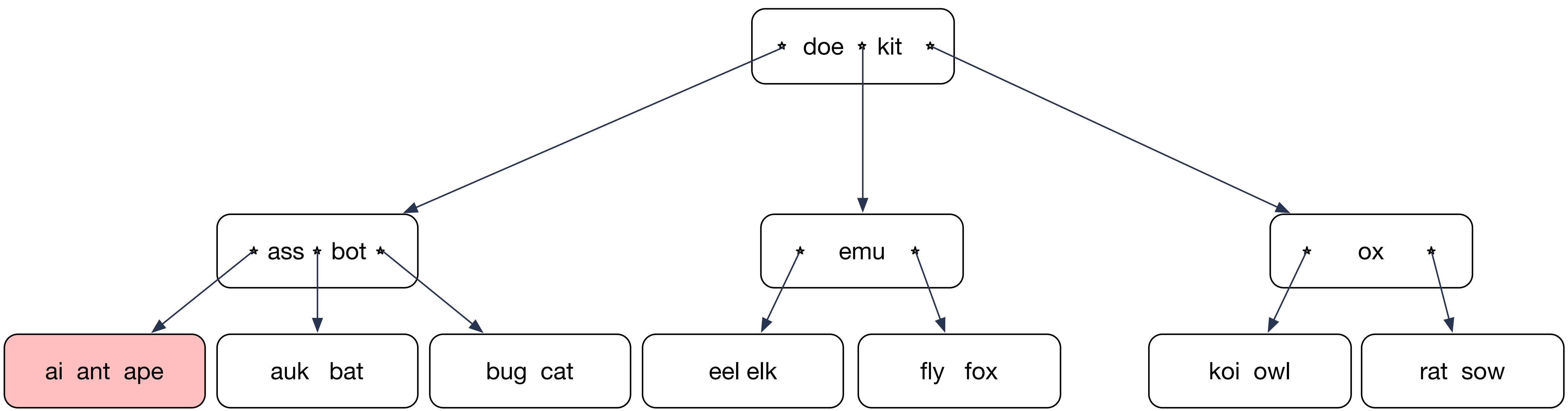


B-tree



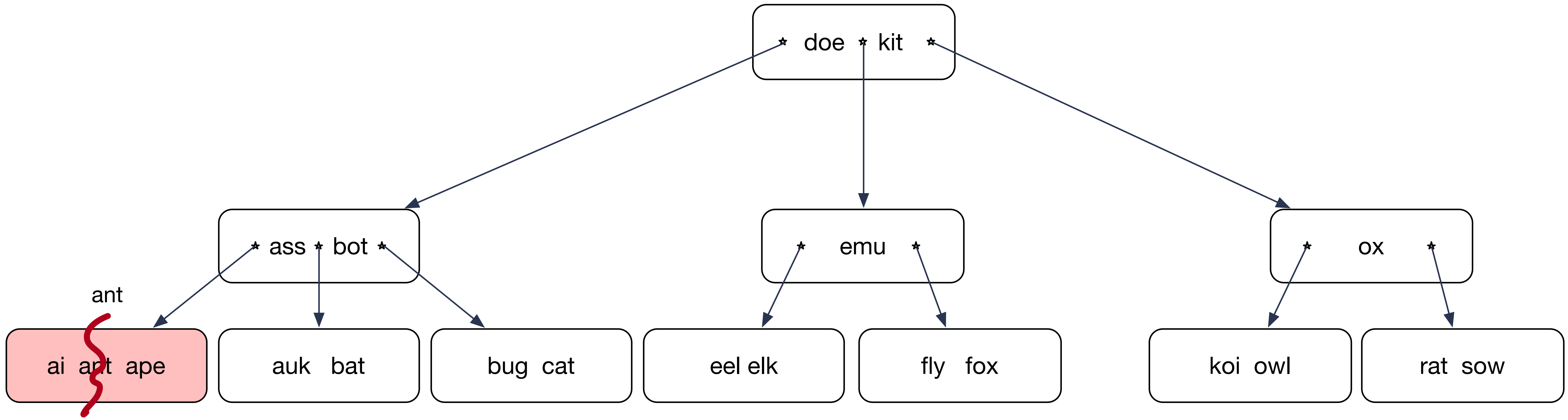
Now insert “ai”

B-tree



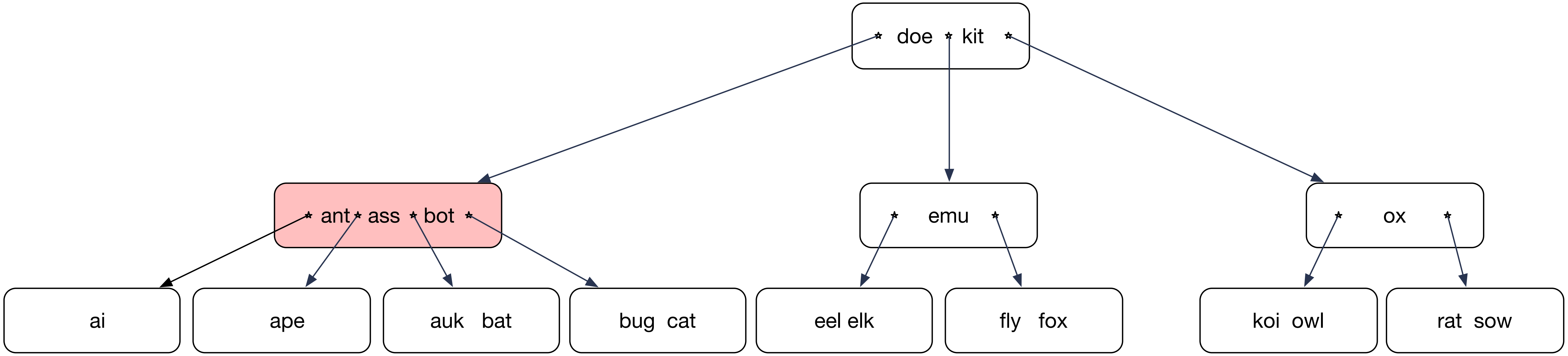
Insert creates an overflowing node
Only one neighboring sibling, but that one is full
Split!

B-tree



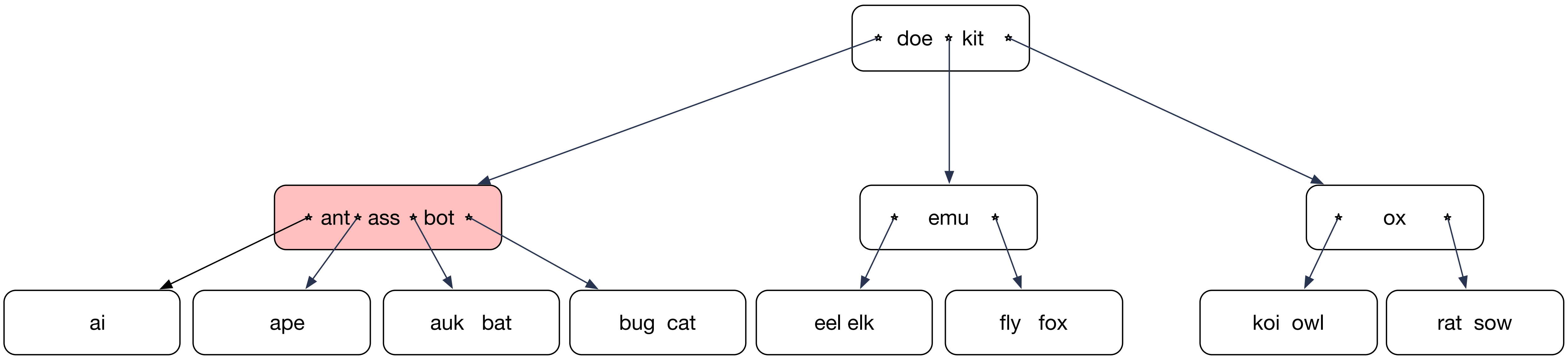
Middle key moves up

B-tree



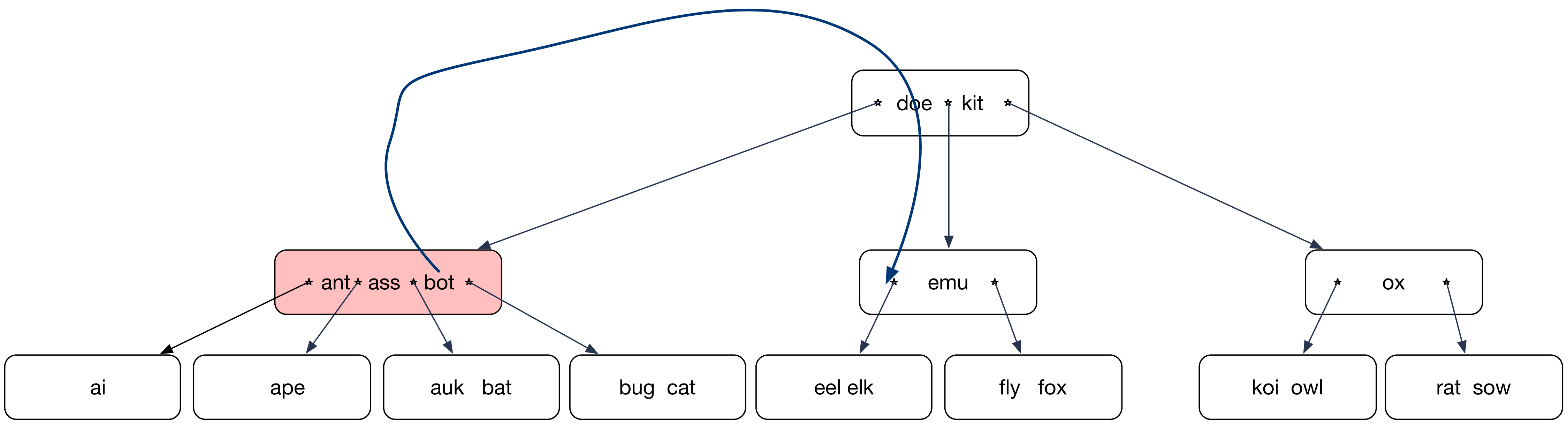
Unfortunately, this gives another overflow
But this node has a right sibling not at full capacity

B-tree



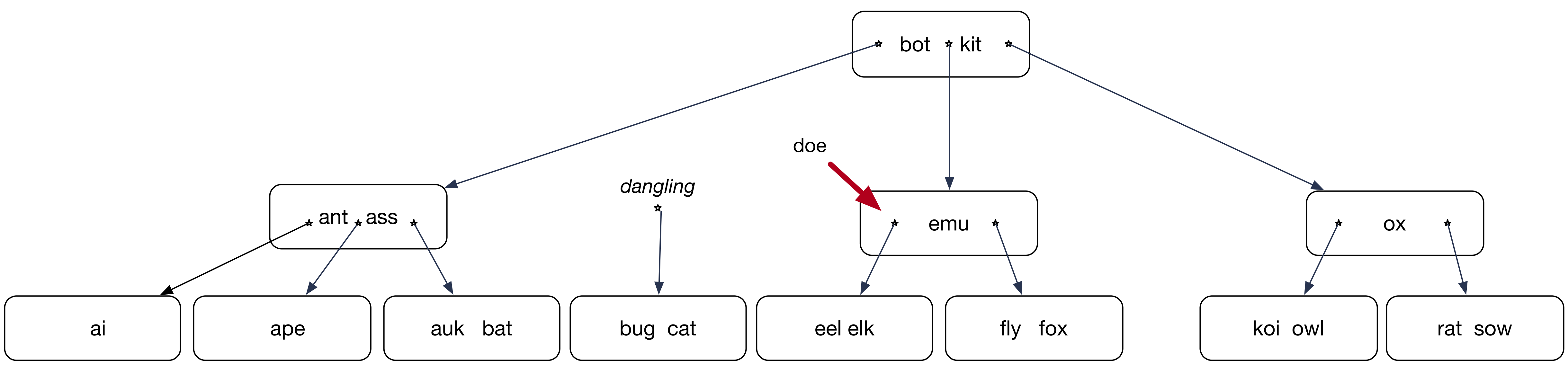
Right rotate:
Move “bot” up
Move “doe” down
Reattach nodes

