Data Structures

Algorithms

Types of Data Structures

- Organize data to make access / processing fast
  - Speed depends on the internal organization
  - Internal organization allows different types of accesses

- Problems:
  - Large data is nowadays distributed over several data centers
  - Need to take advantage of storage devices

Types of Data Structures

- Internal Memory
    - DRAM: fast access, byte addressable
- Storage
    - Hard Disk Drives
        - Data in blocks
        - Decent for streaming (consecutive blocks)
        - Bad for random access (~10 msec per access)
    - Solid State Disks
        - Data in blocks (called pages)
        - Decent access times (~1msec per access)

Types of Data Structures

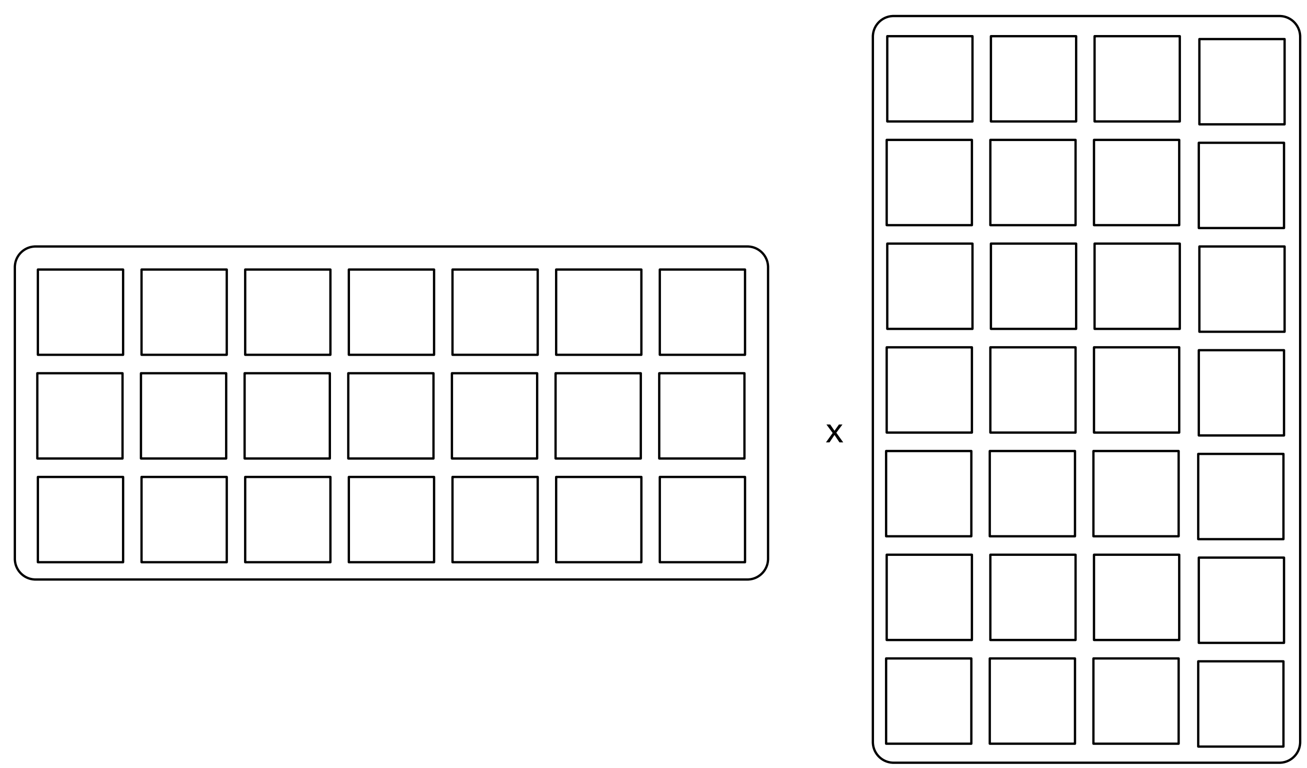
- Thread safe:
    - Several threads can safely access data structure
    - Need collaboration between threads
        - Implemented with locks
        - Implemented without locks
            - Difficult to do
            - Needs atomic instructions: Hardware and compiler support

Types of Data Structures

- Caches can make big performance differences
  - Cache aware algorithms
    - Get the parameter of the caches
  - Cache oblivious algorithms
    - Work well for all cache sizes
  - Dumb algorithms
    - Do not pay attention to caches at all
    - Frequent surprises with bad performance

Example

- Multiplying two big, non-dense matrices
  - Cache aware:
    - Break matrices into subsquares
    - Three subsquares fit comfortably into cache



Example

- Cache Oblivious
  - Use a Divide and Conquer Algorithm that subdivides the sub-squares repeatedly
  - Only **cold** cache misses when a new sub-square needs to be loaded into cache.

