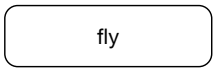
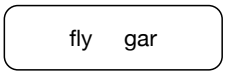
Solutions B-tree Problems

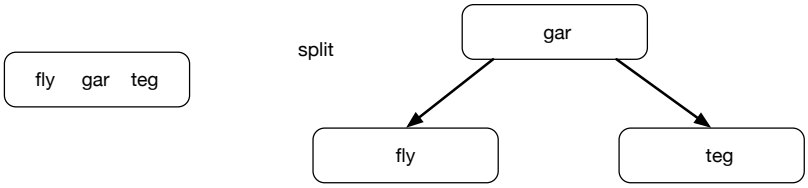
Insert fly:



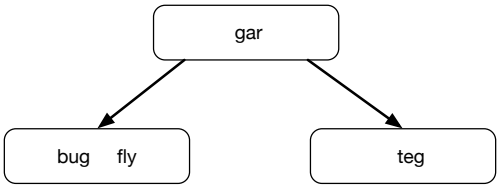
Insert gar:



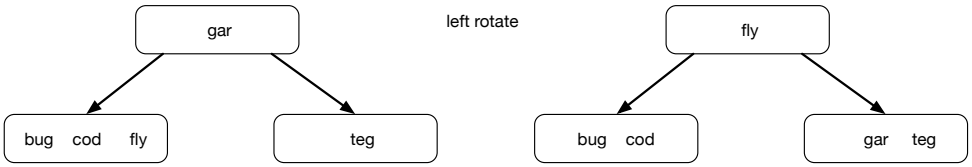
Insert teg:



Insert bug:

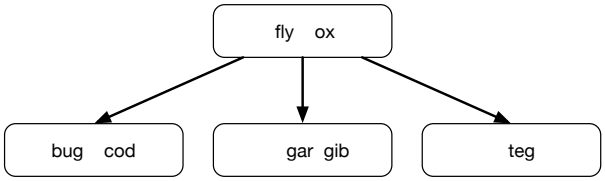


Insert cod:

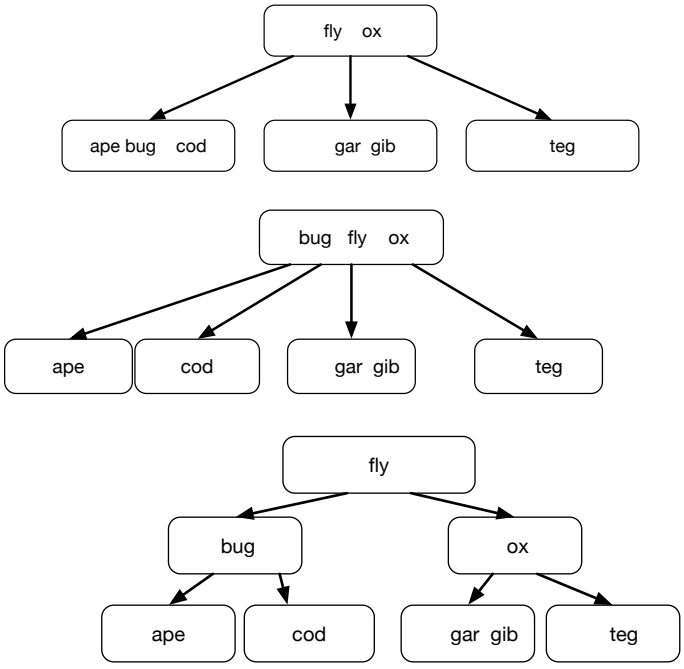


Insert ox:

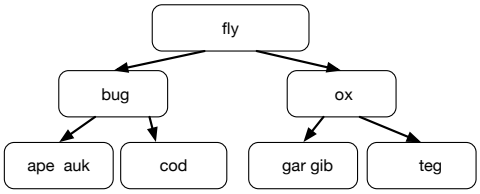
Insert gib:



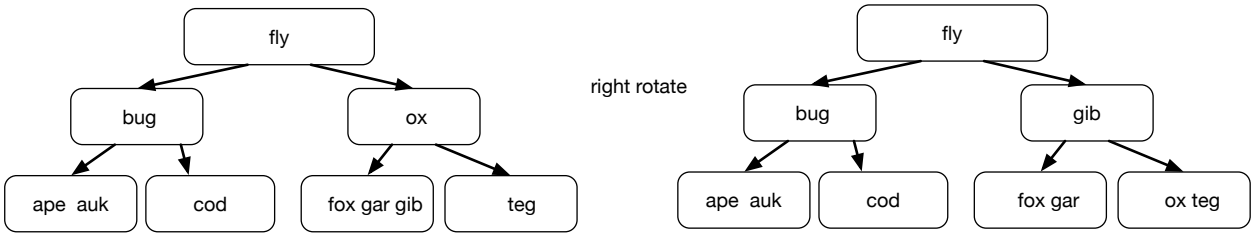
Insert ape leads to a cascade of splits:



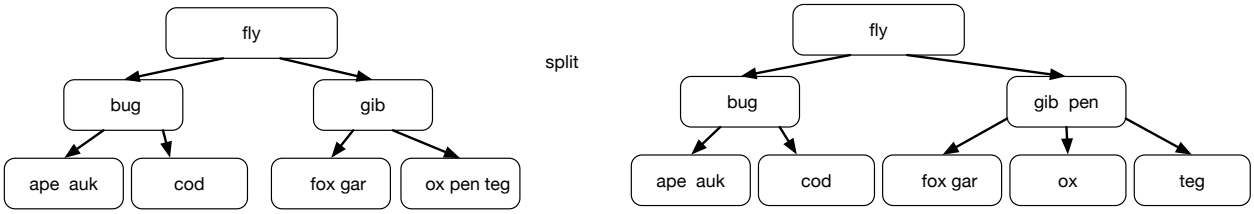
Insert auk:



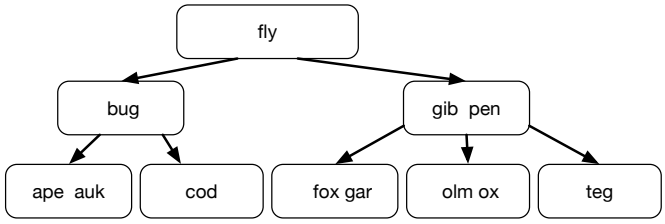
Insert fox leads to a right rotate:



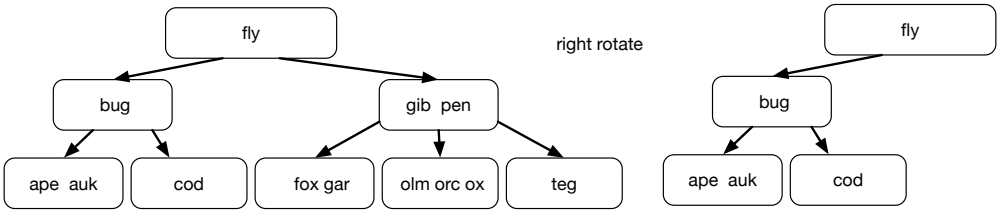
Insert pen comes with a split:



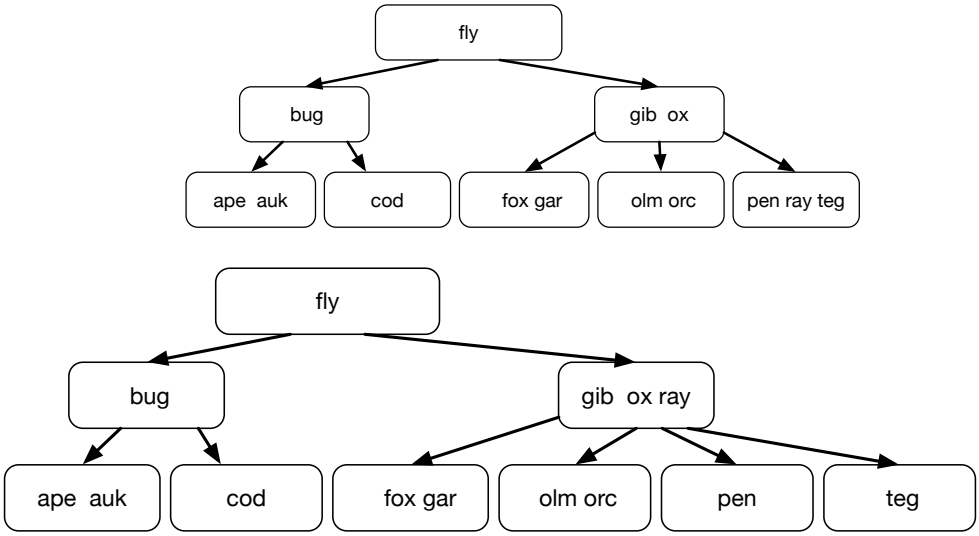
Insert olm:

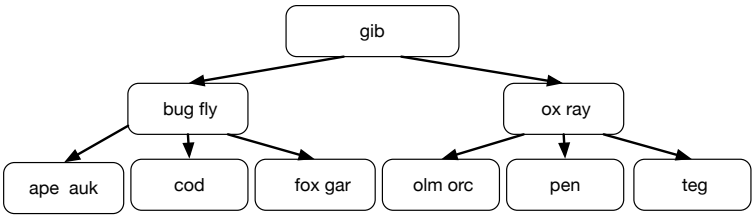


Insert orc

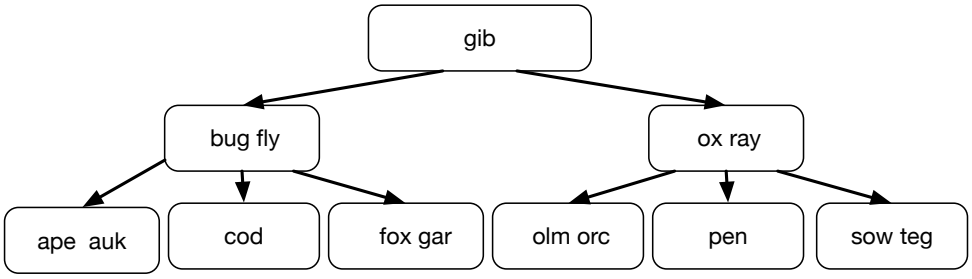


Insert ray leads to a split and then a left rotate

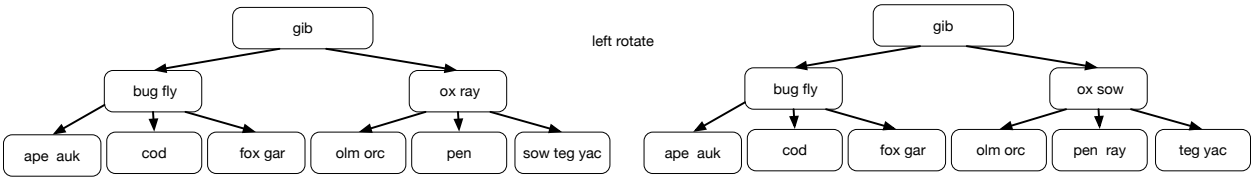




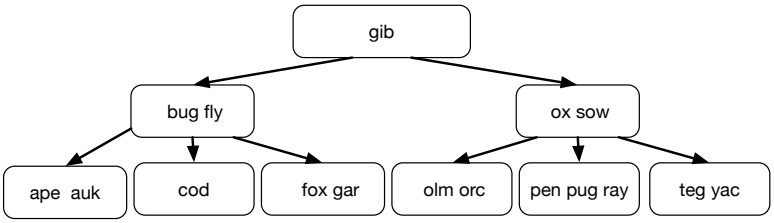
Insert sow

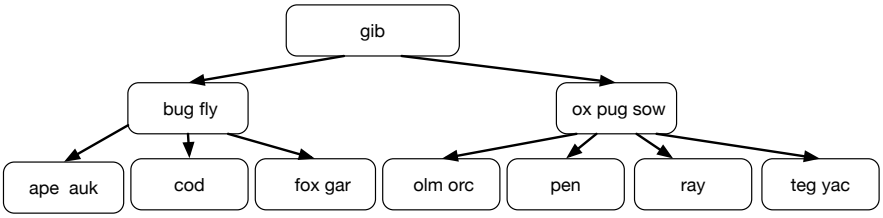


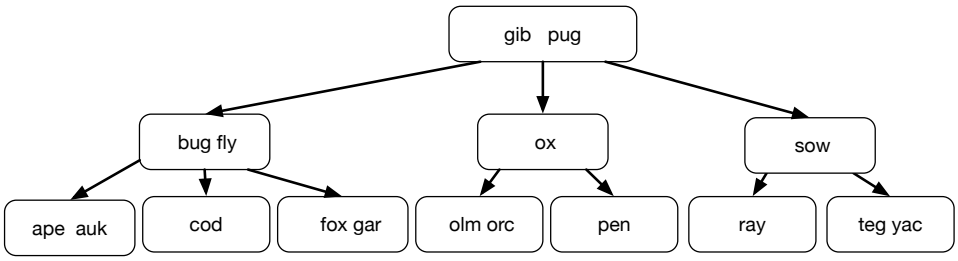
Insert yak yields a left rotate:



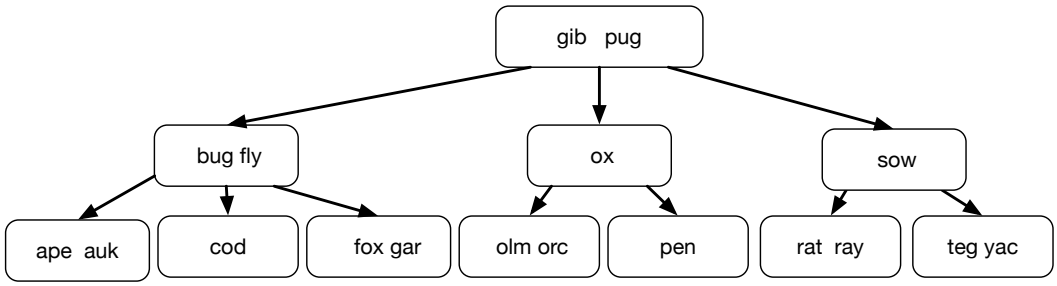
Insert pug leads to a cascade of splits:



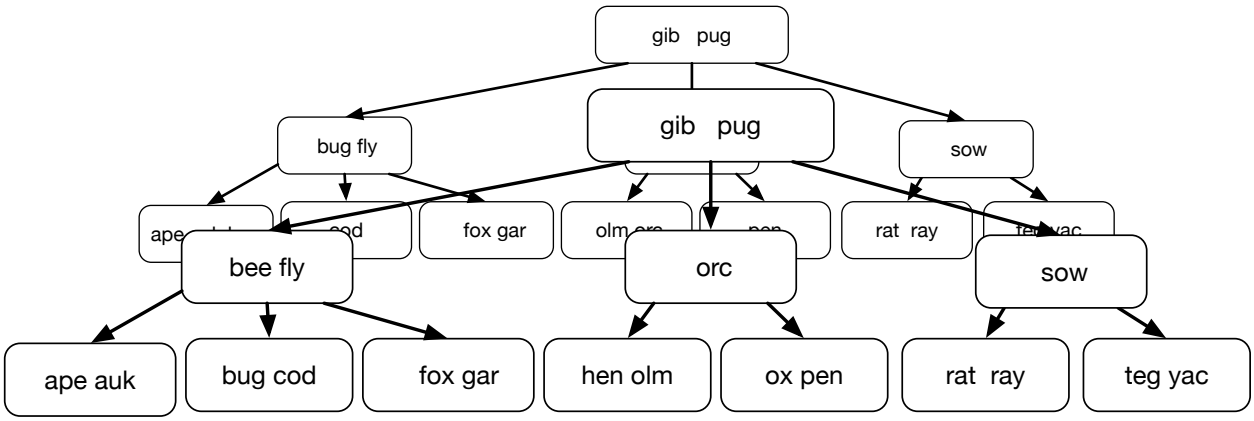




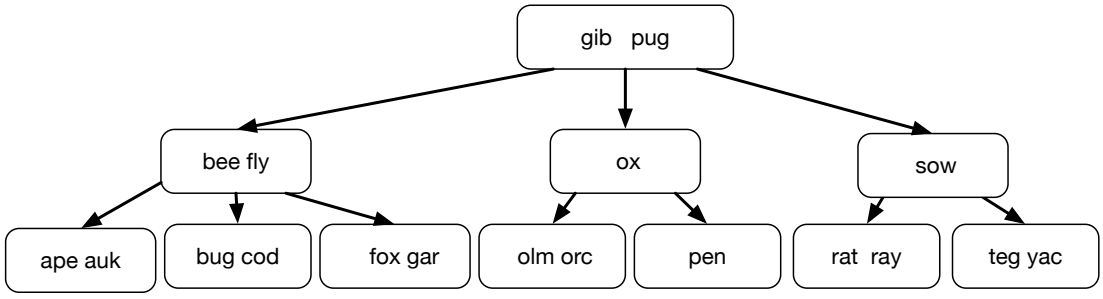
Insert rat



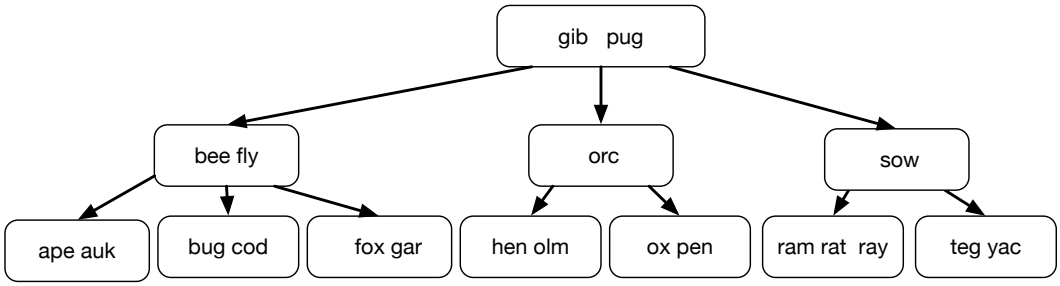
Insert bee leads to a right rotate

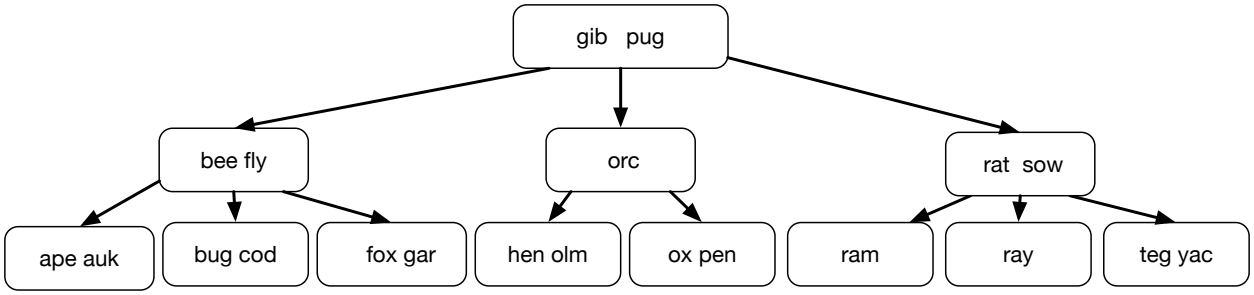


Insert hen

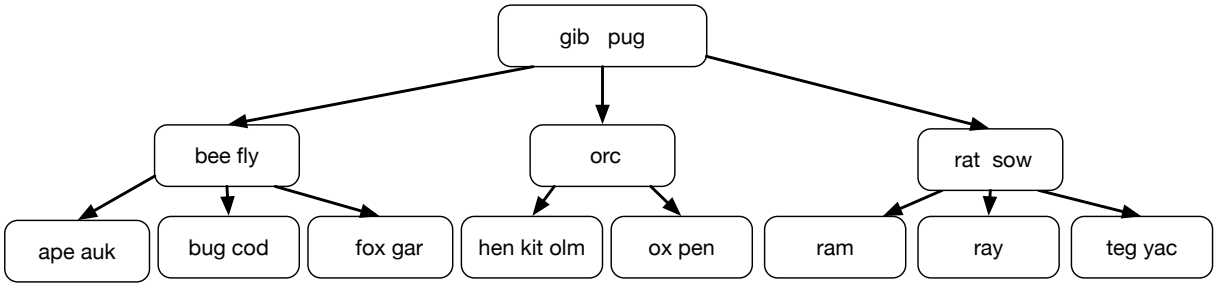


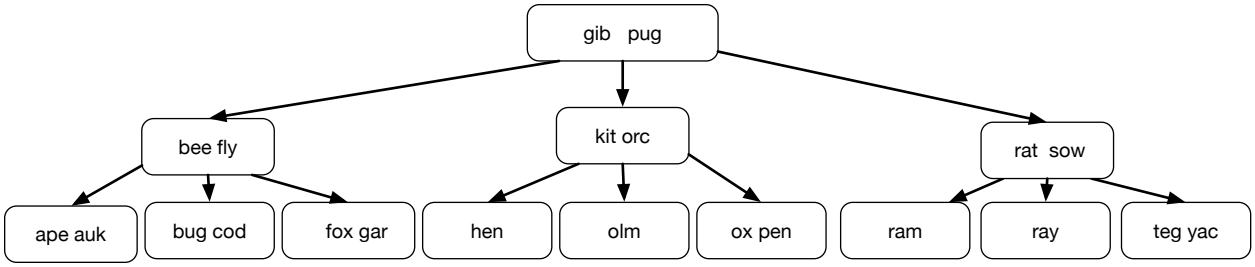
Insert ram is done with a split



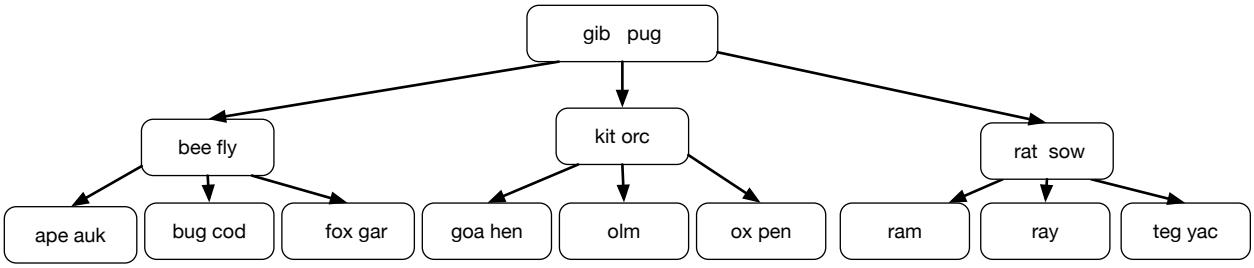


Insert kit

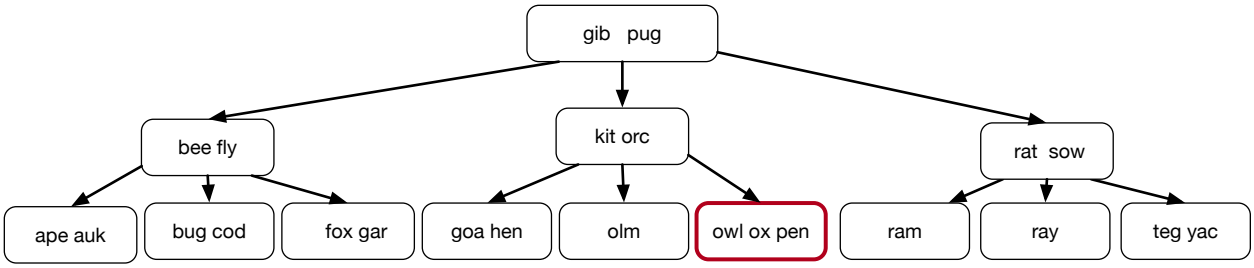


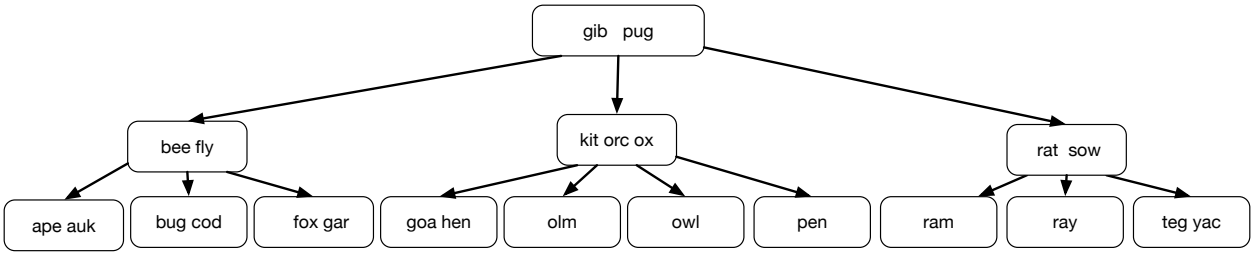


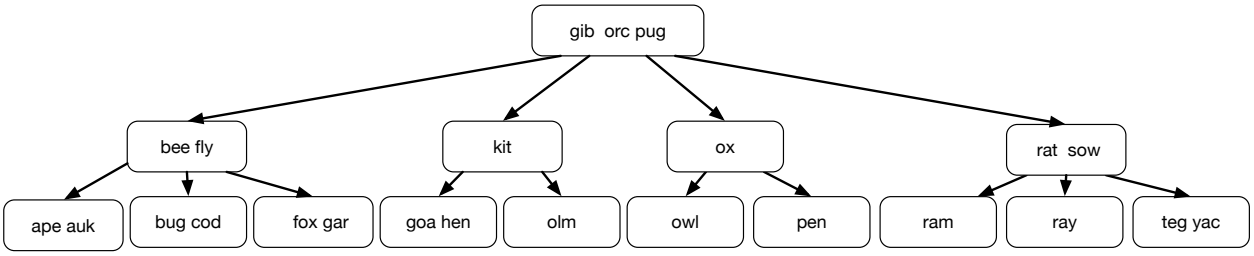
Insert goa

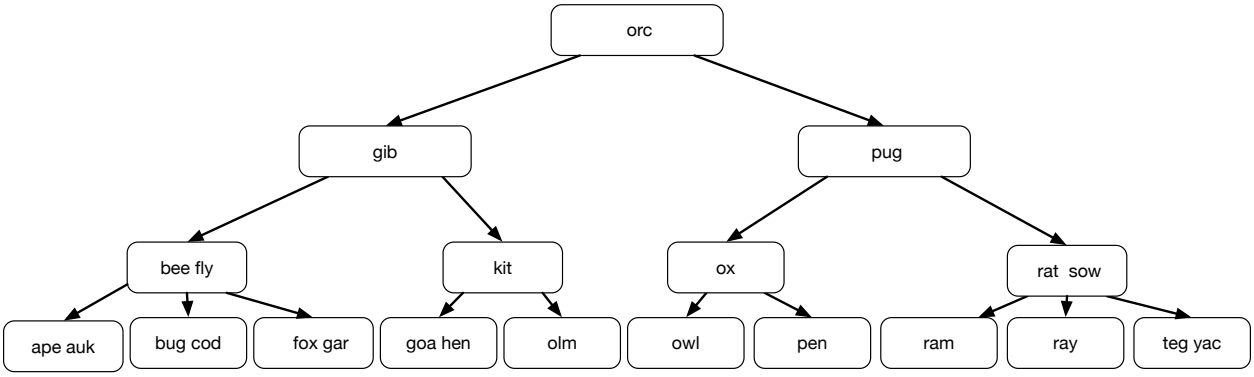

gib pug
bee fly
kit orc
rat sow
ape auk
bug cod
fox gar
goa hen
olm
ox pen
ram
ray
teg yac