B-tree



Move “bot” up
Move “doe” down
Reattach the dangling node

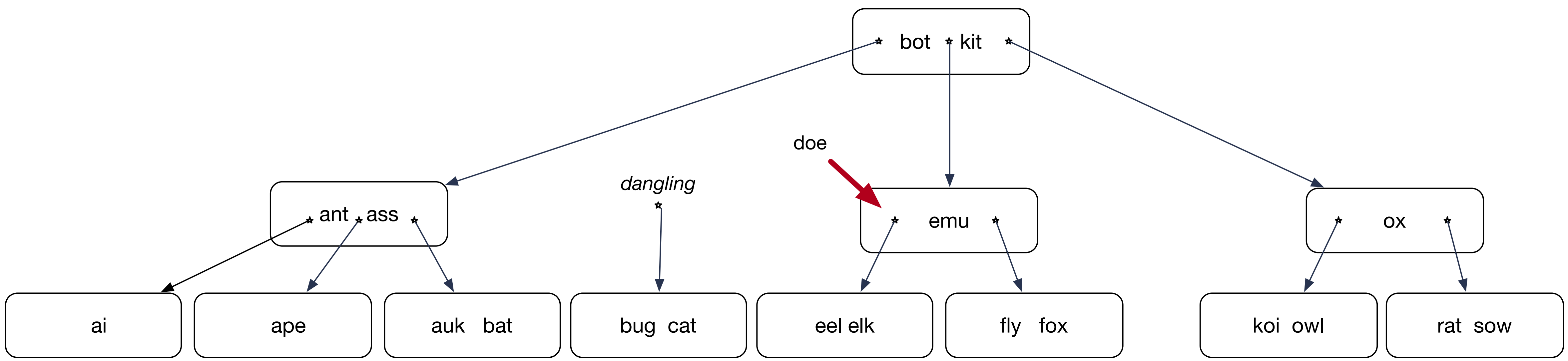
B-tree



“bot” had moved up and replaced doe

The “emu” node needs to receive one key and one pointer

B-tree



bot kit
doe
dangling
ant ass
emu
ox
ai
ape
auk bat
bug cat
eel elk
fly fox
koi owl
rat sow

B-tree

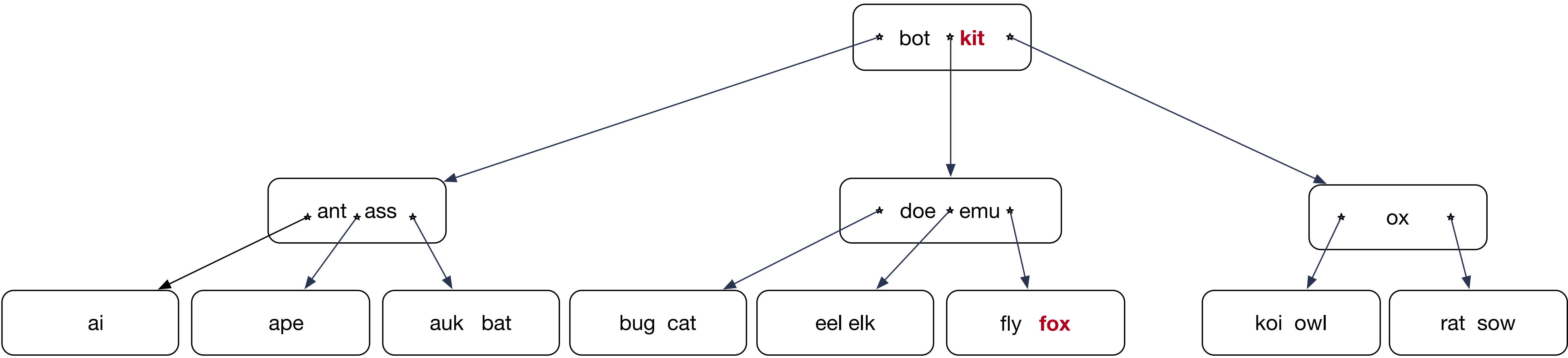
- Deletes
  - Usually restructuring not done because there is no need
  - Underflowing nodes will fill up with new inserts

B-tree

- Implementing deletion anyway:
  - Can only remove keys from leaves
  - If a delete causes an underflow, try a rotate into the underflowing node
  - If this is not possible, then merge with a sibling
    - A merge is the opposite of a split
  - This can create an underflow in the parent node
    - Again, first try rotate, then do a merge

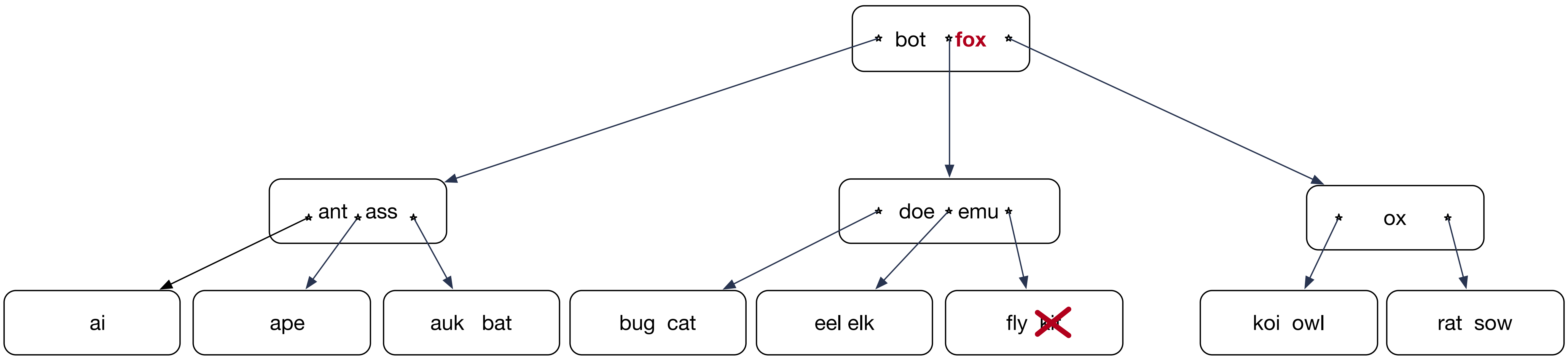
B-tree

Delete “kit”



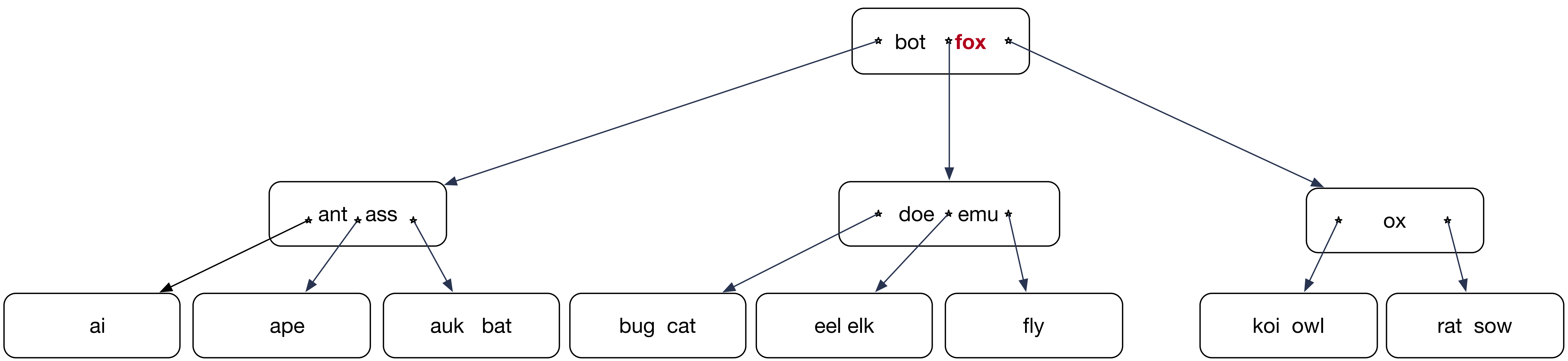
Delete “kit”
“kit” is in an interior node.
Exchange it with the key in the leave immediately before “fox”

B-tree



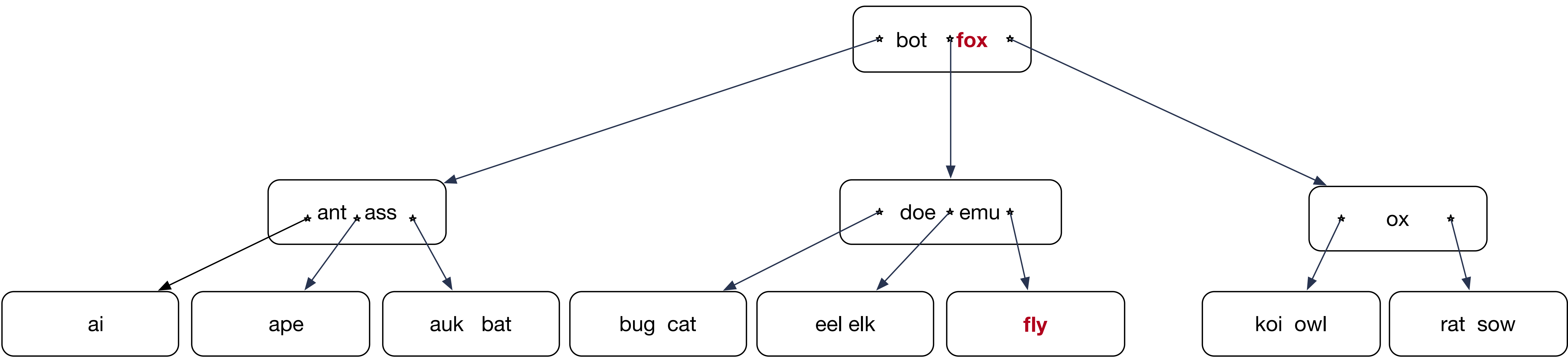
After interchanging “fox” and “kit”, can delete “kit”

B-tree



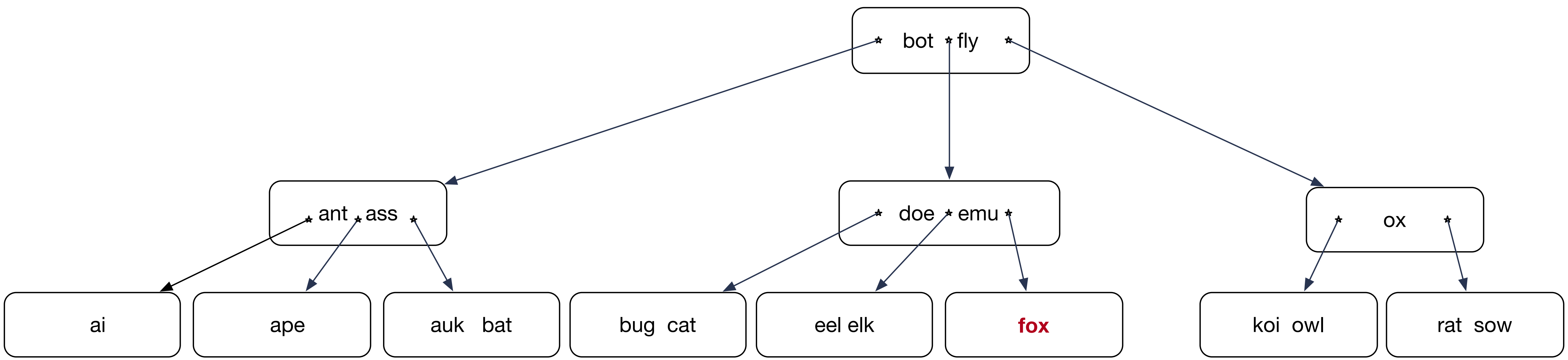
Now delete “fox”

B-tree



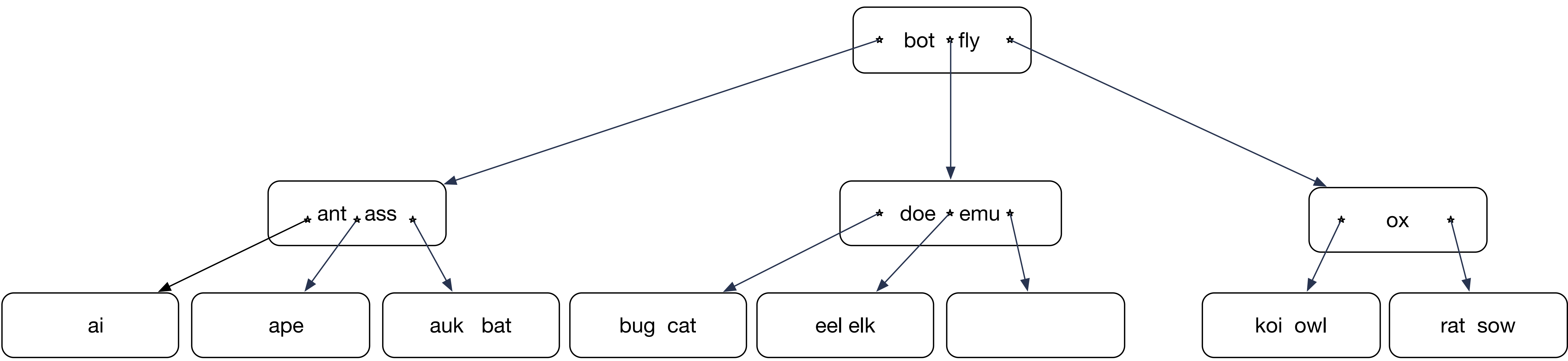
Step 1: Find the key. If it is not in a leaf
Step 2: Determine the key just before it, necessarily in a leaf
Step 3: Interchange the two keys