Types of Data Structure

- Dictionary — Key - Value Store
  - CRUD operations: create, read, update, delete
  - Solutions differ regarding read and write speeds

Types of Data Structure

- Range Queries (Big Table, RP)
    - CRUD and range operation

Types of Data Structure

- Priority queue:
    - Insert, retrieve minimum and delete it

Types of Data Structure

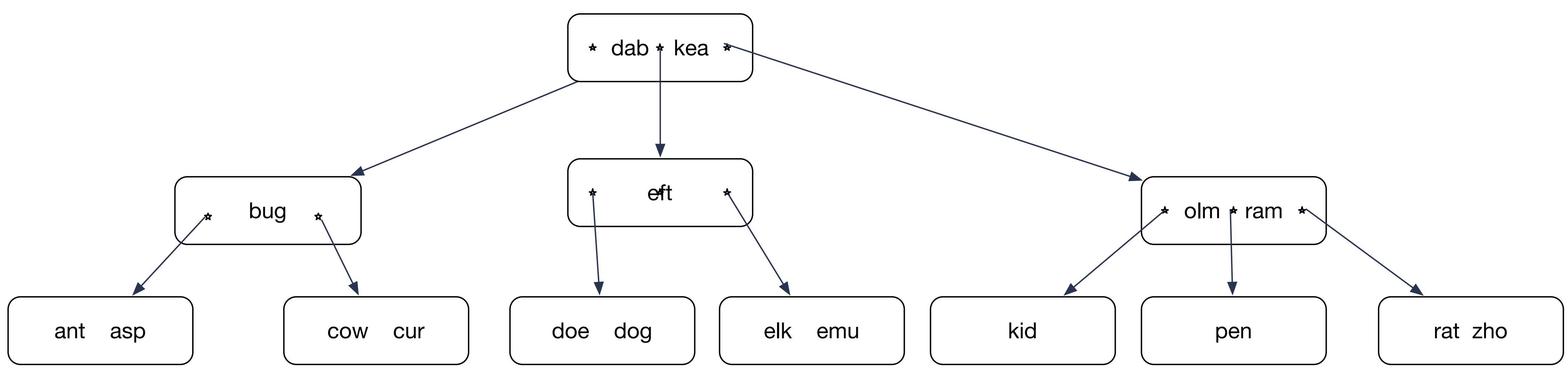
- Log:
  - Append, Read

B-Trees

- B-trees: In-memory data structure for CRUD and range queries
  - Balanced Tree
  - Each node can have between *d* and 2*d* keys with the exception of the root
  - Each node consists of a sequence of node pointer, key, node pointer, key, …, key, node pointer
  - Tree is ordered.
    - All keys in a child are between the keys adjacent to the node pointer