Insert owl has splits cascading to the top:

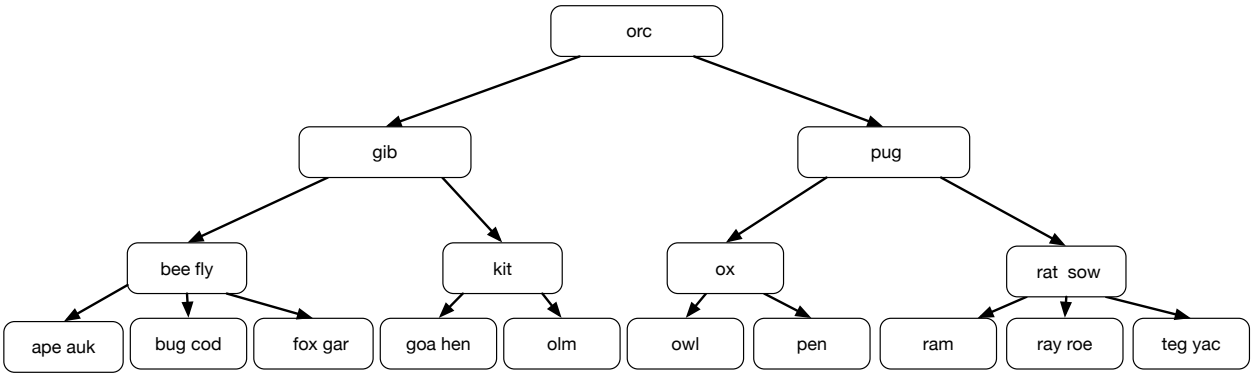




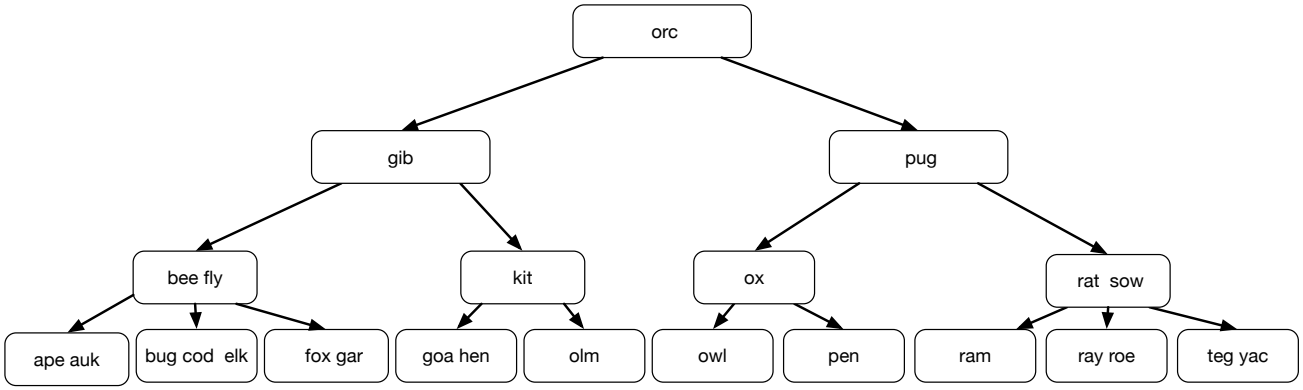




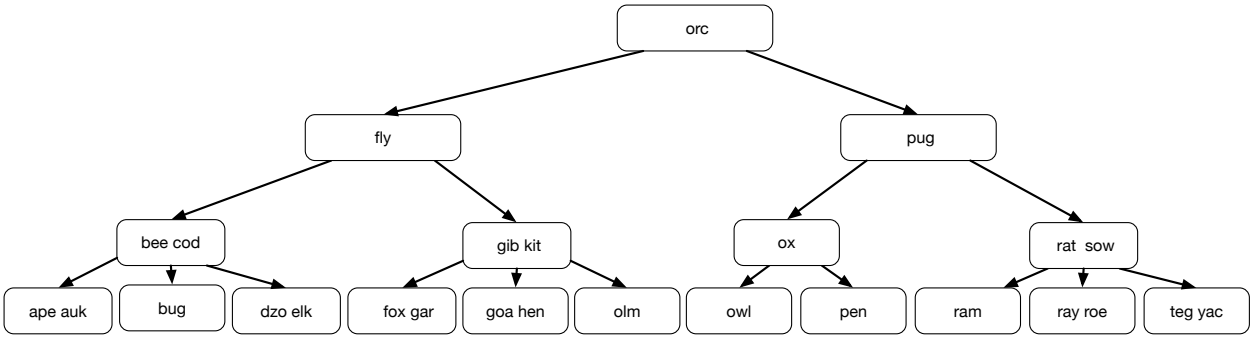
Insert roe



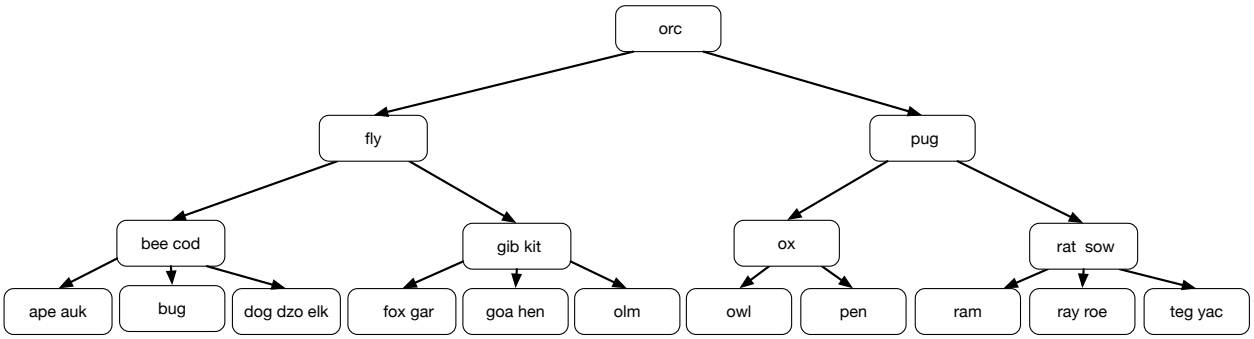
Insert elk

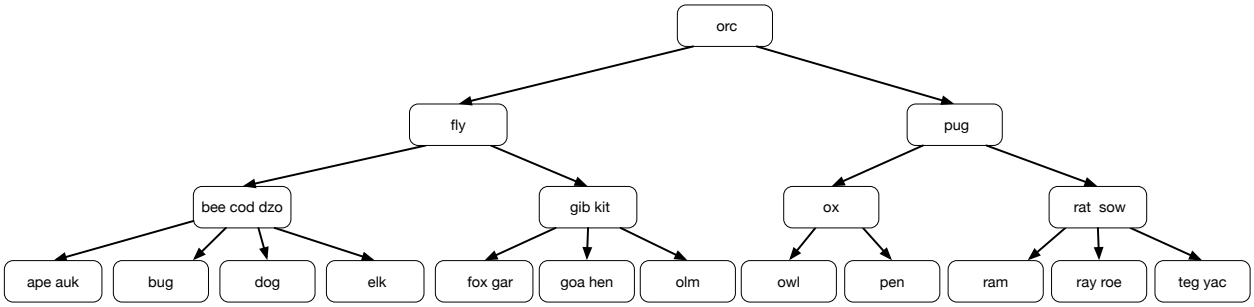


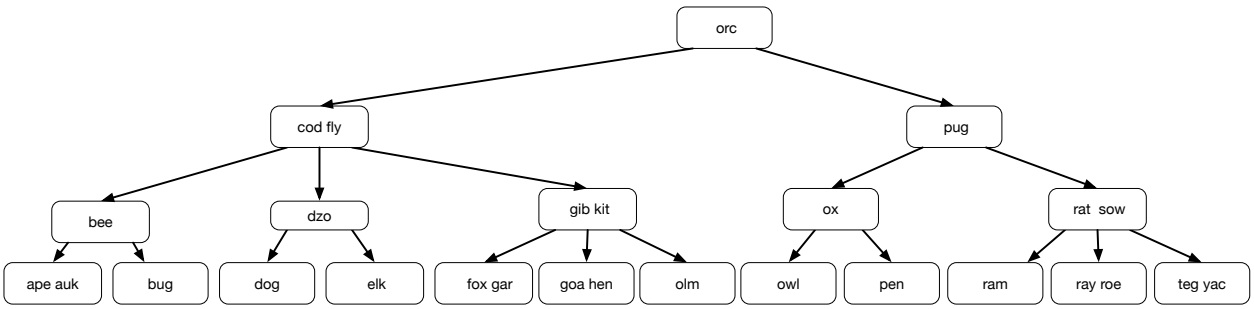
Insert dzo



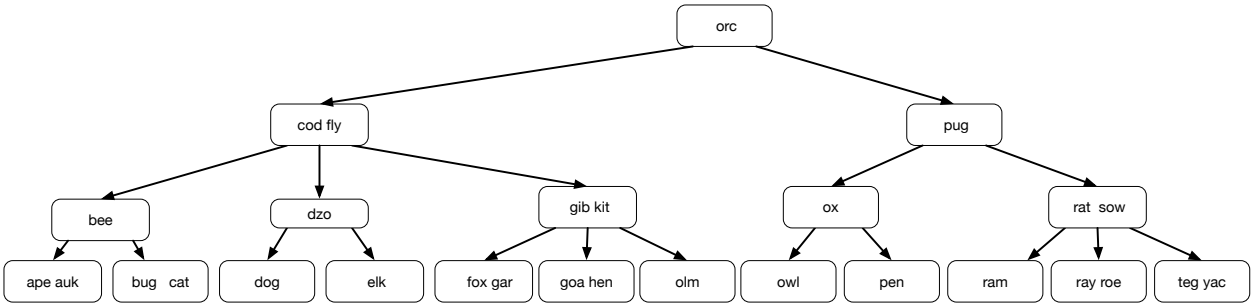
Insert dog leads to two splits



