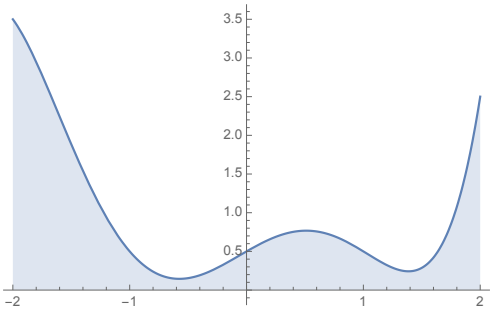
Module 10 Activities: More on Functions

1. Write a function of two arguments, *x* and *y,* that returns the value $\frac{x^2+y^2-1}{x^2+y^2+1}$.

2. Write a fruitless function of two variables, *n*, and *m* that prints out *n* plus signs followed by *m* minus signs.

3. Write a function that calculates the following function: $b(x)=\frac{16x(\pi-x)}{5\pi^2-4x(\pi-x)}$. The constant π is obtained by importing math (import math) and then using math.pi. This is Bhaskara I's sine approximation function from around 650. Compare it with the sine function for $x \in \{-\pi/2, -\pi/3, -\pi/4, 0, \pi/4, \pi/3, \pi/2\}$.

4. The *integral* of a function between *a* and *b* is the area between the graph of the function, the x-axis, and the lines through *a* and *b.* It is denoted as $\int_a^b f(x)dx$ and a great part of Mathematical Analysis is devoted to studying integrals. You do not really need to understand integrals, just implement some formula about them. There are many ways to approximate the integral of a function numerically.



(a) Three-point Trapezoid Rule:

$$\int_a^b f(x)dx \approx \frac{b-a}{4}\left(f(a)+2f(\frac{a+b}{2})+f(b)\right).$$

Create a function of *a, b*, and *f* that calculates the approximate integral using this formula. Then calculate the approximations for the following integrals and compare with their exact values.

a) $\int_0^1 x^2dx = 1/3$ (Hint: define a function square)

b) $\int_0^1 \exp(x)dx = 1.71828$ (Hint: import math and use math.exp)

c) $\int_0^1 \sqrt{x}dx = 0.666667$ (Hint: import math and use math.sqrt)

Solution:

We have a function of three arguments, a, b, and func:

```
def simple_trap(a, b, func):
```

To implement the formula we build the sum step by step. We then return the sum multiplied by $(b-a)/4$. The whole function is

```
def simple_trap(a, b, func):
    accu = func(a)
    mid = (a+b)/2
    accu += 2*func(mid)
    accu += func(b)
    return (b-a)*accu/4
```

To try it out, we import math and define the square function. We then call the function as

```
print(simple_trap(0,1,sq))
print(simple_trap(0,1,math.exp))
print(simple_trap(0,1,math.sqrt))
```

(b) Four-point Trapezoid Rule:

$$\int_a^b f(x)dx = \frac{(b-a)}{6}\left(f(a) + 2f(a+\frac{b-a}{3}) + 2f(a+2\frac{b-a}{3}) + f(b)\right)$$

Then try the formula out on the same integrals as before.