B-tree



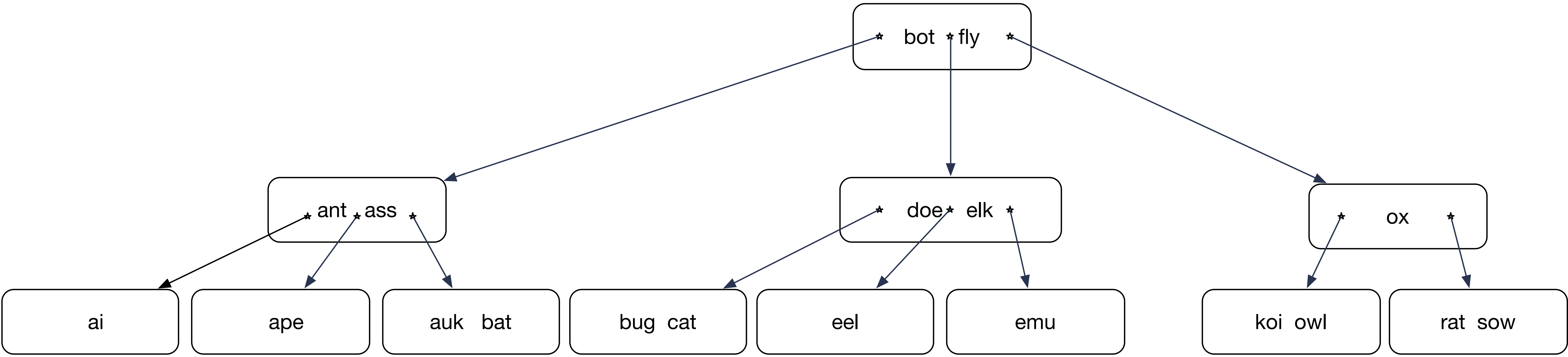
Step 4: Remove the key now from a leaf

B-tree



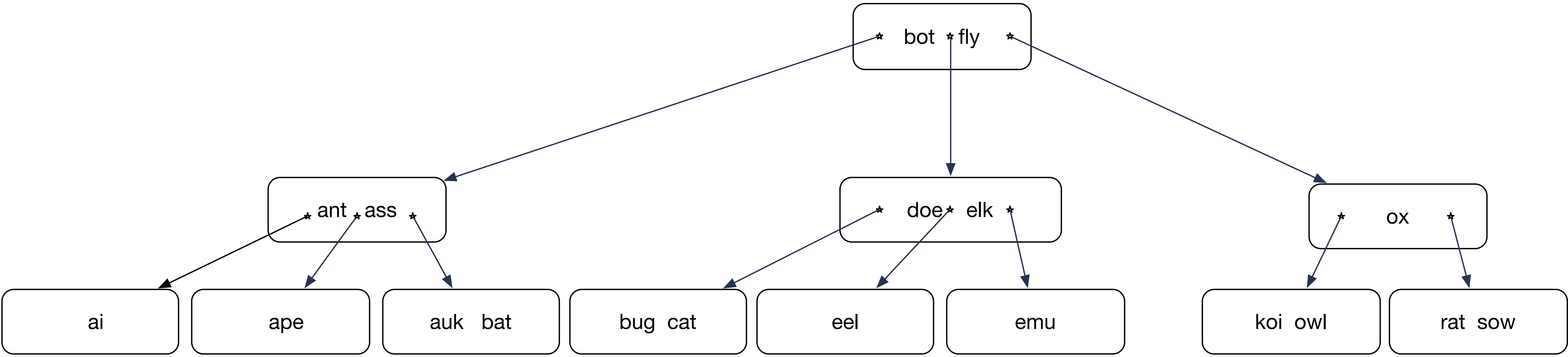
This causes an underflow
Remedy the underflow by right rotating from the sibling

B-tree



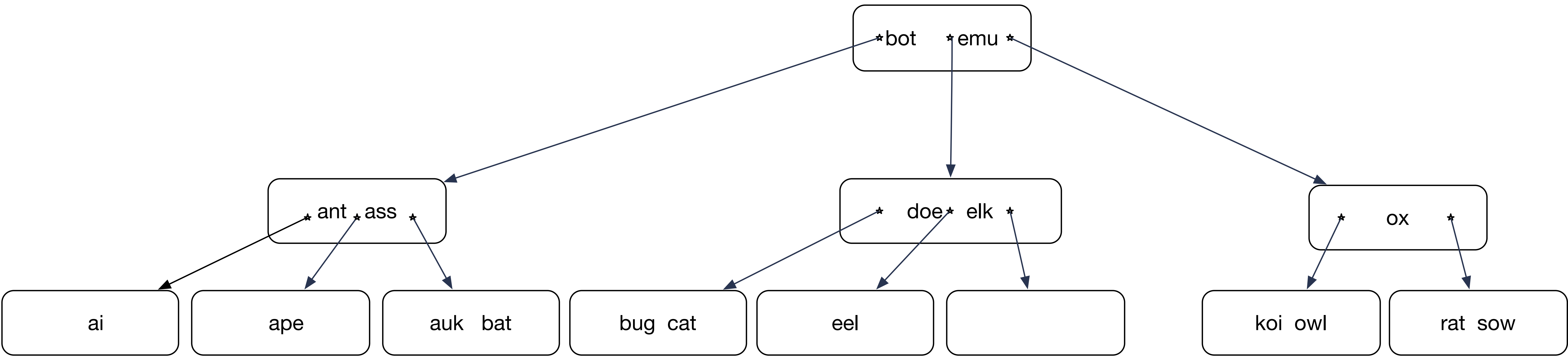
Everything is now in order

B-tree



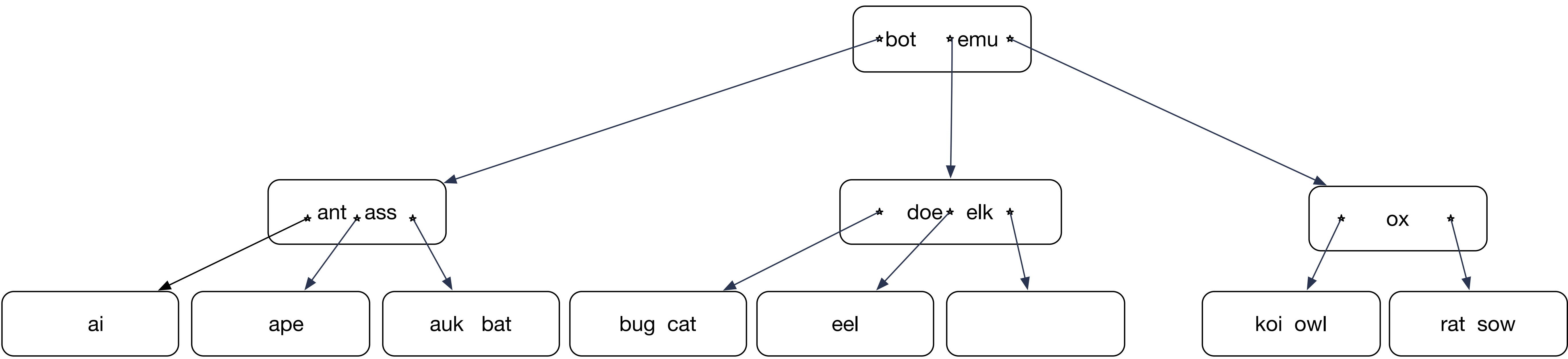
Now delete fly

B-tree



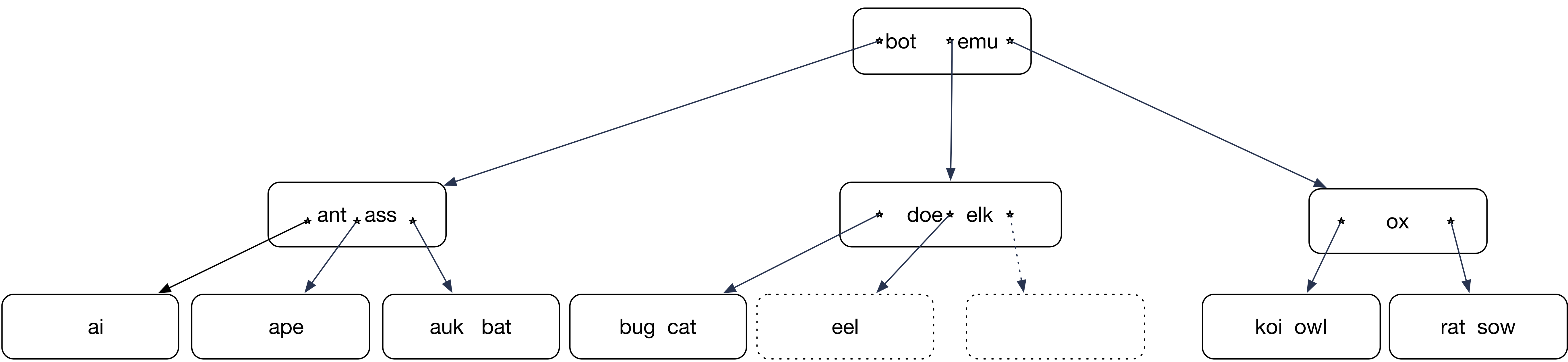
Switch “fly” with “emu”
remove “fly” from the leaf
Again: underflow

B-tree



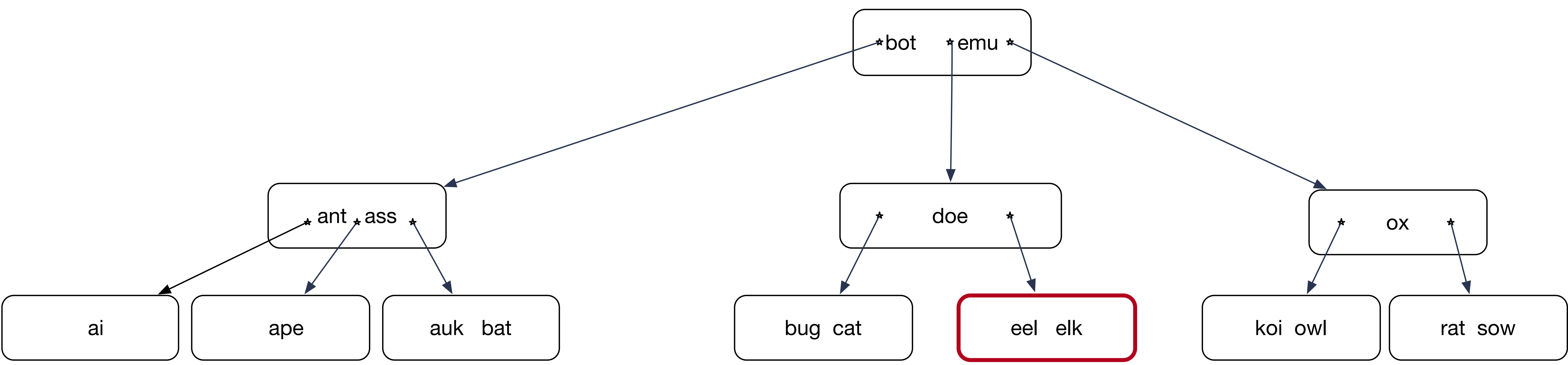
Cannot left-rotate: There is no left sibling
Cannot right-rotate: The right sibling has only one key
Need to merge: Combine the two nodes by bringing down "elk"

B-tree



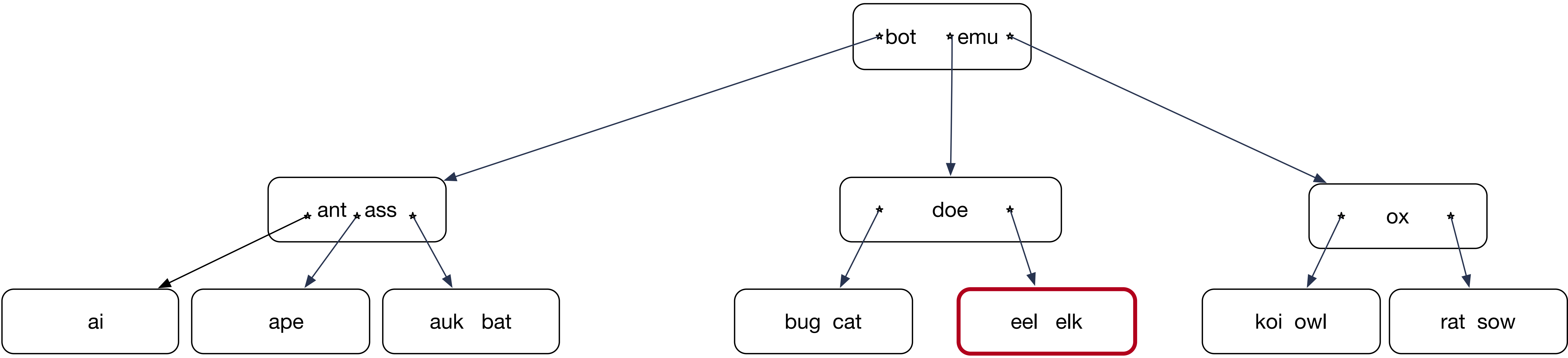
We can merge the two nodes because the number of keys combined is less than 2 *k*