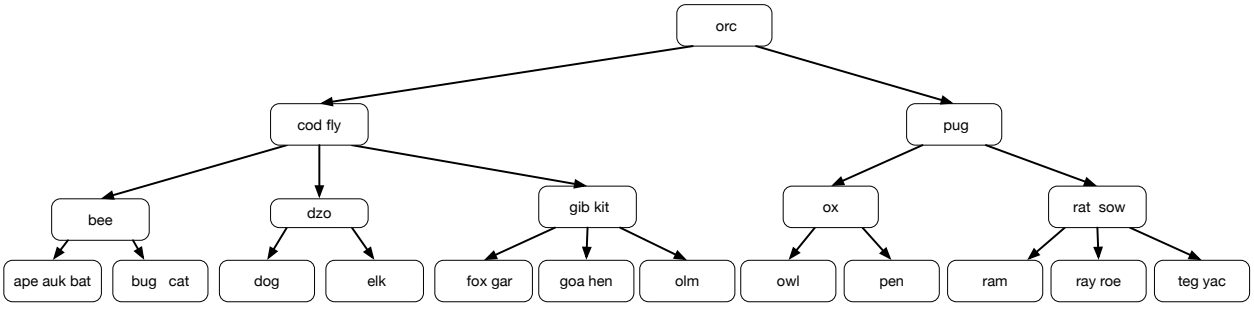


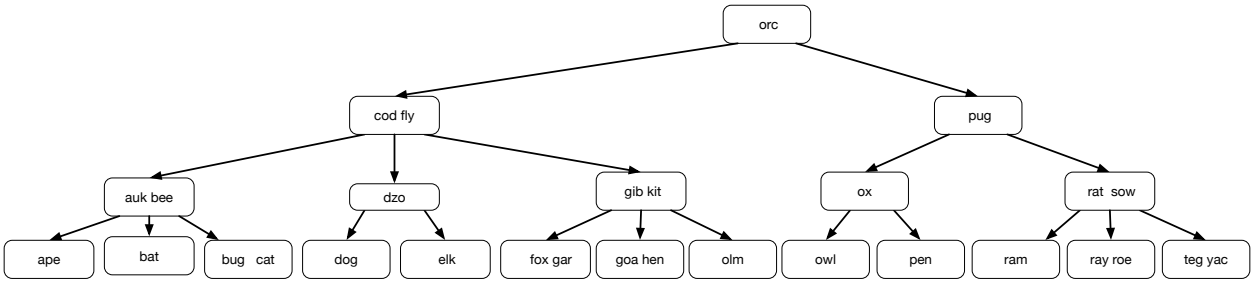


Insert cat

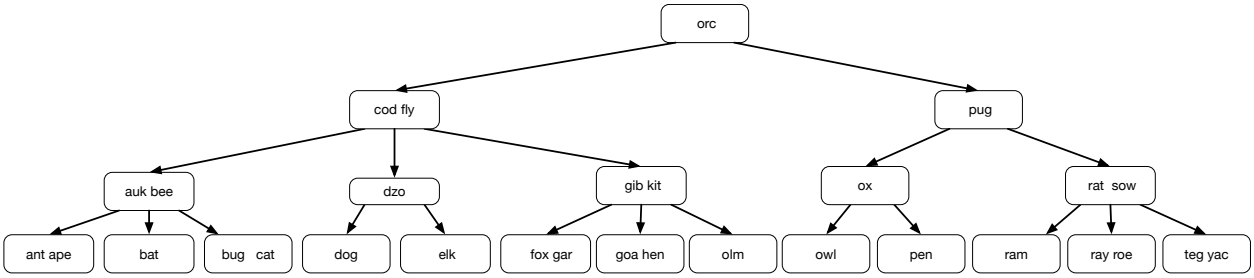


Insert bat



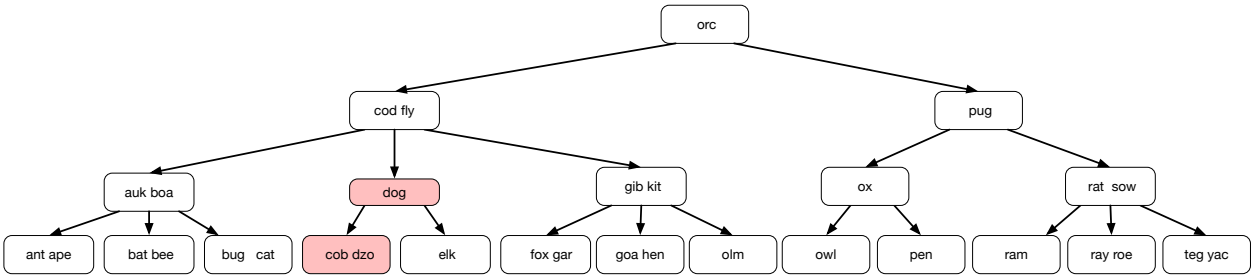


Insert ant

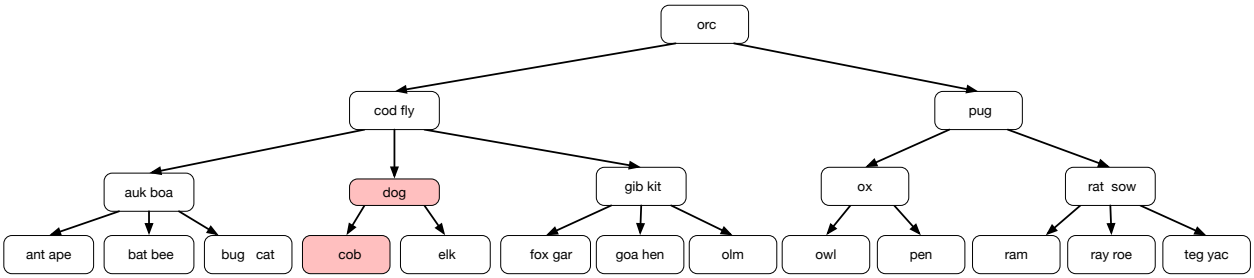

orc
cod fly
pug
auk bee
dzo
gib kit
ox
rat sow
ant ape
bat
bug cat
dog
elk
fox gar
goa hen
olm
owl
pen
ram
ray roe
teg yac

Delete Problem:

