Homework 8 Solutions

Problem 1:

To determine the optimal solution, we start in the last row and last column. Since the value there is equal to the cell in the second-last column and last row, we know that item G is not used in the optimal solution we are generating. We consult the last row in the column for F. There we find that the cell to the left has a smaller value. This means that item F has been included. We can find the cell from which we obtained the solution by moving up the E-column by the weight of F, which is two. We proceed, traversing the cells highlighted in the table below. We now can read of an optimal solution. (It is possible that there is another optimal solution resulting from a tie.) It is {A, B, C, E, F}.

Capacity	{A}	{A,B}	{A,B,C}	{A.B,C,D}	{A,B,…,E}	{A,B,…,F}	{A,B,…,G}
0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0
2	0	0	0	0	3	3	3
3	0	0	6	6	6	6	6
4	0	8	8	8	8	8	8
5	11	11	11	11	11	11	11
6	11	11	11	11	11	11	11
7	11	11	14	14	14	14	14
8	11	11	17	17	17	17	17
9	11	19	19	19	19	19	19
10	11	19	19	19	20	20	20
11	11	19	19	22	22	22	22
12	11	19	25	25	25	25	25
13	11	19	25	25	25	25	25
14	11	19	25	25	28	28	28
15	11	19	25	30	30	30	30
16	11	19	25	30	30	31	31

Problem 2:

Capacity	{A}	{A,B}	{A,B,C}	{A.B,C,D}	{A,B,…,E}	{A,B,…,F}
0	0	0	0	0	0	0
1	0	0	0	0	0	2
2	0	0	0	0	5	5
3	0	0	0	9	9	9
4	0	0	14	14	14	14
5	0	30	30	30	30	30
6	0	30	30	30	30	32
7	28	30	30	30	35	35
8	28	30	30	39	39	39
9	28	30	44	44	44	44
10	28	60	60	60	60	60
11	28	60	60	60	60	62
12	28	60	60	60	65	65
13	28	60	60	69	69	69
14	56	60	74	74	74	74
15	56	90	90	90	90	90
16	56	90	90	90	90	92

The solution proceeds similarly to before, but if the value is not copied from the previous column, then we need to check for multiple insertions. In this case, in the F-column, we did not inherit the value from the preceding column, and we need to investigate how we could have gotten it. We therefore need to go up 1 row, 2 rows, 3 rows, etc, since the weight of item F is only 1. When we come to the B-column, we need to investigate whether we came from row 10, row 5, or row 0, as the weight of the B-item is 5. The optimal solution according to the colored tableau is one F and three B items.

Problem 3:

Instead of using the tableau, we proceed step by step. Label the matrices A to F. First, we look at the costs of multiplying to adjacent matrices:

AB 150 multiplications

BC 450 multiplications

CD 270 multiplications
DE 189 multiplications
EF 210 multiplications

Now we look at all ways to multiply three adjacent matrices:

$(AB)C$ $\underline{3 \times 10 \times 9 + 150 = 420}$ <u>multiplications</u>
$A(BC)$ $3 \times 5 \times 9 + 450 = 585$ multiplications

$(BC)D$ $5 \times 9 \times 3 + 450 = 585$ multiplications
$B(CD)$ $\underline{5 \times 10 \times 3 + 270 = 420}$ <u>multiplications</u>

$(CD)E$ $\underline{10 \times 3 \times 7 + 270 = 480}$ <u>multiplications</u>
$C(DE)$ $10 \times 9 \times 7 + 270 = 900$ multiplications

$(DE)F$ $9 \times 7 \times 10 + 189 = 819$ multiplications
$D(EF)$ $\underline{9 \times 3 \times 10 + 210 = 480}$ <u>multiplications</u>

The next step looks at products of four consecutive matrices.

$A(BCD)$ $3 \times 5 \times 3 + 420 = 465$ multiplications,
(where we now that the right had is calculated as $B(CD)$)
$(AB)(CD)$ $3 \times 10 \times 3 + 150 + 270 = 510$ multiplications
$(ABC)D$ $3 \times 9 \times 3 + 420 = 501$ multiplications

$B(CDE)$ $5 \times 10 \times 7 + 480 = 830$ multiplications
$(BC)(DE)$ $5 \times 9 \times 7 + 450 + 189 = 954$ multiplications
$(BCD)E$ $5 \times 10 \times 7 + 420 = 770$ multiplications

$C(DEF)$ $10 \times 9 \times 10 + 480 = 1380$ multiplications
$(CD)(EF)$ $10 \times 3 \times 10 + 270 + 210 = 780$ multiplications
$(CDE)F$ $10 \times 7 \times 10 + 480 = 1180$ multiplications

Next to last, we multiply 5 consecutive matrices.

$A(BCDE)$ $3 \times 5 \times 7 + 770 = 875$ multiplications
$(AB)(CDE)$ $3 \times 10 \times 7 + 150 + 480 = 840$ multiplications
$(ABC)(DE)$ $3 \times 9 \times 7 + 420 + 189 = 798$ multiplications
$(ABCD)E$ $3 \times 3 \times 7 + 465 = 528$ multiplications

$B(CDEF)$ $5 \times 10 \times 10 + 780 = 1280$ multiplications
$(BC)(DEF)$ $5 \times 9 \times 10 + 450 + 480 = 1380$ multiplications
$(BCD)(EF)$ $5 \times 3 \times 10 + 420 + 210 = 780$ multiplications
$(BCDE)F$ $5 \times 7 \times 10 + 770 = 1120$ multiplications.

Lastly, we have five ways to multiply the six matrices:

$A(BCDEF)$	$3 \times 5 \times 10 + 780 = 930$ multiplications
$(AB)(CDEF)$	$3 * 10 * 10 + 150 + 780 = 1230$ multiplications
$(ABC)(DEF)$	$3 \times 9 \times 10 + 420 + 480 = 1170$ multiplications
$(ABCD)(EF)$	$3 \times 3 \times 7 + 465 + 210 = 738$ multiplications
$(ABCDE)F$	$3 \times 7 \times 10 + 528 = 738$ multiplications.

We have a tie. The first option is given by $(ABCD) \cdot (EF) = (A \cdot (BCD)) \cdot (EF) =$ $\big(A \cdot (B \cdot (B \cdot C) \cdot D)\big) \cdot (EF)$, the second option is given by $(ABCDE)F = \big((ABCD) \cdot E\big) \cdot F = \Big(\big(A(BCD)\big) \cdot E\Big) \cdot F = \Big(\big(A \cdot (B \cdot (CD)\big) \cdot E\Big) \cdot F$