B-tree



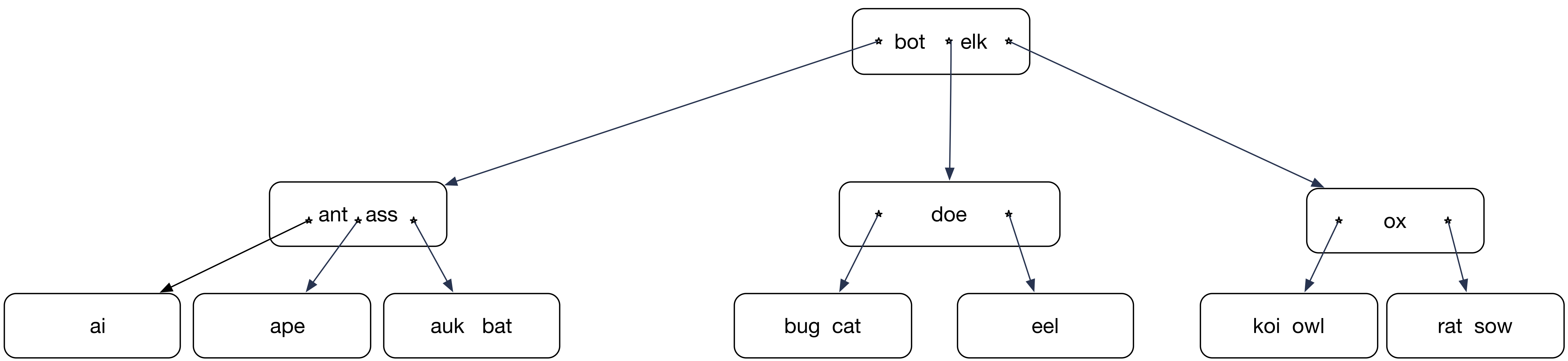
bot
emu
ant
ass
doe
ox
ai
ape
auk bat
bug cat
eel elk
koi owl
rat sow

B-tree



Delete "emu"

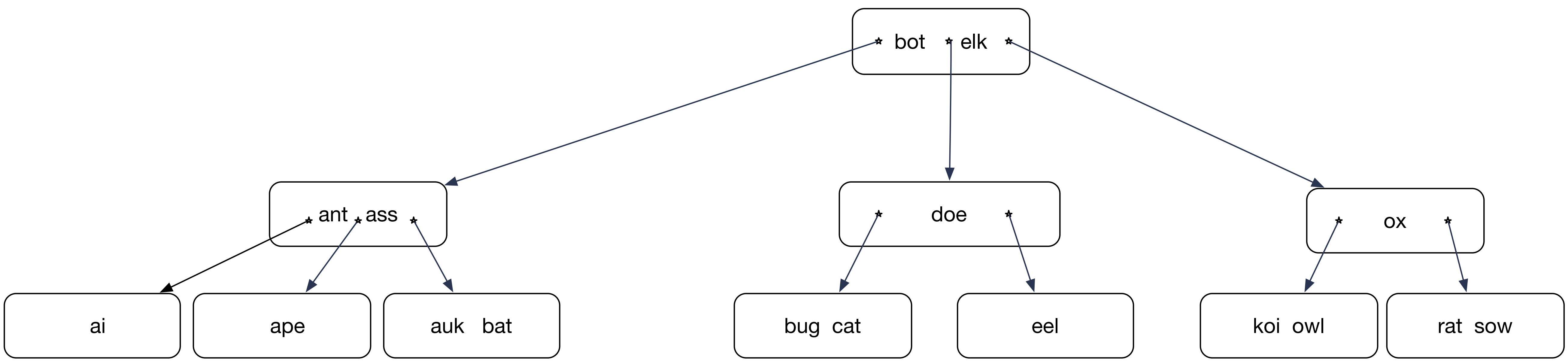
B-tree



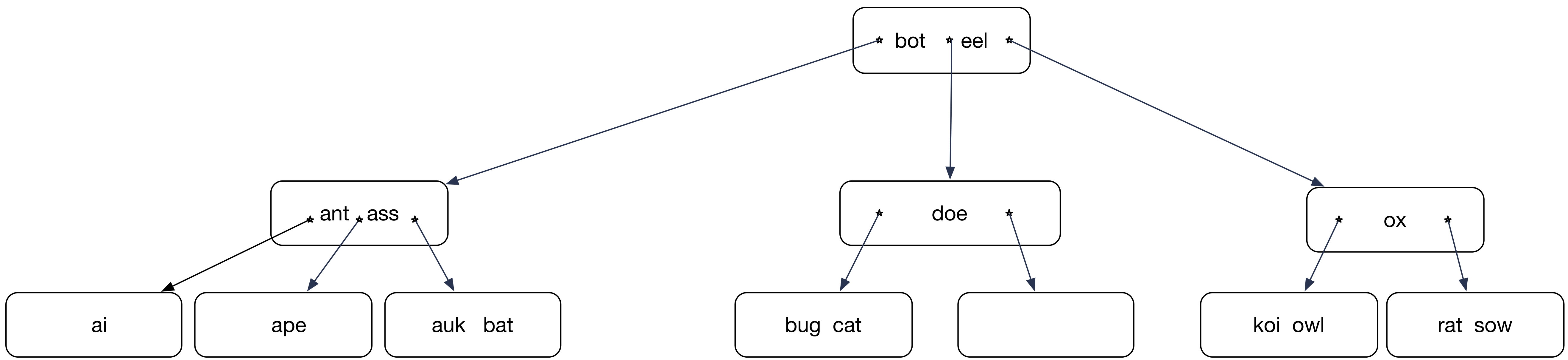
Switch predecessor, then delete from node

B-tree

Now delete "elk"

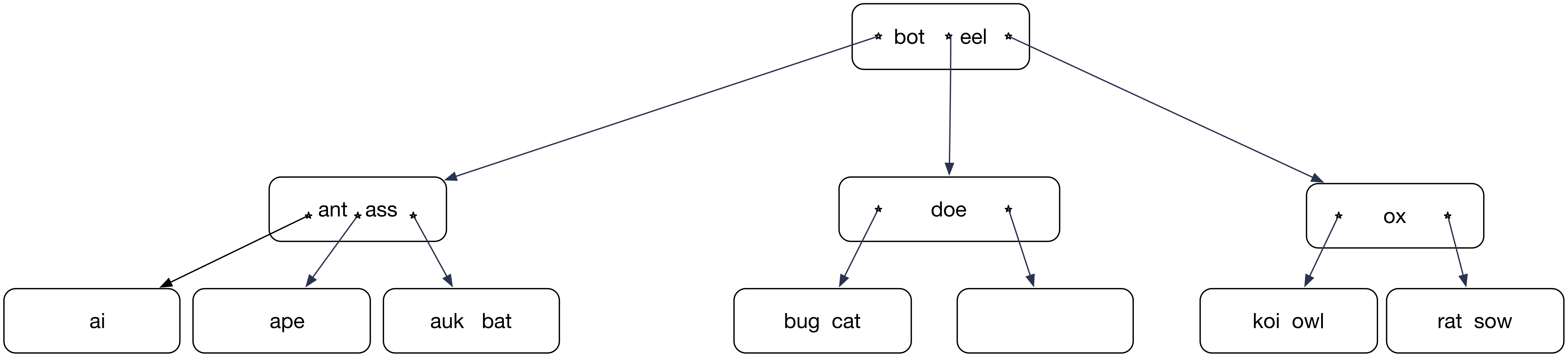


B-tree



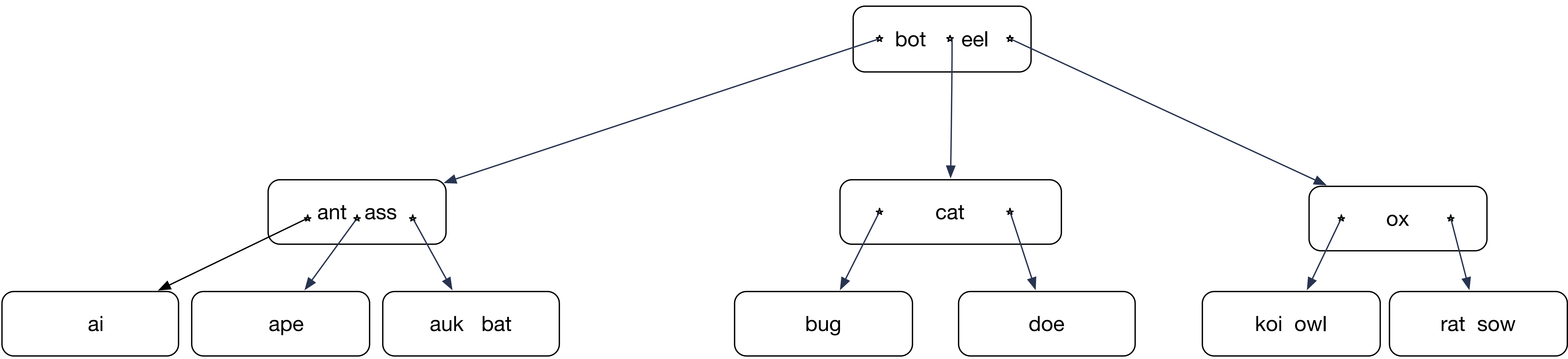
Results in an underflow

B-tree



Results in an underflow
But can rotate a key into the underflowing node

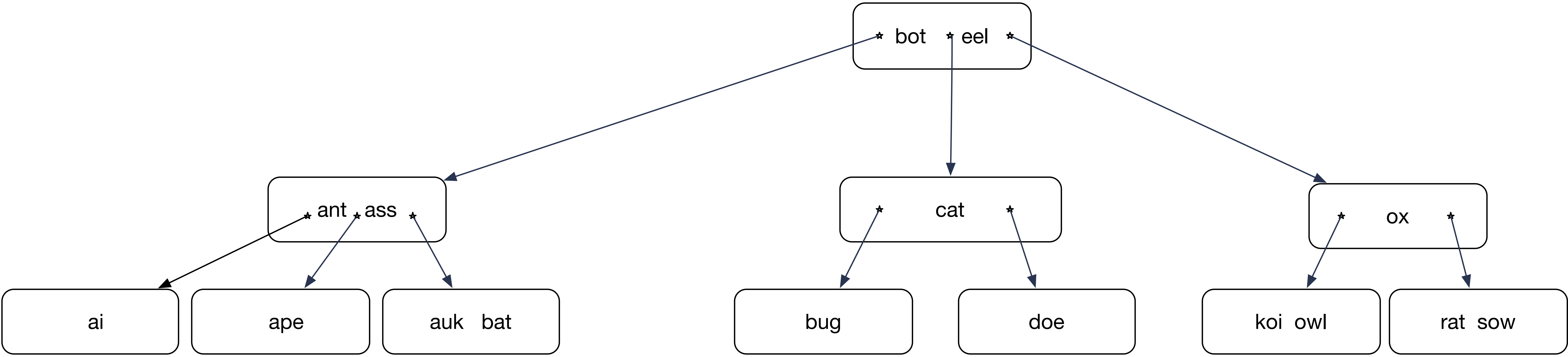
B-tree



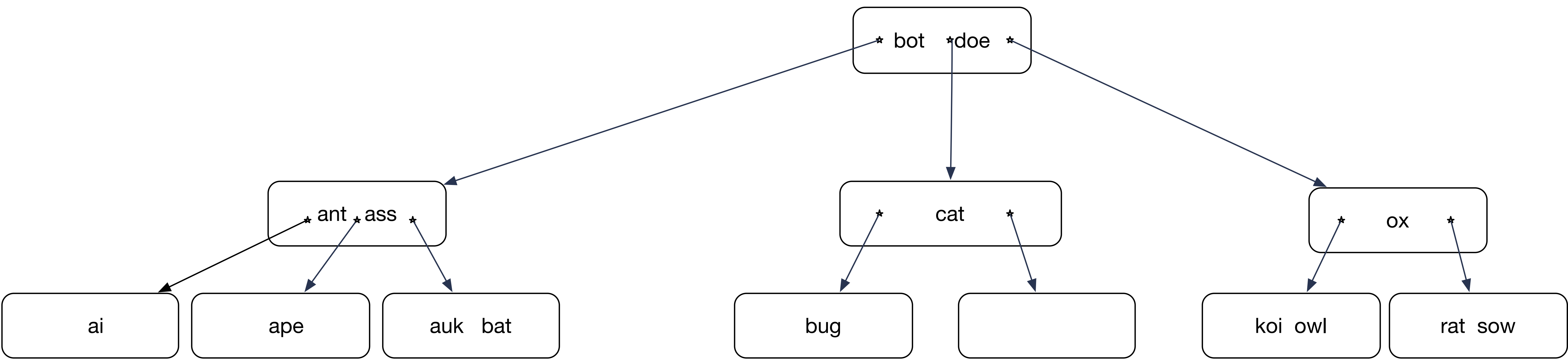
Result after left-rotation

B-tree

“Now delete “eel”