B-Trees

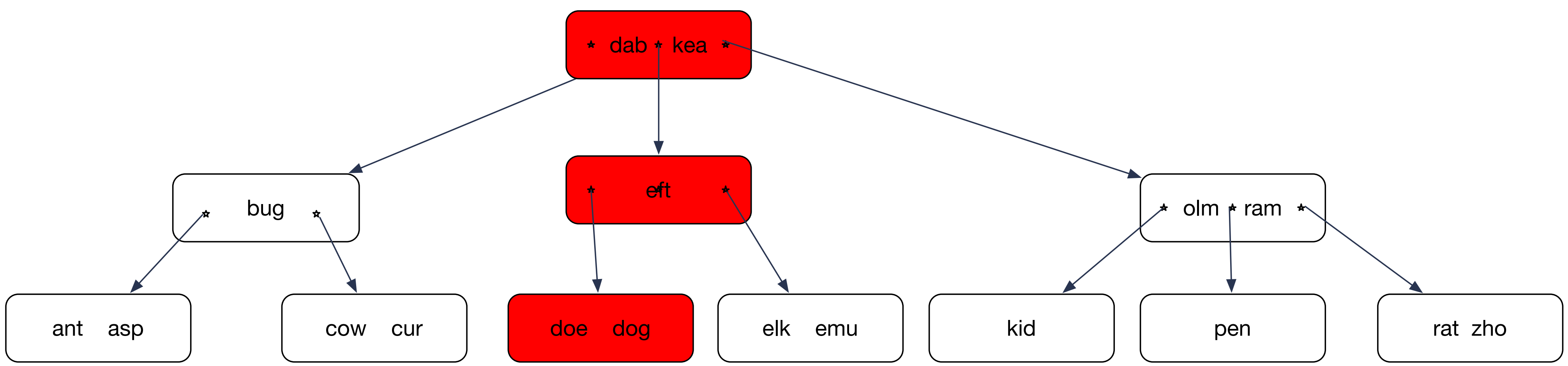
- Example: 2-3 tree: Each node has two or three children



B-Trees

- Read dog:
  - Load root, determine location of dog in relation to the keys
  - Follow middle pointer
  - Follow pointer to the left
  - Find “dog”

