Midterm preparation

(1) Show the loop invariant for Bubblesort given in the comments of the following pseudo-code

```
BUBBLESORT(A)
    for i = 1 to A.length - 1
    // the elements A[1]…A[i-1] are the i-1 smallest
    // elements of the array in order
        for j = A.length downto i + 1
            if A[j] < A[j - 1]
                exchange A[j] with A[j - 1]
```

(2) Give the asymptotic comparison (o, Θ, Ω) of the following pairs of functions of n.

    (a) $n \exp(n), \quad \exp(n^2)$

    (b) $\dfrac{n}{\sqrt{n+1}}, \quad \log(\sqrt{(n)})$

(3) Apply the Master Theorem on the following recurrences, if possible. Indicate when and why the MT does not apply.

    (a) $T(n) = 27T(n/3) + n^2 \log(n)$
    (b) $T(n) = 2T(n-1) + 1$
    (c) $T(n) = 2T(n/4) + \sqrt{n}$

(4) Given n numbers, we want to extract the top 3 (maximum three elements). Give a recurrence for the following (bad) algorithm: If $n \leq 3$, return all elements. If $n \leq 5$, use any constant time algorithm to find the top 3. If $n > 5$, use a linear time algorithm to find the median of the numbers. Then partition around the median. Because there are at least six numbers, the upper part of the partition contains the top 3. Recursively, apply the algorithm to the upper part of the partition.