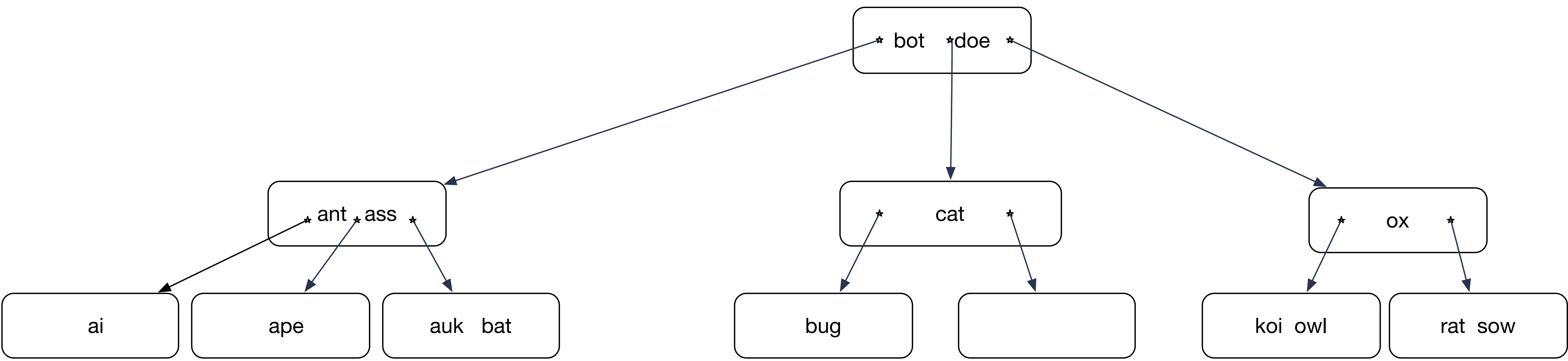


B-tree



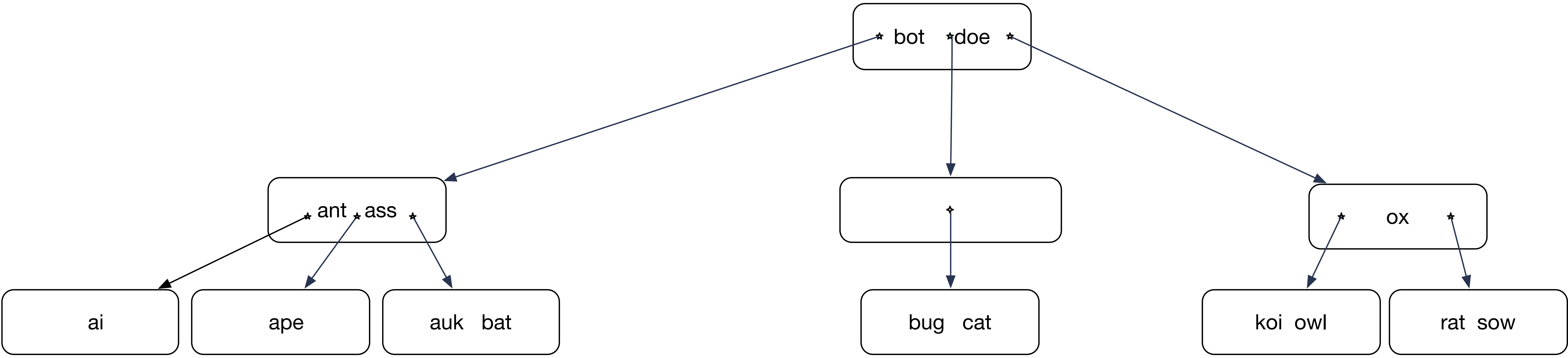
Interchange "eel" with its predecessor
Delete "eel" from leaf:
Underflow

B-tree



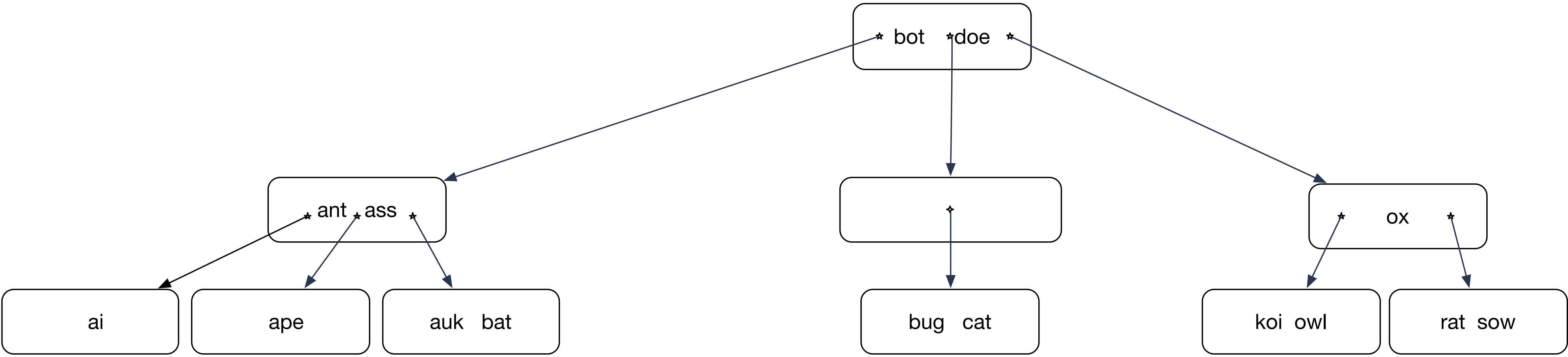
Need to merge

B-tree



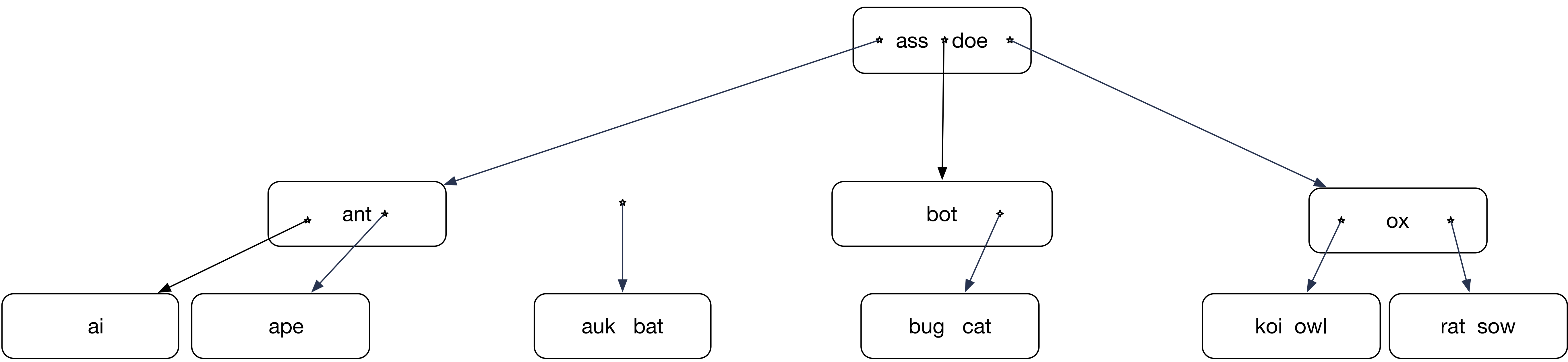
Merge results in another underflow
Use right rotate
(though merge with right sibling
is possible)

B-tree



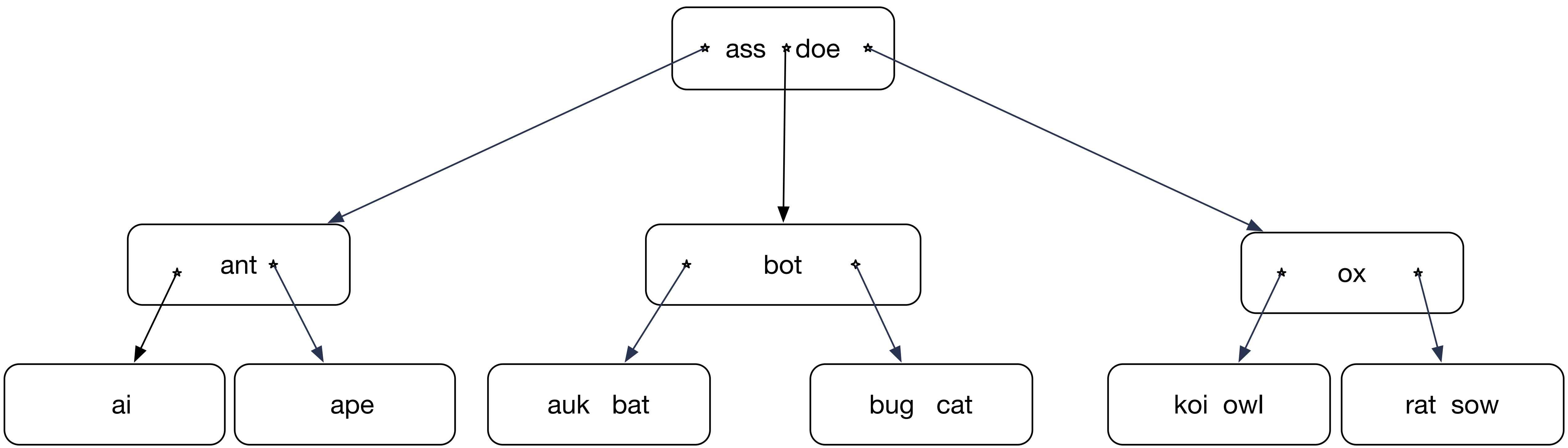
“ass” goes up, “bot” goes down
One node is reattached

B-tree



Reattach node

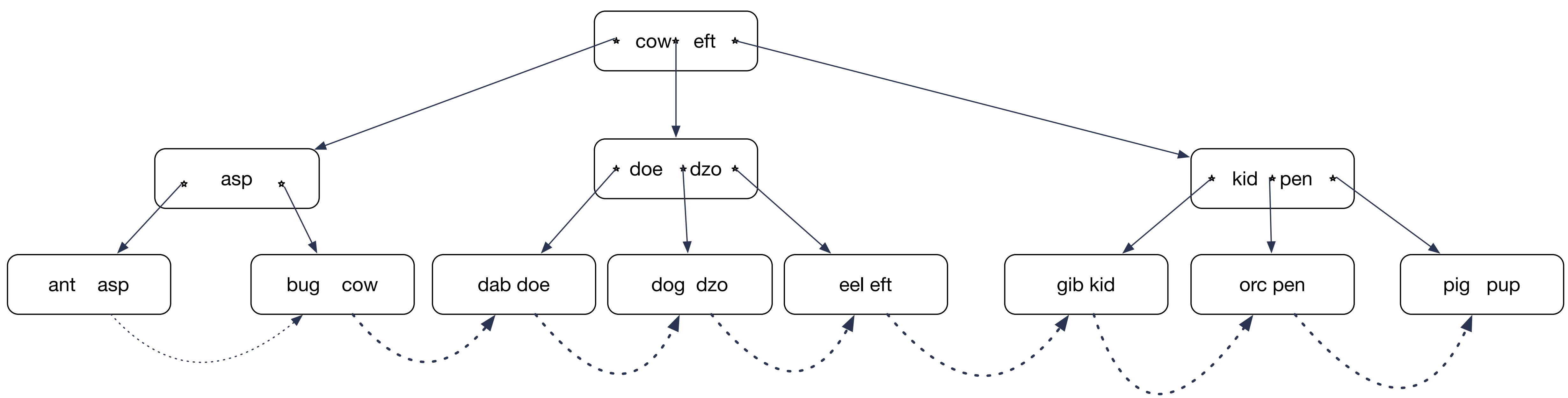
B-tree



In real life

- Use B+ tree for better access with block storage
  - Data pointers / data are only in the leaf nodes
  - Interior nodes only have keys as signals
  - Link leaf nodes for faster range queries.

