Laboratory 2

1. Define a function of *n* that calculates $\sum_{i=1}^{n} \frac{i}{i^2+1}$.

2. Find the number of integers between 1 and $11 \cdot 13 \cdot 17 \cdot 19$ that satisfy the congruences $x^2 \equiv 3 \pmod{11}$, $x^3 \equiv 8 \pmod{13}$, $x^4 \equiv 13 \pmod{17}$, $x^5 \equiv 9 \pmod{19}$.

3. Find the smallest integer *n* such that $\sum_{i=1}^{n} \frac{i}{i^2+1}$ is greater than 4. Do not use the function defined in Exercise 1 other than for checking.

4. A simple approximate integration formula for the integral $\int_a^b f(x)dx$ uses the average of a function f at the *n* points $a + \delta/2, a + \delta/2 + \delta, a + \delta/2 + 2\delta, \ldots,$ $a + \delta/2 + (n-1)\delta(= b - \delta/2)$ where $\delta = \frac{b-a}{n}$. Implement this as a function `appint(f, a, b, n)` that returns $\frac{(b-a)}{n} \sum_{i=0}^{n-1} f(a + \frac{\delta}{2} + i \cdot \delta)$

5. An approximation formula for the derivative of a function uses a small value $\delta = 0.000001$ and gives $f'(x) \approx \frac{f(x+\delta) - f(x-\delta)}{2\delta}$. Implement this as a function `appder(f, x)`.