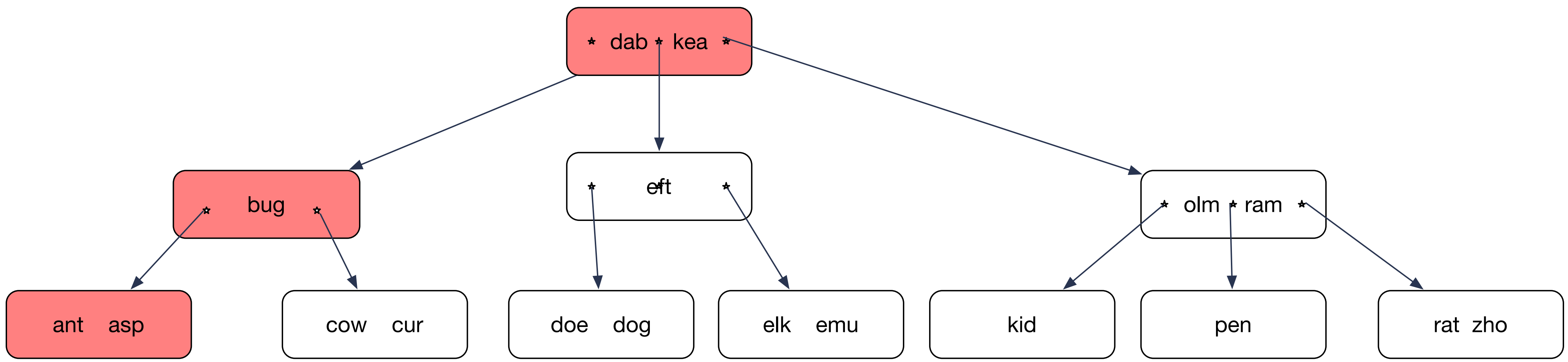
B-Trees



dab
kea
bug
eft
olm
ram
ant asp
cow cur
doe dog
elk emu
kid
pen
rat zho

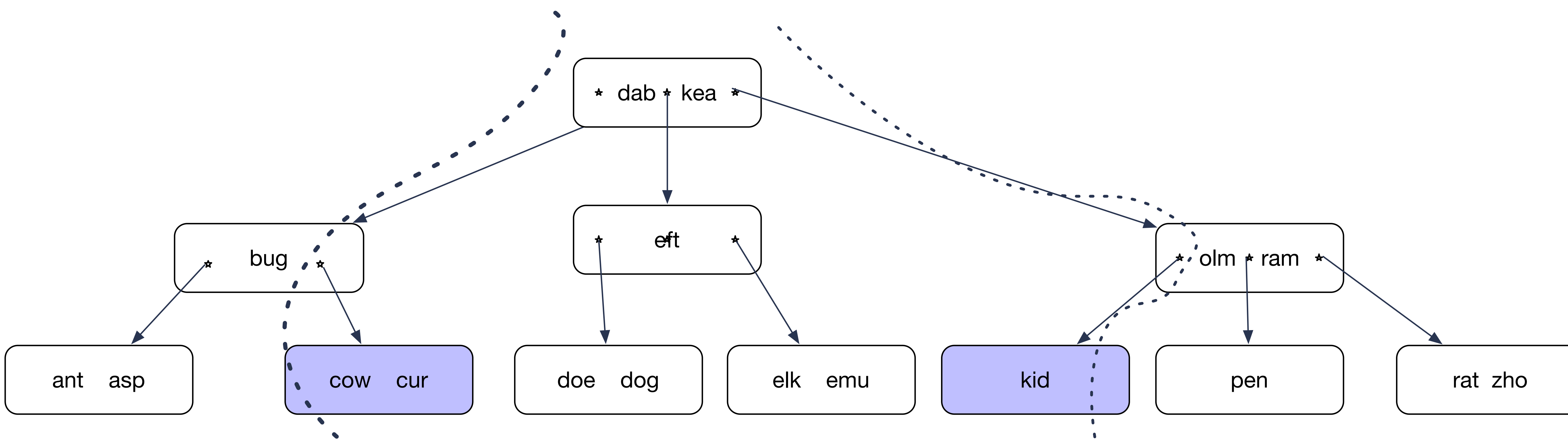
B-Trees

- Search for “auk” :