Switch dzo with predecessor dog

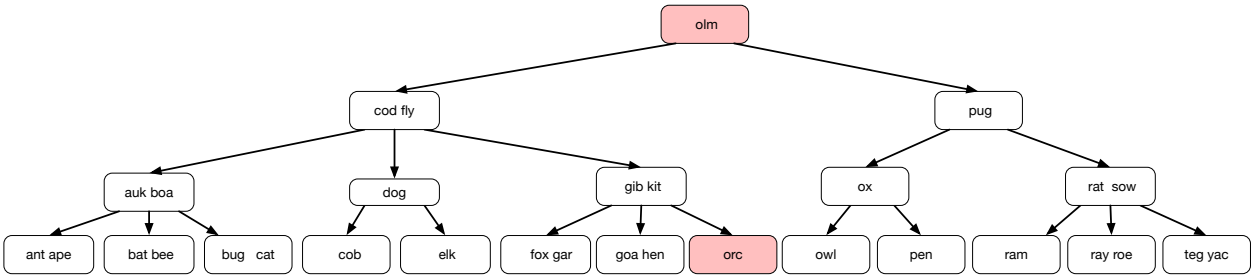


Delete dzo

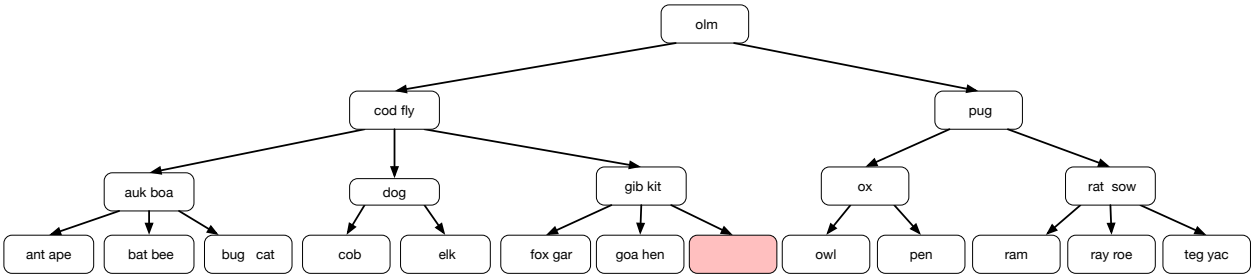


No restructuring is needed

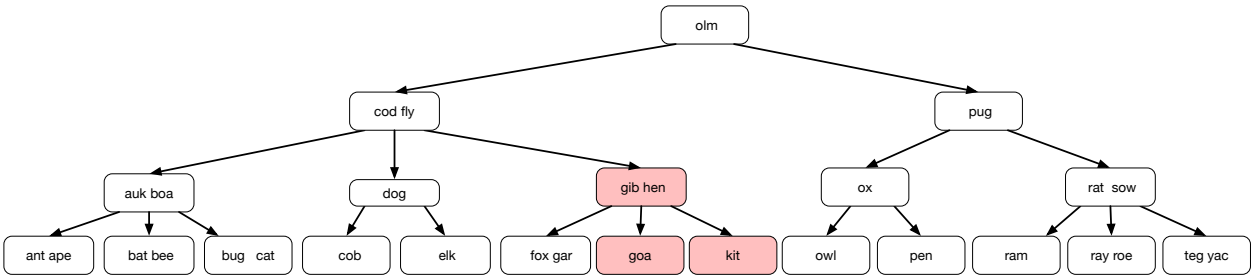
Deleting orc. First, switch orc with the predecessor.



From the resulting tree, delete orc.

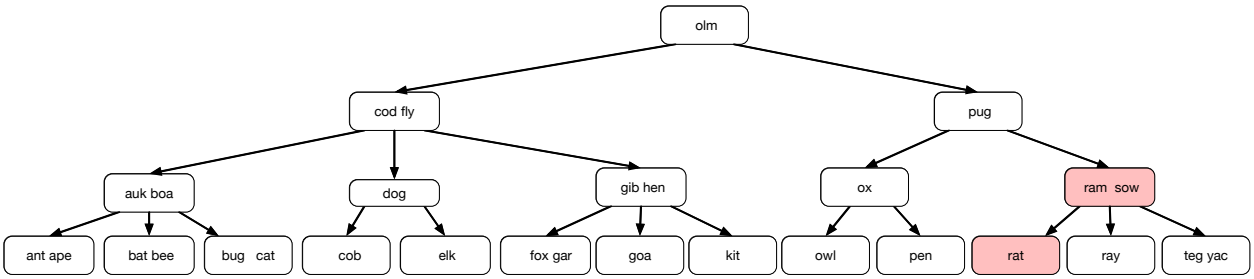


Since there is an underflow, we need to do something. Use a right rotate.

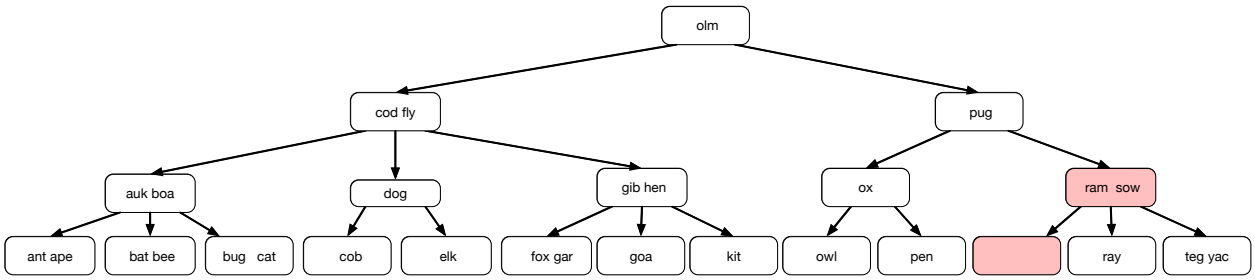


Deleting roe. The key is already in a leaf, so we can just remove it. The resulting tree has no underflow, so we are done.

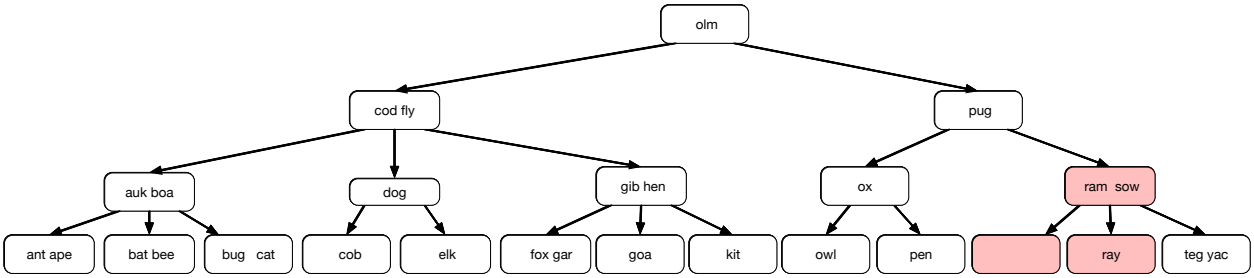
Deleting 'rat'. We switch with the predecessor.



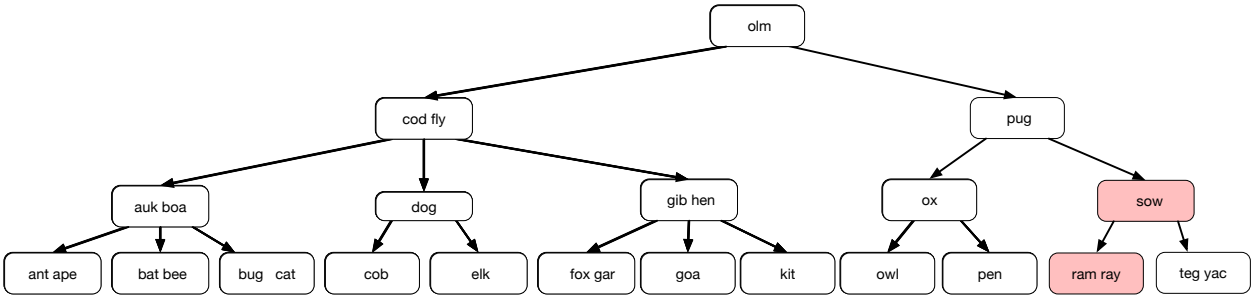
We then delete 'rat' from the leaf. This gives an underflow. A rotate is not possible.



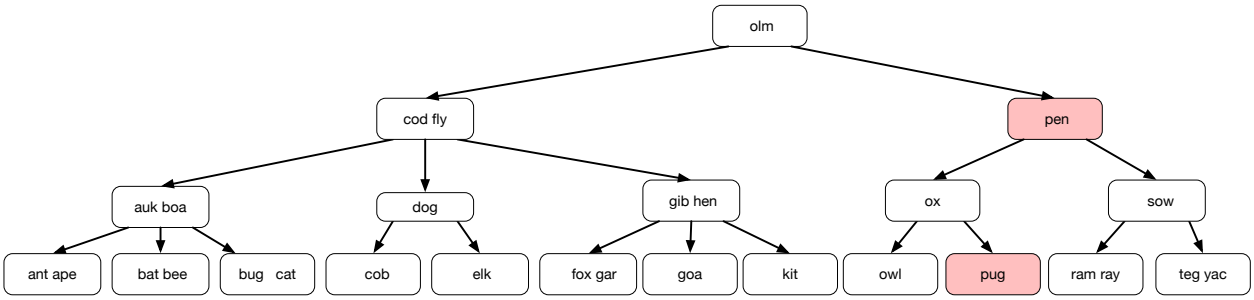
We therefore undo a split. This is called a merge. We involve three nodes.



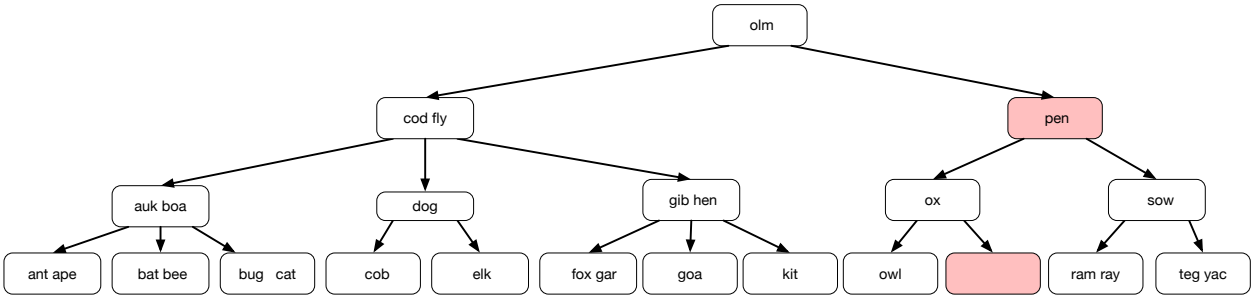
The keys in the left node (of which there are none), the separating key in the top node, and the keys in the right node are combined in a single node.



