Homework 12 Solutions

Problem 1:

We create $N \times N$ Boolean variables $v_{i,j}$ that are true if there is a knight on square (i, j) and false else. A knight moves by two rows up or down and one column left or right, or by moving one row up or down and two columns left or right. Thus, if $V_{i,j} = 1$, then $V_{i+1,j+2}$, $V_{i+1,j+2}$, $V_{i-1,j+2}$, $V_{i-1,j+2}$, $V_{i+2,j+1}$, $V_{i+2,j-1}$, $V_{i-2,j+1}$ and $V_{i-2,j-1}$ have to be false.

Since $A \implies \neg B$ is equivalent $\neg A \vee \neg B$, we can implement a knights assignment using 2-literal clauses.

In order to use Pycosat, we need to enumerate the variables starting with 1. We also want to display the results. Thus:

```
N=8

def v(i, j):
    return N * i + j + 1

def coordinates(x):
    x = x-1
    return (x//N, x%N)
```

Problem 2:

1. $v_{1,1} = 1, v_{n,n} = 1$
2. $\forall j \neq 1,n : \; v_{i,2} \vee v_{i,3} \vee \ldots \vee v_{i,n-1} \vee v_{i,n}$
3. $\forall k \, \forall i, j, i \neq j : \; \neg(v_{k,i} \wedge v_{k,j})$ where the clause in CNF is $\neg v_{k,i} \vee \neg v_{k,j}$
4. $\forall l \in \{2,3,\ldots,n-1\} : \; v_{2,l} \vee v_{3,l} \vee \ldots \vee v_{n-2,l} \vee v_{n-1,l}$
5. $\forall l \in \{2,3,\ldots,n-1\} \; \forall i, j \in \{2,3,\ldots,n-1\}, i \neq j : \; v_{i,l} \Rightarrow \neg v_{j,l}$ where the clause in CNF is $\neg v_{i,l} \vee \neg v_{j,l}$
6. $\forall j \in \{1,\ldots n\} \forall k \in \{1,\ldots n\} \forall l \in \{1,\ldots,n-1\} : (j,k) \notin E : \; v_{j,l} \Rightarrow \neg v_{k,l+1}$