Worksheet 1 Solutions:

(1) Diagram (a) does not represent a finite state machine. In State A, there is a transition on 'a' to both State B and State C. Therefore, the transition function is not a function!

Diagram (b) does describe a finite state machine with transition table on the right. Because we can write a transition table, the transition function δ is indeed a function. We can check the other components. The set of state is finite and equals $\{A, B, C\}$. There is one starting state A. The set of accepting states is $\{C\}$.

	A	**B**	**C**
a	B	A	B
b	B	A	B
c	C	C	B

(2) The input alphabet is $\{a, c, g, t\}$. The transition table is on the right.

$\hat{\delta}(C, ccat) = B$ by chasing arrows. We also chase arrows to determine whether strings are accepted. If we process $aatttgttg$, we start in A and then go to C (on the second a), to B on t, to B on t, to B on t, to A on g, to C on t, to B on t, to g on A, which is the accepting state. Processing $aattttt$ gets us from A to B to C to B to B to B to B to B, which is **not** an accepting state.

	A	**B**	**C**
a	B	C	A
c	A	A	A
g	A	A	A
t	C	B	B

(3) We have

$$
\begin{aligned}
\hat{\delta}(C,110110) &= \delta(\hat{\delta}(C,11011),0) \\
&= \delta(\delta(\hat{\delta}(C,1101),1),0) \\
&= \delta(\delta(\delta(\hat{\delta}(C,110),1),1),0) \\
&= \delta(\delta(\delta(\delta(\hat{\delta}(C,11),0),1),1),0) \\
&= \delta(\delta(\delta(\delta(\delta(\hat{\delta}(C,1),1),0),1),1),0) \\
&= \delta(\delta(\delta(\delta(\delta(C,1),1),0),1),1),0) \\
&= \delta(\delta(\delta(\delta(D,1),0),1),1),0) \\
&= \delta(\delta(\delta(E,0),1),1),0) \\
&= \delta(\delta(\delta(C,1),1),0) \\
&= \delta(\delta(D,1),0) \\
&= \delta(E,0) \\
&= C.
\end{aligned}
$$

(4) The input alphabet of the new deterministic automaton is $\{0,1\}$. If we start in A, we only have one transition on 0 to B, but then an ϵ-transition from B to C (which we can but do not have to take), therefore in the new deterministic automaton:

$$\delta(\{A\},0) = \{B, C\}\ .$$

$$\delta(\{A\},1) = \{\}\ .$$

Similarly, if we start in B, we can take an ϵ-transition before or after another transition. Therefore:

$$\delta(\{B\},0) = \{B, C\}$$

and

$$\delta(\{B\},1) = \{D\}\ .$$

Finally, there is no transition out of C on 0, and only one possibility to transition on 1, so that

$$\delta(\{C\},0) = \{\},$$

$$\delta(\{C\},1) = \{D\}\ .$$