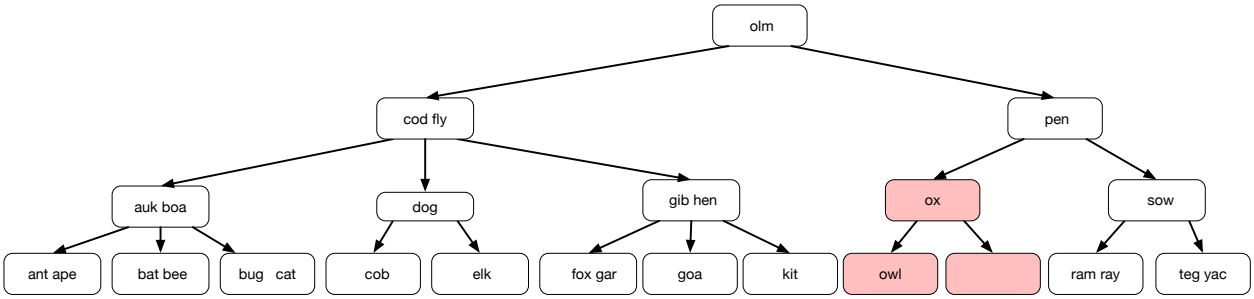
Delete pug. We first swap pug with its predecessor pen.



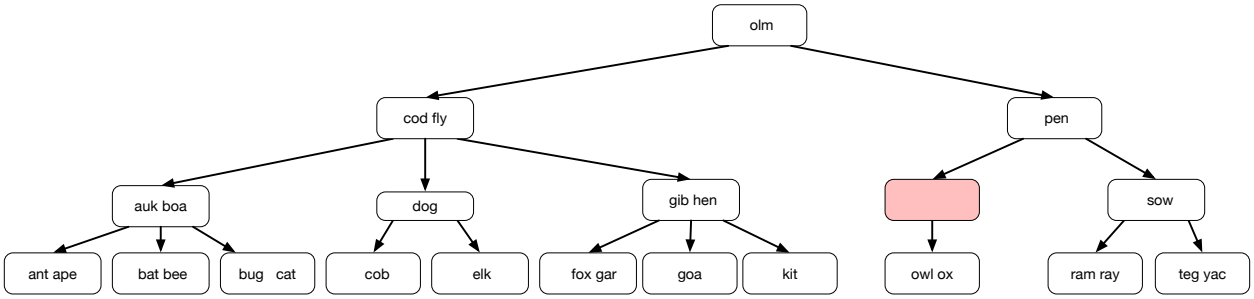
We then delete pug. This gives an underflow.



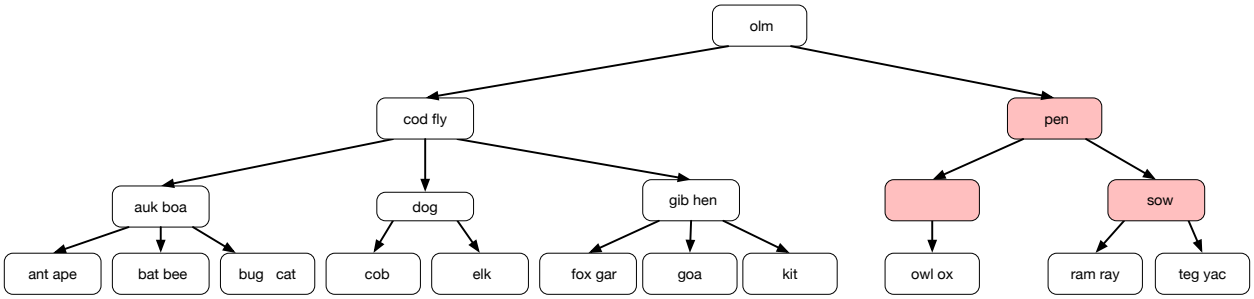
We therefore need to do a merge of three nodes. Remember that the tree needs to be balanced. Therefore, we cannot just replace the three nodes with a single node.



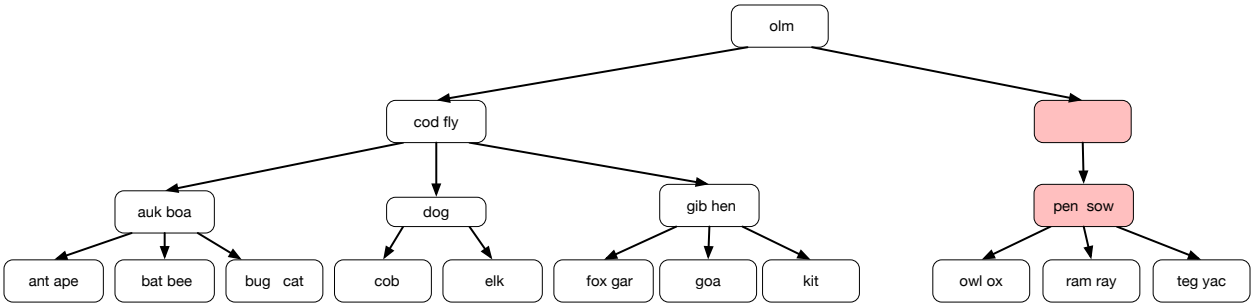
The result is a lower level node with an underflow.



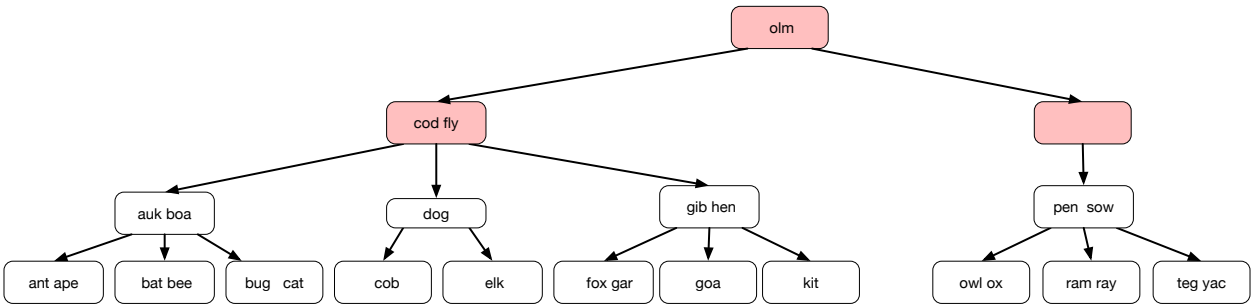
To deal with this underflow, we first try rotates, but this does not work. There is only one sibling, which is at minimum capacity. Therefore, we merge again three nodes.



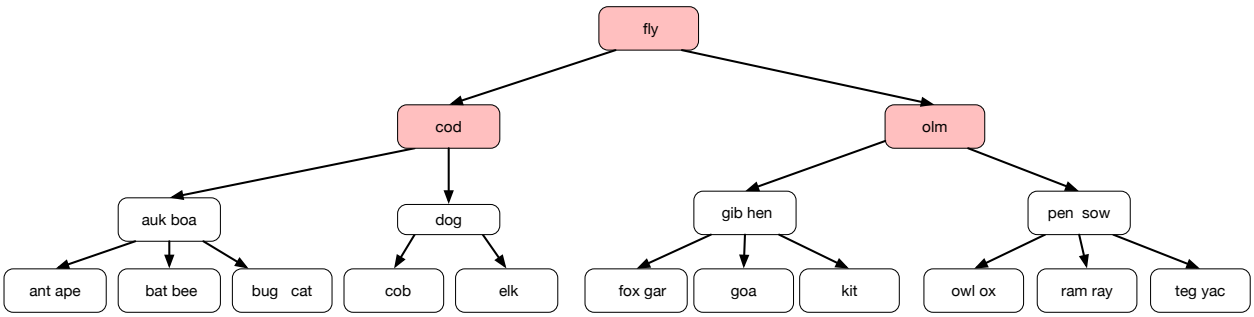
This moves the underflow upstairs:



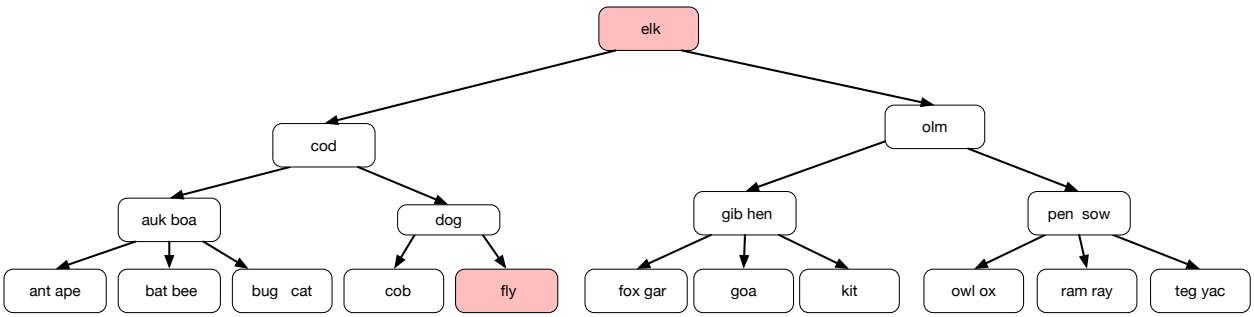
Now we can do a left rotate with the left sibling.

