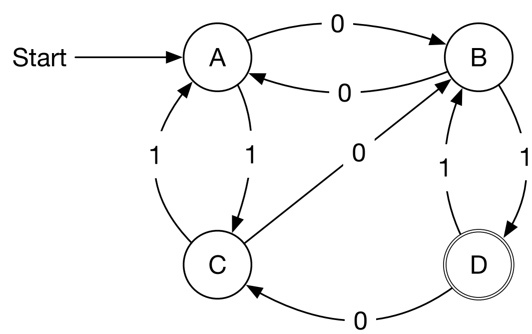
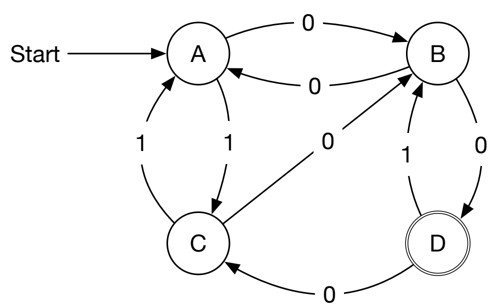
Homework 1

Due February 3, 2020

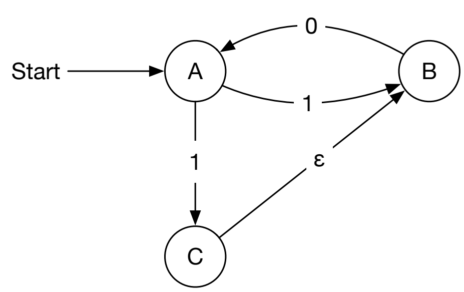
1. Given the following DFA, determine $\hat{\delta}(A, 01001)$ by applying the recursive definition.



2. Translate the following Non-deterministic Finite Automaton (NFA) into a Deterministic Finite Automaton (DFA). Give the set of states, the transition table, the starting state and the accepting state(s).

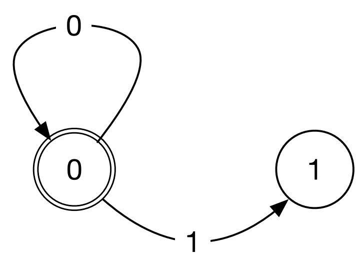


3. Translate the following NFA with ϵ-moves into a DFA. Give the set of states, the transition table, the starting state and the accepting state(s).

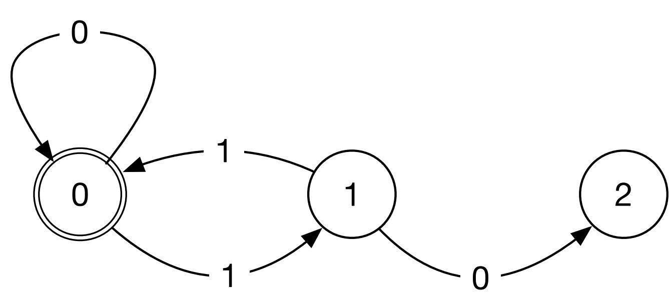


4. Create a DFA that accepts a binary string (with many letters) if and only if the corresponding integer is divisible by 5. Create a diagram.

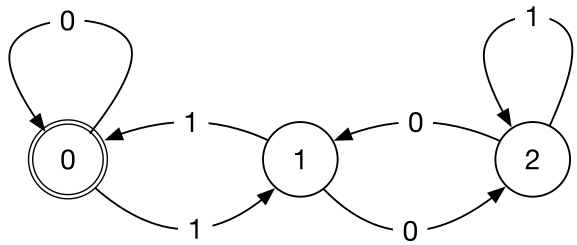
Hint: We create a DFA that accepts a binary string if and only if the corresponding integer is divisible by 3. We label the states by 0, 1, and 2. The DFA is such that if the binary string processed so far is congruent to i mod 3, then the DFA is in state i. Assume that we are currently in State 0. If *n* is the number represented by the part of the binary string processed so far, then $n \equiv 0 \pmod 3$. If the next digit is '0', then the next number is $2n$ as adding a 0 to a binary integer just multiplies with 2. Since $n = 3x$ with some integer $x \in \mathbb{N}_0$, $2n = 2 \cdot 3 \cdot x \equiv 0 \pmod 3$. If the next binary number is 1, then the new value after processing the 1 is $2n + 1 = 2 \cdot 3 \cdot x + 1 \equiv 1 \pmod 3$. This gives us



Now assume that we are in State 1. Then the processed string corresponds to an integer $n \equiv 1 \pmod 3$ and therefore, $n = 3 \cdot x + 1$ for an $x \in \mathbb{N}_0$. If the next binary digit is 0, then the value after processing this zero is $2n = 2 \cdot (3 \cdot x + 1) = 6x + 2 \equiv 2 \pmod 3$, i.e. the processing of 0 corresponds to a transition to State 2. If the next binary digit is however1, then the value afterwords is $2n + 1 = 2 \cdot (3x + 1) + 1 = 6x + 3 = (2x + 1)3 \equiv 0 \pmod 3$ and so we have a state transition to State 0:



If we are in State 2, then the number seen so far can be written as $3x + 2$ with an integer $x \in \mathbb{N}_0$. If the next digit is zero, then the value after processing the next digit is $2n$ and since $2n = 2(3x + 2) = 6x + 4 = 3(2x + 1) + 1 \equiv 1 \pmod 3$, processing the 0 corresponds to a transition to State 1. If the next digit is one, then the value becomes $2n + 1$ and since we have $2n + 1 = 2(3x + 2) + 1 = 6x + 4 + 1 = 6x + 5 = 3(2x + 1) + 2 \equiv 2 \pmod 3$, processing the next digit corresponds to a state transition to itself.



Finally, we need to determine the starting state. If the first digit is a 0, then we end up in State 0, If the second digit is a 1, then we end up in State 1. Therefore, we can make State 0 the accepting state using the mathematical convention that the empty string corresponds to 0.