B+ Tree



cow eft
asp
doe dzo
kid pen
ant asp
bug cow
dab doe
dog dzo
eel eft
gib kid
orc pen
pig pup

B+ Tree

- Real life B+ trees:
  - Interior nodes have many more keys (e.g. 100)
  - Leaf nodes have as much data as they can keep
  - Need few levels:
    - Fast lookup

Hashing

- Central idea of hashing:
  - Calculate the location of the record from the key
  - Hash functions:
    - Can be made indistinguishable from random function
      - SH3, MD5, …
      - Often simpler
        - ID modulo slots

Hashing

- Can lead to collisions:
    - Two different keys map into the same address
    - Two ways to resolve:
        - **Open Addressing**
            - Have a rule for a secondary address, etc.
        - **Chaining**
            - Can store more than one datum at an address

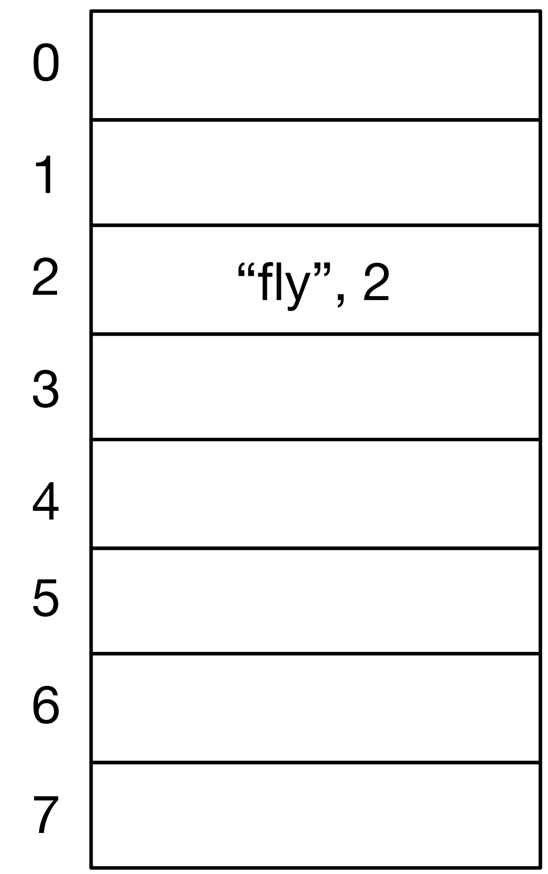
Hashing

- Open addressing example:
  - Linear probing: Try the next slot

Hashing Example

```
def hash(a_string):
    accu = 0
    i = 1
    for letter in a_string:
        accu += ord(letter)*i
        i+=1
    return accu % 8
```

Insert “fly”



Hashing Example

```
def hash(a_string):
    accu = 0
    i = 1
    for letter in a_string:
        accu += ord(letter)*i
        i+=1
    return accu % 8
```

0	
1	
2	“fly”, 2
3	“gnu”, 2
4	
5	
6	
7	

Insert “gnu”

`hash(“gnu”) —> 2`

Since spot 2 is taken, move to the next spot

Hashing Example

```
def hash(a_string):
    accu = 0
    i = 1
    for letter in a_string:
        accu += ord(letter)*i
        i+=1
    return accu % 8
```

0	
1	
2	“fly”, 2
3	“gnu”, 2
4	“hog”, 3
5	
6	
7	

Insert “hog”

`hash(“hog”) –> 3`

Since spot is taken, move to the next

Hashing Example


```
def hash(a_string):
    accu = 0
    i = 1
    for letter in a_string:
        accu += ord(letter)*i
        i+=1
    return accu % 8
```

Looking for “gnu”

```
hash("gnu") -> 2
```

0	
1	
2	“fly”, 2
3	“gnu”, 2
4	“hog”, 3
5	
6	
7	“pig”, 7

Try out location 2. Occupied, but not by “gnu”

Hashing Example


```
def hash(a_string):
    accu = 0
    i = 1
    for letter in a_string:
        accu += ord(letter)*i
        i+=1
    return accu % 8
```

0	
1	
2	“fly”, 2
3	“gnu”, 2
4	“hog”, 3
5	
6	
7	“pig”, 7

Looking for “gnu”

`hash(“gnu”) —> 2`

Try out location 3. Find “gnu”

Hashing Example


```
def hash(a_string):
    accu = 0
    i = 1
    for letter in a_string:
        accu += ord(letter)*i
        i+=1
    return accu % 8
```

0	
1	
2	“fly”, 2
3	“gnu”, 2
4	“hog”, 3
5	
6	
7	“pig”, 7

Looking for “ram”

```
hash(“ram”) -> 3
```

Look at location 3: someone else is there
Look at location 4: someone else is there
Look at location 5: nobody is there, so if it were in the dictionary, it would be there

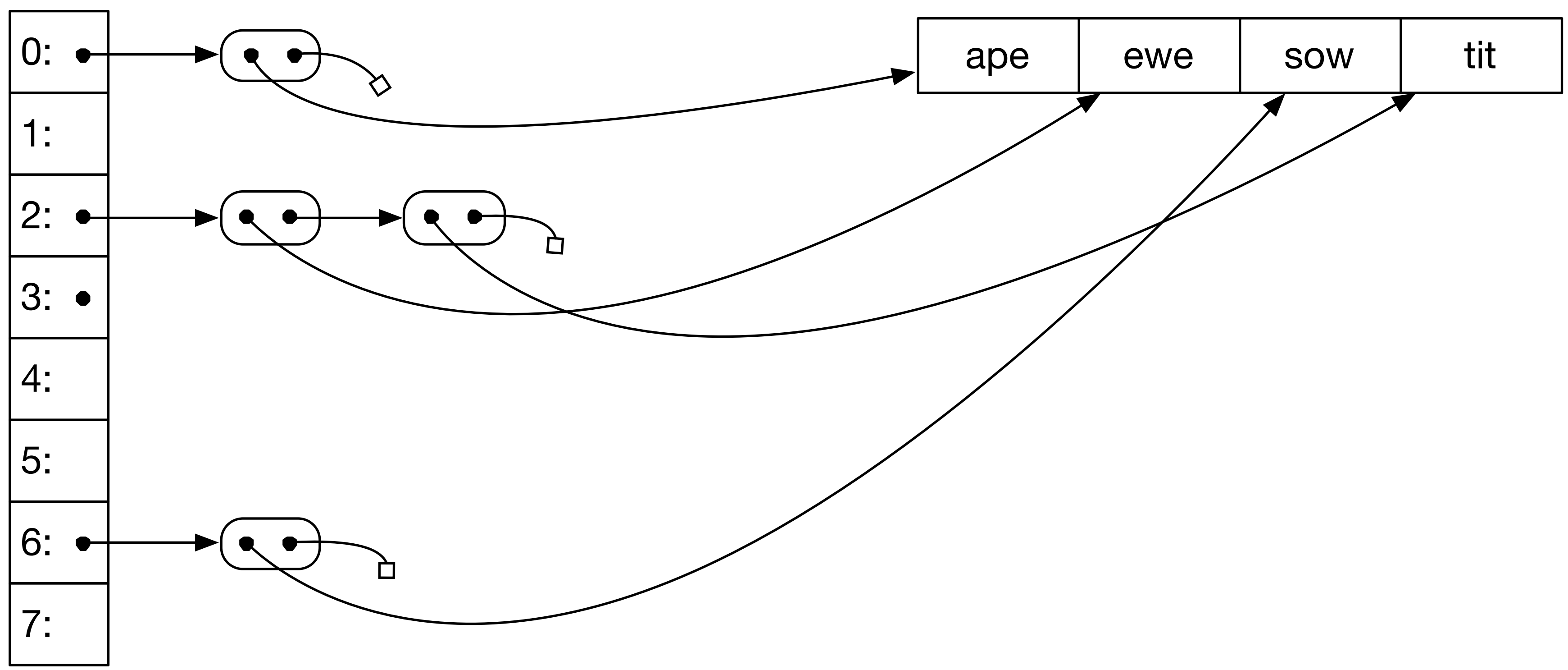
Hashing

- Linear probing leads to convoys:
  - Occupied cells tend to coalesce
- Quadratic probing is better, but might perform worse with long cache lines
- Large number of better versions are used:
  - Passbits
  - Cuckoo hashing
    - Uses two hash functions
  - Robin Hood hashing …

Hashing

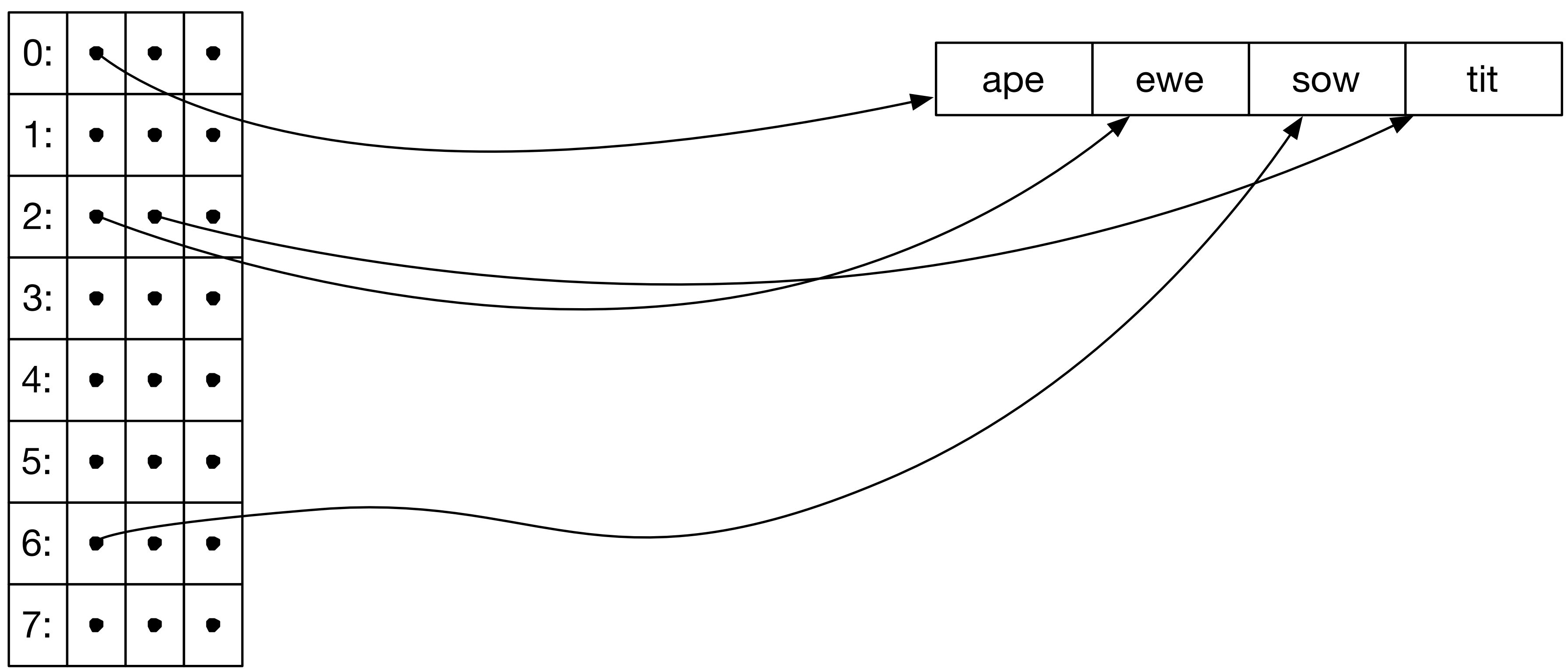
- Chaining
  - Keep data mapped to a location in a “bucket”
    - Can implement the bucket in several ways
      - Linked List

Hashing



Chaining Example with linked lists

Hashing Example



Chaining Example with an array of pointers (with overflow pointer if necessary)

Hashing Example

0:	ape	null	null
1:	null	null	null
2:	ewe	tit	null
3:	null	null	null
4:	null	null	null
5:	null	null	null
6:	sow	null	null
7:	null	null	null

Chaining with fixed buckets
Each bucket has two slots and a pointer to an overflow bucket

Hashing

- Extensible Hashing:
    - Load factor α = Space Used / Space Provided
    - Load factor determines performance
    - Idea of extensible hashing:
        - Gracefully add more capacity to a growing hash table

Linear Hashing

Linear Hashing

- Extensible Hashing:
  - Uses a lot of metadata to reflect history of splitting
    - But only splits buckets when they are needed
  - Linear Hashing
    - Splits buckets in a predefined order
    - Minimal meta-data
    - Sounds like a horrible idea, but …

Linear Hashing

- Assume a hash function that creates a large string of bits
  - We start using these bits as we extend the address space
  - Start out with a single bucket, Bucket 0
  - All items are located in Bucket 0

Bucket 0:

19, 28, 33

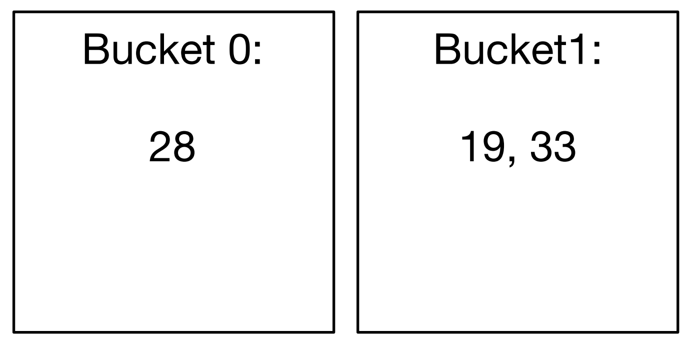
Items with keys 19, 28, 33

Linear Hashing

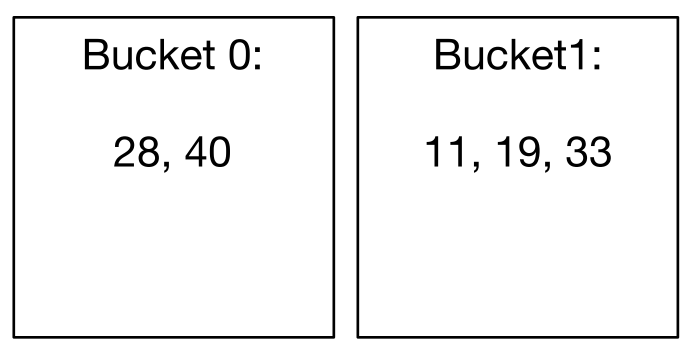
- Eventually, this bucket will overflow
  - E.g. if the load factor is more than 2
  - Bucket 0 splits
  - All items in Bucket 0 are rehashed:
    - Use the last bit in order to determine whether the item goes into Bucket 0 or Bucket 1
    - Address is $h_1(c) = c \pmod 2$

Linear Hashing

- After the split, the hash table has two buckets:



- After more insertions, the load factor again exceeds 2



Linear Hashing

- Again, the bucket splits.
  - But it has to be Bucket 0

Bucket 0:	Bucket1:	Bucket 2:
28, 40	11, 19, 33	

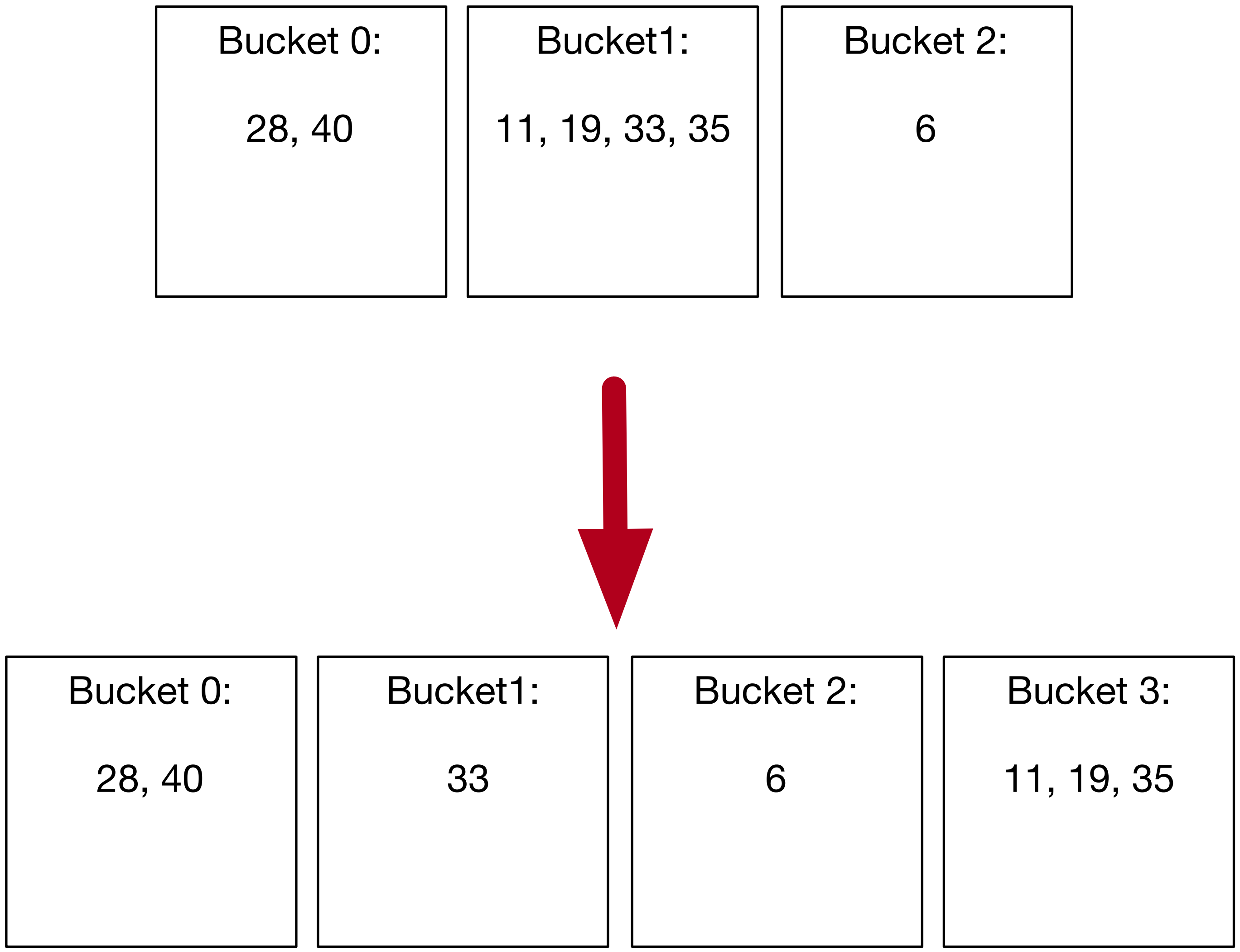
  - For the rehashing, we now use two bits, i.e.

$$h_2(c) = c \pmod 4$$

    - But only for those items in Bucket 0

Linear Hashing

- After some more insertions, Bucket 1 will split



Linear Hashing

- The state of a linear hash table is described by the number N of buckets
  - The level l is the number of bits that are being used to calculate the hash
  - The split pointer s points to the next bucket to be split
  - The relationship is

$$N = 2^l + s$$

    - This is unique, since always $s < 2^l$

Linear Hashing

- Addressing function
  - The address of an item with key c is calculated by

```
def address(c):
   a = hash(c) % 2**l
   if a < s:
      a = hash(c) % 2**(l+1)
   return a
```

  - This reflects the fact that we use more bits for buckets that are already split

Linear Hashtable Evolution

$$N = 1 = 2^0 + 0$$

```
def address(c):
   a = hash(c) % 2**l
   if a < s:
     a = hash(c) % 2**(l+1)
   return a
```

```
Number of buckets: 1
Split pointer: 0
Level: 0
```

Bucket 0:

19, 28, 33

Linear Hashtable Evolution

$N = 2 = 2^1 + 0$

```
Number of buckets: 2
Split pointer: 0
Level: 1
```

```
def address(c):
   a = hash(c) % 2**l
   if a < s:
     a = hash(c) % 2**(l+1)
   return a
```

Bucket 0:	Bucket1:
28	19, 33

```
Add items with hashes 40 and 11
This gives an overflow and we split Bucket 0
```

Linear Hashtable Evolution

$N = 3 = 2^1 + 1$

```
Number of buckets: 3
Split pointer: 1
Level: 1
```

```
def address(c):
    a = hash(c) % 2**l
    if a < s:
        a = hash(c) % 2**(l+1)
    return a
```

Bucket 0:	Bucket1:
28, 40	11, 19, 33

```
split Bucket 0
Create Bucket 2
Use new hash function on items in Bucket 0
```

Bucket 0:	Bucket1:	Bucket 2:
28, 40	11, 19, 33	

```
No items were moved
```

Linear Hashtable Evolution

$N = 3 = 2^1 + 1$

```
Number of buckets: 3
Split pointer: 1
Level: 1
```

```
def address(c):
   a = hash(c) % 2**l
   if a < s:
      a = hash(c) % 2**(l+1)
   return a
```

Bucket 0:	Bucket1:	Bucket 2:
28, 40	11, 19, 33	

Add items 6, 35

Bucket 0:	Bucket1:	Bucket 2:
28, 40	11, 19, 33, 35	6

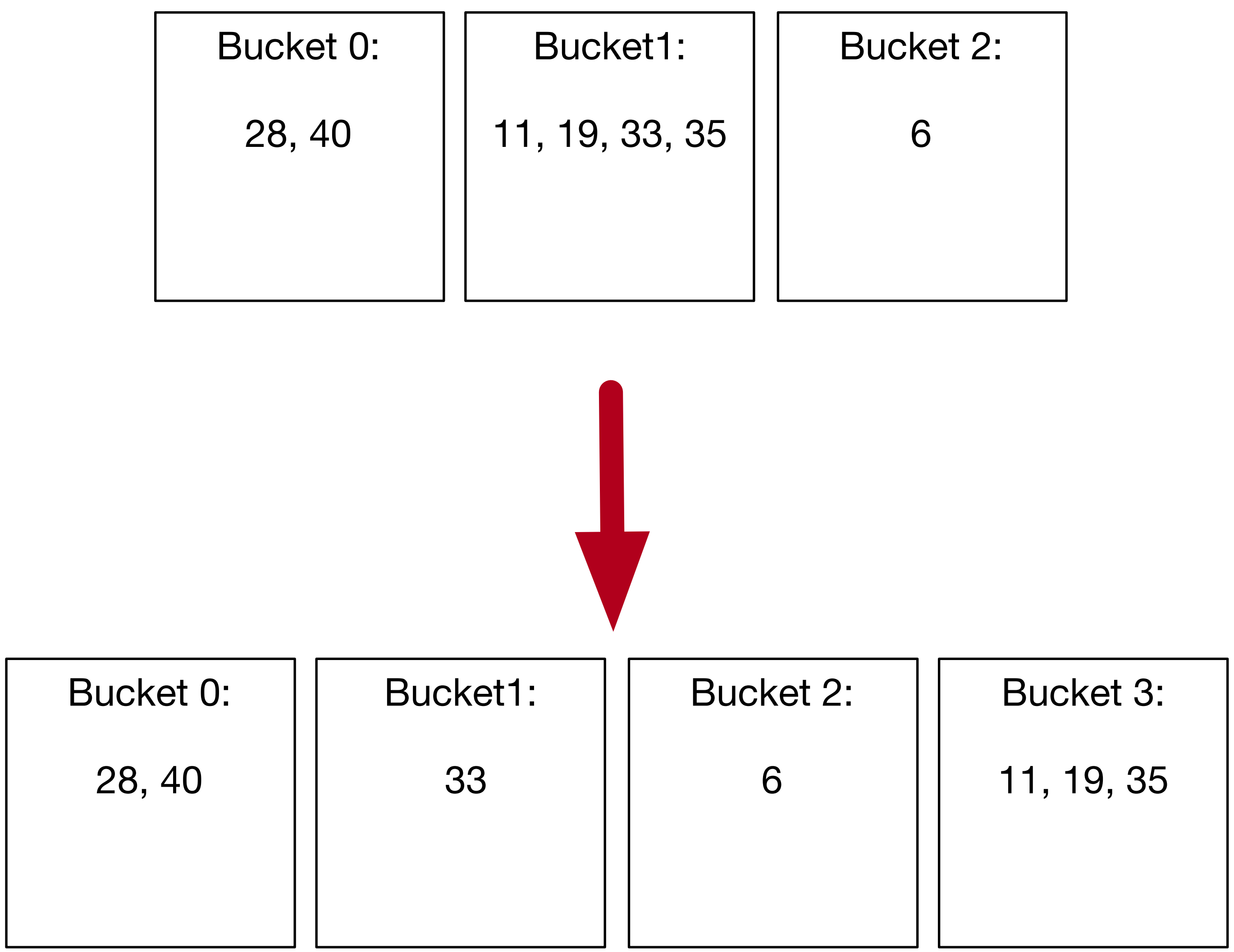
Because of overflow, we split Bucket 1

Linear Hashtable Evolution

$$N = 4 = 2^2 + 0$$

```
Number of buckets: 4
Split pointer: 0
Level: 2
```

```
def address(c):
    a = hash(c) % 2**l
    if a < s:
        a = hash(c) % 2**(l+1)
    return a
```



Linear Hashtable Evolution

$N = 4 = 2^2 + 0$

```
Number of buckets: 4
Split pointer: 0
Level: 2
```

```
def address(c):
   a = hash(c) % 2**l
   if a < s:
      a = hash(c) % 2**(l+1)
   return a
```