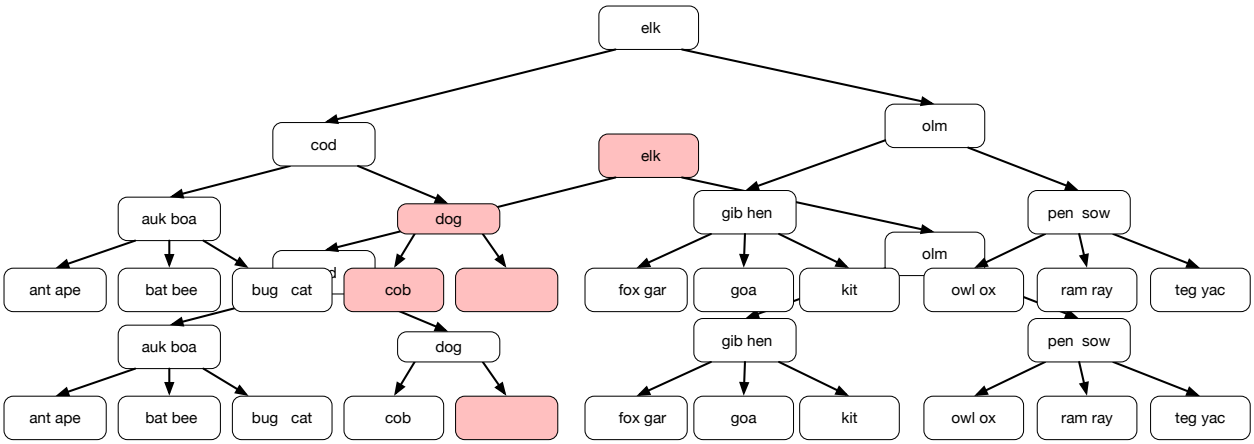


In this rotate, fly goes up and olm goes down. The node with gib and hen is reattached



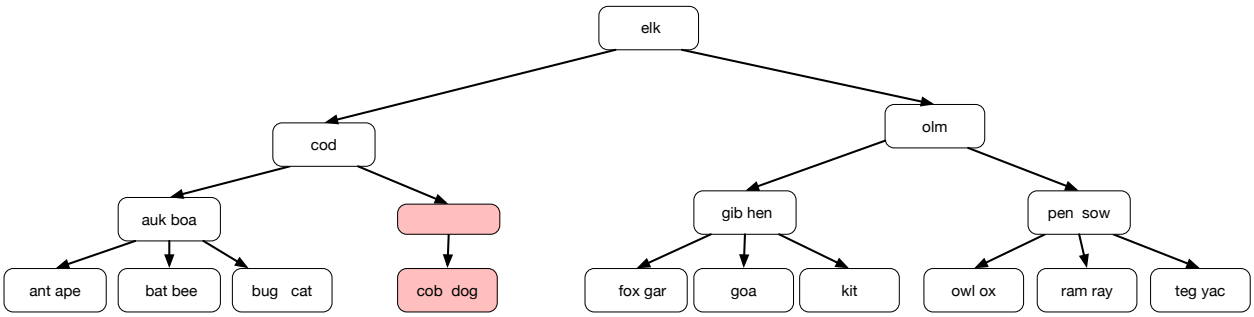
Deleting fly. We swap with its predecessor.



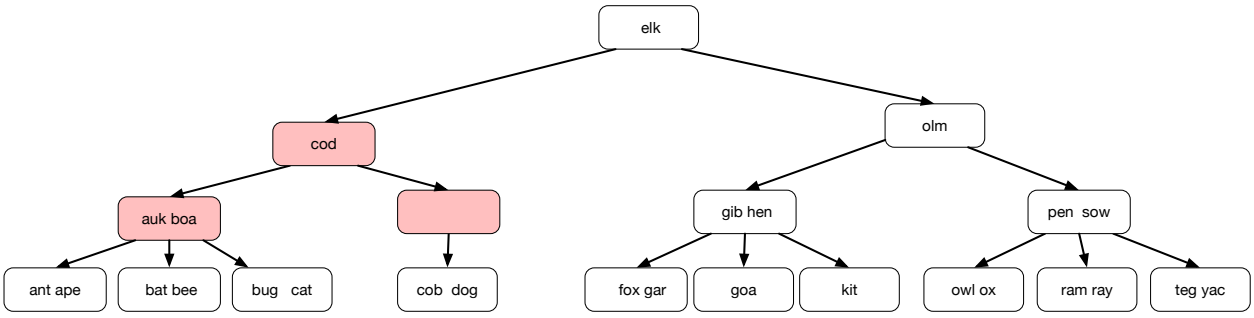


We delete fly from the leaf.

The resulting underflow cannot be dealt with by a rotation, so we need to merge (undo a split).

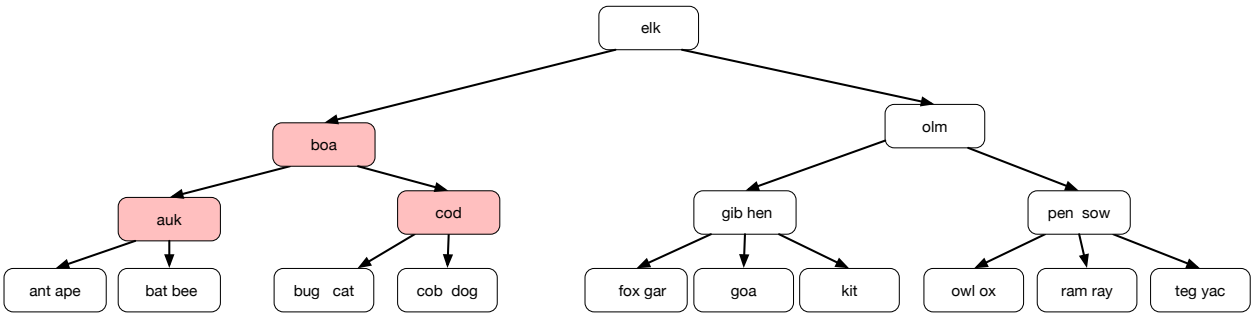


Again, we merge the two children, not the parent, who becomes empty.

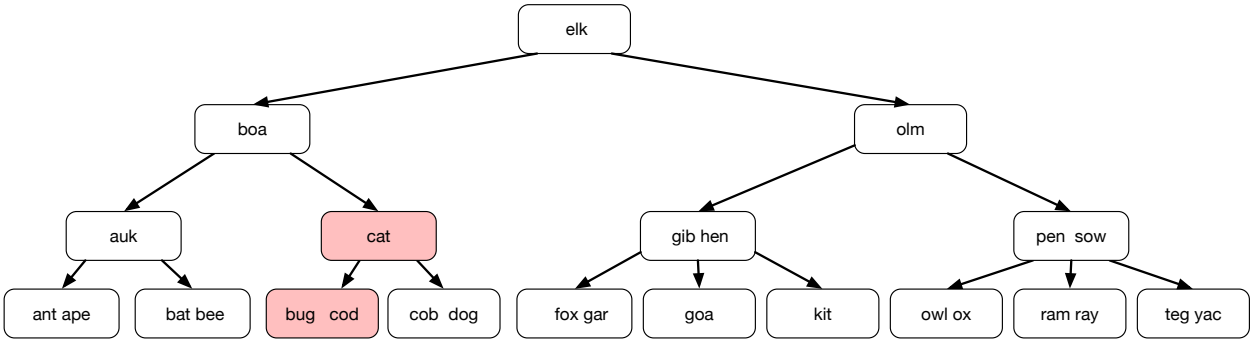


We can solve the underflow with a right rotate.

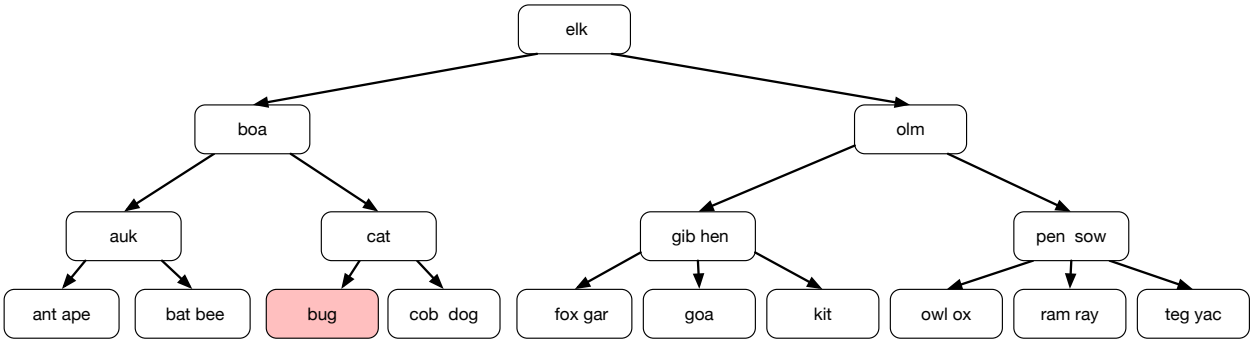
This means that boa goes up, cod goes down and one node is reattached.



Deleting 'cod'. We swap with the predecessor, 'cat'.



And then delete it from the leaf.



No further restructuring is needed.