Sample Midterm 2

1. Use the format instruction to print out the table for the logistic function

$$x \mapsto L(x) = \frac{2}{1 + \exp(-2(x - 1))}$$

for x = 0, 0.1, 0.2, ...   3.9, 4.0. The function values should be given with three digits accuracy. You need to repeat the following partial print-out.

```
 x       logistic
-------------------
0.0       0.238
0.1       0.284
0.2       0.336
0.3       0.396
0.4       0.463
0.5       0.538
0.6       0.620
```

2. Write a function that opens up a text file in mode "latin-1" and then counts the frequency of characters following immediately the character 'a'. For example, if a line of the text file reads "Alice was beginning to get very tired of sitting by her sister on the", then we count only 's' because there is only one 'a'. After reading the next line "bank, and of having nothing to do. Once or twice she had peeped into the", we count 'n' and 'd'. Then write a program that neatly displays the frequency of the small letters of the alphabet following an 'a' in "alice.txt".

3. Write a function that calculates the sum $\displaystyle\sum_{i=1,\ i \neq 2 \ (\mathrm{mod}\ 3)}^{n} \frac{i}{i + 5}.$

   The expression means that you skip over indices i that have remainder 2 when divided by 3. With other words, the sum starts out as

$$\frac{1}{6} + \frac{3}{8} + \frac{4}{9} + \frac{6}{10} + \frac{7}{11} + \frac{9}{12} + \ldots.$$

   You can see that we skipped over i=2, 5, 8, ...

4. Use list comprehension to create a list of all squares of numbers between 1 and 1000 that have remainder 1 when divided by 4.

5. Use set comprehension to create the set of all numbers between 1 and 1000 that are divisible by a square.

6. Pythagorean triples are triples of integer numbers a, b, and c such that $a^2 + b^2 = c$. Euclid's formula generates all important Pythagorean triples. Given two positive numbers m and n that are both odd, the Pythagorean triple is given by $a = mn$, $b = \dfrac{n^2 - m^2}{2}$, and $c = \dfrac{m^2 + n^2}{2}$. Use set comprehension to create all Pythagorean triples with Euclid's formula when $1 \le m < n < 30$. (Hint: `range(1,51,2)` generates all odd numbers 1, 3, 5, ... between 1 and 50.)