Bucket 0:	Bucket1:	Bucket 2:	Bucket 3:
28, 40	33	6	11, 19, 35

```
Now add keys 8, 49
```

Bucket 0:	Bucket1:	Bucket 2:	Bucket 3:
28, 40, 8	33, 49	6	11, 19, 35

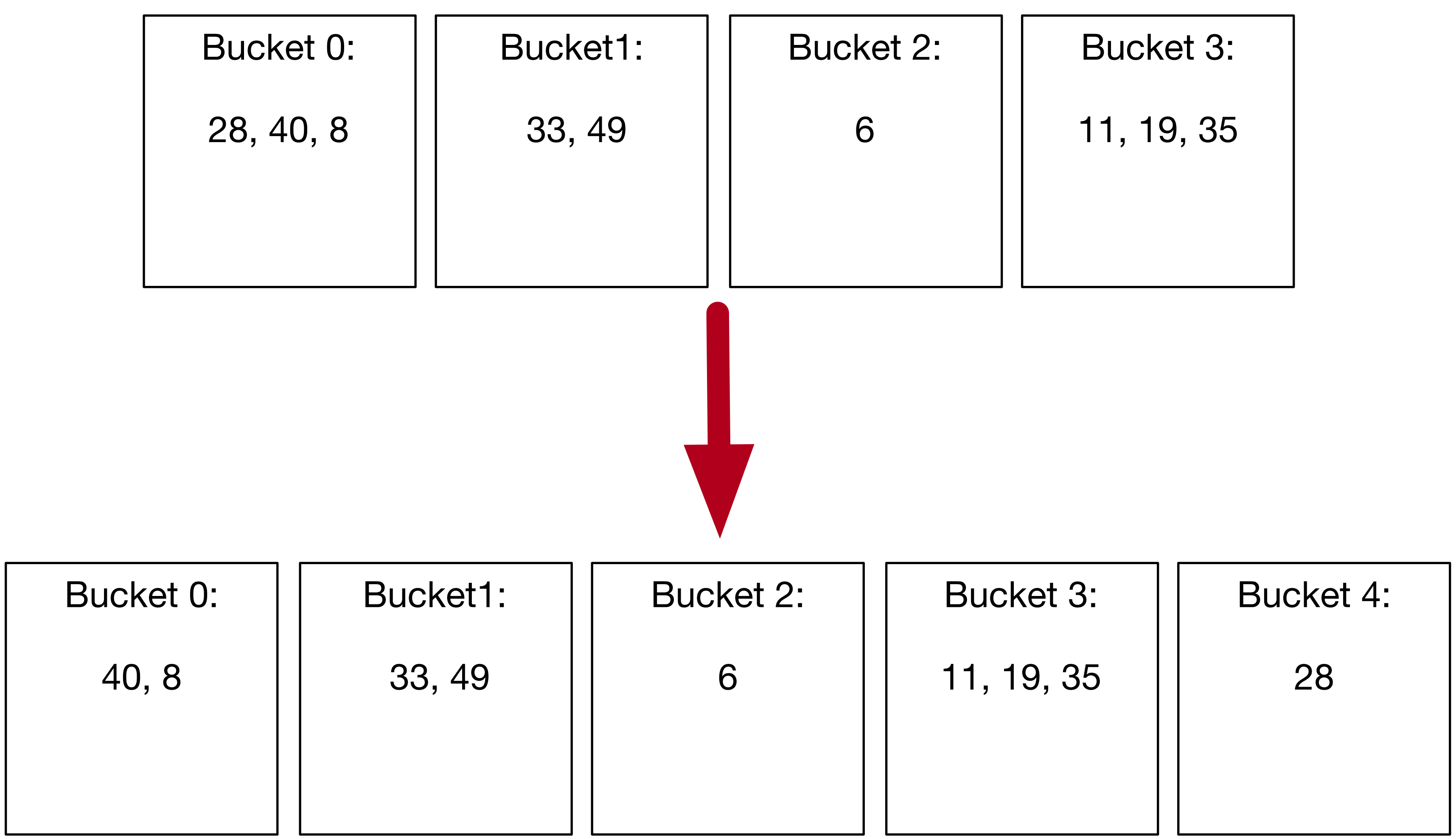
```
Creates an overflow!
Need to split!
```

Linear Hashtable Evolution

$N = 5 = 2^2 + 1$

```
Number of buckets: 1
Split pointer: 1
Level: 2
```

```
def address(c):
   a = hash(c) % 2**l
   if a < s:
      a = hash(c) % 2**(l+1)
   return a
```




```
Create Bucket 4.
Rehash Bucket 0.
```

Linear Hashtable Evolution

$N = 5 = 2^2 + 1$

```
Number of buckets: 5
Split pointer: 1
Level: 2
```

```
def address(c):
   a = hash(c) % 2**l
   if a < s:
      a = hash(c) % 2**(l+1)
   return a
```

Bucket 0:	Bucket1:	Bucket 2:	Bucket 3:	Bucket 4:
40, 8	33, 49	6	11, 19, 35	28

`Add keys 9, 42`

Bucket 0:	Bucket1:	Bucket 2:	Bucket 3:	Bucket 4:
40, 8	9, 33, 49	6, 42	11, 19, 35	28

`Creates an overflow!`
`Need to split!`

Linear Hashtable Evolution

$$N = 6 = 2^2 + 2$$

```
Number of buckets: 1
Split pointer: 2
Level: 2
```

```
def address(c):
    a = hash(c) % 2**l
    if a < s:
        a = hash(c) % 2**(l+1)
    return a
```

Bucket 0:	Bucket1:	Bucket 2:	Bucket 3:	Bucket 4:
40, 8	9, 33, 49	6, 42	11, 19, 35	28

Split

Bucket 0:	Bucket1:	Bucket 2:	Bucket 3:	Bucket 4:	Bucket 5:
40, 8	9, 33, 49	6, 42	11, 19, 35	28	

No item actually moved, but average load factor is now again under 2.

Linear Hashtable Evolution

$$N = 6 = 2^2 + 2$$

```
Number of buckets: 6
Split pointer: 2
Level: 2
```

```
def address(c):
   a = hash(c) % 2**l
   if a < s:
      a = hash(c) % 2**(l+1)
   return a
```

Bucket 0:	Bucket1:	Bucket 2:	Bucket 3:	Bucket 4:	Bucket 5:
40, 8	9, 33, 49	6, 42	11, 19, 35	28	

`add 5,10`

Bucket 0:	Bucket1:	Bucket 2:	Bucket 3:	Bucket 4:	Bucket 5:
40, 8	9, 33, 49	6, 10, 42	11, 19, 35	28	5

Linear Hashtable Evolution

$$N = 7 = 2^2 + 3$$

```
Number of buckets: 7
Split pointer: 3
Level: 2
```

```
def address(c):
   a = hash(c) % 2**l
   if a < s:
      a = hash(c) % 2**(l+1)
   return a
```

Bucket 0:	Bucket1:	Bucket 2:	Bucket 3:	Bucket 4:	Bucket 5:
40, 8	9, 33, 49	6, 10, 42	11, 19, 35	28	5

Bucket 0:	Bucket1:	Bucket 2:	Bucket 3:	Bucket 4:	Bucket 5:	Bucket 6:
40, 8	9, 33, 49	10, 42	11, 19, 35	28	5	6

Linear Hashtable Evolution

$$N = 7 = 2^2 + 3$$

```
Number of buckets: 7
Split pointer: 3
Level: 2
```

```
def address(c):
   a = hash(c) % 2**l
   if a < s:
      a = hash(c) % 2**(l+1)
   return a
```

Bucket 0:	Bucket1:	Bucket 2:	Bucket 3:	Bucket 4:	Bucket 5:	Bucket 6:
40, 8	9, 33, 49	10, 42	11, 19, 35	28	5	6

`add 92, 74`

Bucket 0:	Bucket1:	Bucket 2:	Bucket 3:	Bucket 4:	Bucket 5:	Bucket 6:
40, 8	9, 33, 49	10, 42, 74	11, 19, 35	28, 92	5	6

Linear Hashtable Evolution

$N = 8 = 2^3 + 0$

```
Number of buckets: 8
Split pointer: 0
Level: 3
```

```
def address(c):
    a = hash(c) % 2**l
    if a < s:
        a = hash(c) % 2**(l+1)
    return a
```

Bucket 0:	Bucket1:	Bucket 2:	Bucket 3:	Bucket 4:	Bucket 5:	Bucket 6:
40, 8	9, 33, 49	10, 42, 74	11, 19, 35	28, 92	5	6

Bucket 0:	Bucket1:	Bucket 2:	Bucket 3:	Bucket 4:	Bucket 5:	Bucket 6:	Bucket 7:
40, 8	9, 33, 49	10, 42, 74	11, 19, 35	28, 92	5	6	

Linear Hashtable Evolution

$$N = 8 = 2^3 + 0$$

```
Number of buckets: 8
Split pointer: 0
Level: 3
```

```
def address(c):
   a = hash(c) % 2**l
   if a < s:
      a = hash(c) % 2**(l+1)
   return a
```

Bucket 0:	Bucket1:	Bucket 2:	Bucket 3:	Bucket 4:	Bucket 5:	Bucket 6:	Bucket 7:
40, 8	9, 33, 49	10, 42, 74	11, 19, 35	28, 92	5	6	

`add 13, 54`

Bucket 0:	Bucket1:	Bucket 2:	Bucket 3:	Bucket 4:	Bucket 5:	Bucket 6:	Bucket 7:
	9, 33, 49	10, 42, 74	11, 19, 35	28, 92	5, 13	6, 54	

Linear Hashtable Evolution

$N = 9 = 2^3 + 1$

```
Number of buckets: 9
Split pointer: 1
Level: 3
```

```
def address(c):
    a = hash(c) % 2**l
    if a < s:
        a = hash(c) % 2**(l+1)
    return a
```

Bucket 0:	Bucket1:	Bucket 2:	Bucket 3:	Bucket 4:	Bucket 5:	Bucket 6:	Bucket 7:
	9, 33, 49	10, 42, 74	11, 19, 35	28, 92	5, 13	6, 54	

Bucket 0:	Bucket1:	Bucket 2:	Bucket 3:	Bucket 4:	Bucket 5:	Bucket 6:	Bucket 7:	Bucket 8:
	9, 33, 49	10, 42, 74	11, 19, 35	28, 92	5, 13	6, 54		40, 8

Linear Hashtable Evolution

$N = 9 = 2^3 + 1$

```
Number of buckets: 9
Split pointer: 1
Level: 3
```

```
def address(c):
   a = hash(c) % 2**l
   if a < s:
      a = hash(c) % 2**(l+1)
   return a
```

Bucket 0:	Bucket1:	Bucket 2:	Bucket 3:	Bucket 4:	Bucket 5:	Bucket 6:	Bucket 7:	Bucket 8:
	9, 33, 49	10, 42, 74	11, 19, 35	28, 92	5, 13	6, 54		40, 8

`add 1, 81`

Bucket 0:	Bucket1:	Bucket 2:	Bucket 3:	Bucket 4:	Bucket 5:	Bucket 6:	Bucket 7:	Bucket 8:
	1, 9, 33, 49, 81	10, 42, 74	11, 19, 35	28, 92	5, 13	6, 54		40, 8

Linear Hashtable Evolution

$$N = 10 = 2^3 + 2$$

```
Number of buckets: 10
Split pointer: 2
Level: 3
```

```
def address(c):
    a = hash(c) % 2**l
    if a < s:
        a = hash(c) % 2**(l+1)
    return a
```

Bucket 0:	Bucket1:	Bucket 2:	Bucket 3:	Bucket 4:	Bucket 5:	Bucket 6:	Bucket 7:	Bucket 8:	Bucket 9:
	1, 33, 49, 81	10, 42, 74	11, 19, 35, 67, 99	28, 92	5, 13	6, 54	39	40, 8	9

Bucket 0:	Bucket1:	Bucket 2:	Bucket 3:	Bucket 4:	Bucket 5:	Bucket 6:	Bucket 7:	Bucket 8:	Bucket 9:	Bucket 10:
	1, 33, 49, 81		11, 19, 35, 67, 99	28, 92	5, 13	6, 54	39	40, 8	9	10, 42, 74

Linear Hashing

- Observations:
  - Buckets split in fixed order
    - 0, 0,1, 0, 1, 2, 3, 0, 1, 2, 3, 4, 5, 6, 7, 0, 1, 2, …, 15, 0, …
    - Address calculation is modulo 2^l, i.e. the *l* least significant bits
    - Buckets 0, 1, …, s-1 and 2***l*, 2***l*+1, … *N*-1 are already split, they have on average half the size of the buckets *s*, *s*+1, …, 2***l*.

Linear Hashing

- Observations:
  - An overflowing bucket is not necessarily split immediately
  - Sometimes, a split leaves all keys in the splitting bucket or moves them all to the new bucket
- On average, a bucket will have α items in them