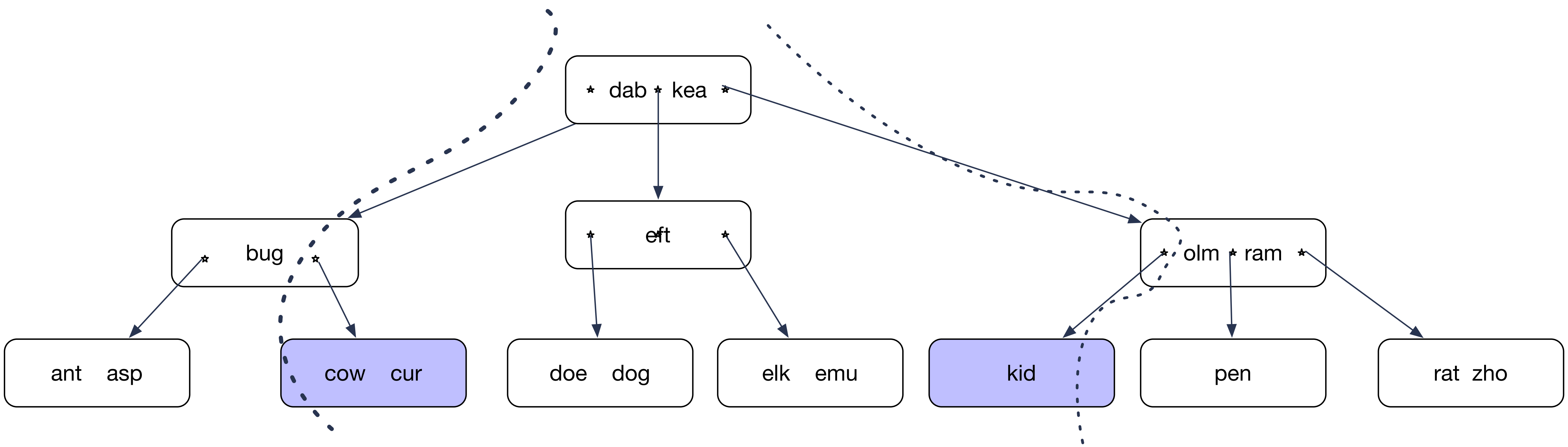
B-Trees

- Range Query c - l
  - Determine location of c and l



B-Trees

- Recursively enumerate all nodes between the lines starting with root



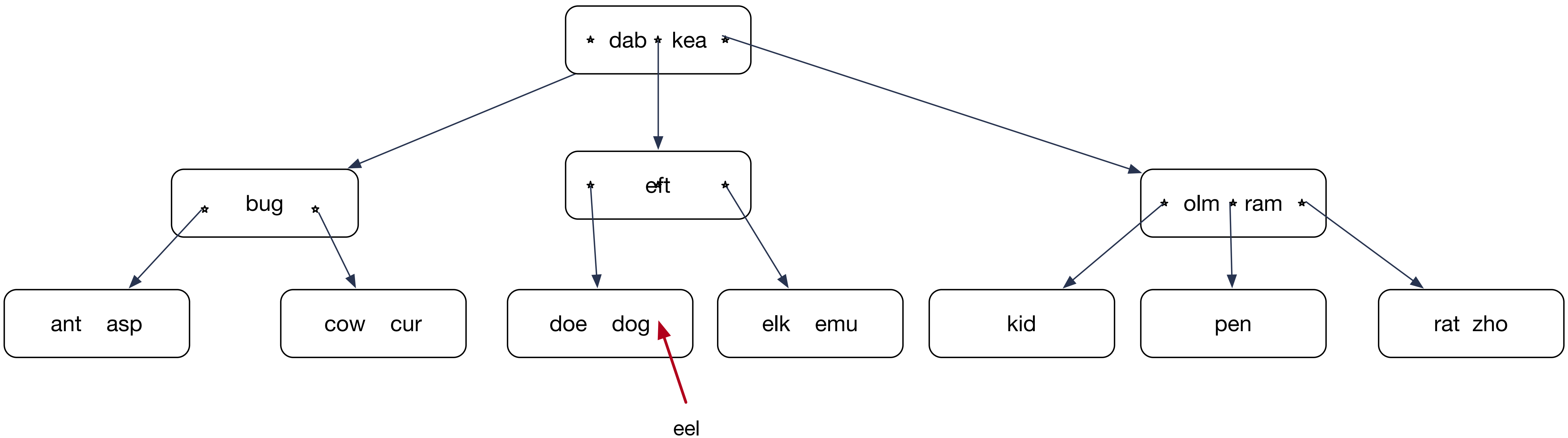
B-trees

- Capacity: With *l* levels, minimum of $1 + 2 + 2^2 + \ldots + 2^l$ nodes:
  - $1(2^{l+1} - 1)$ keys
- Maximum of $1 + 3 + 3^2 + \ldots + 3^l$ nodes
  - $\frac{2}{2}(3^{l+1} - 1)$ keys

B-trees

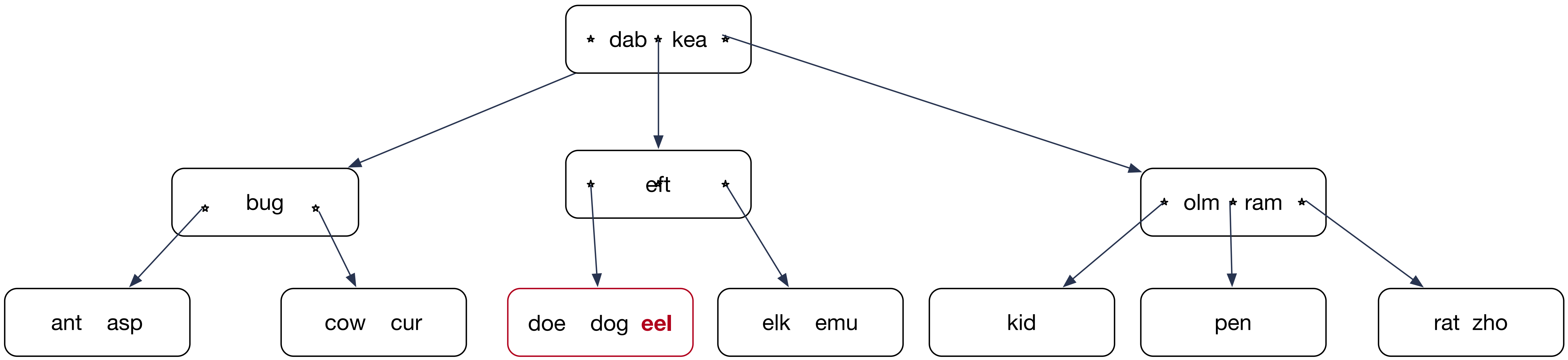
- Inserts:
    - Determine where the key should be located in a leaf
    - Insert into leaf node
    - Leaf node can now have too many keys
    - Take middle node and elevate it to the next higher level
    - Which can cause more “splits”

B-trees



dab kea
bug
eft
olm ram
ant asp
cow cur
doe dog
elk emu
kid
pen
rat zho
eel

B-trees



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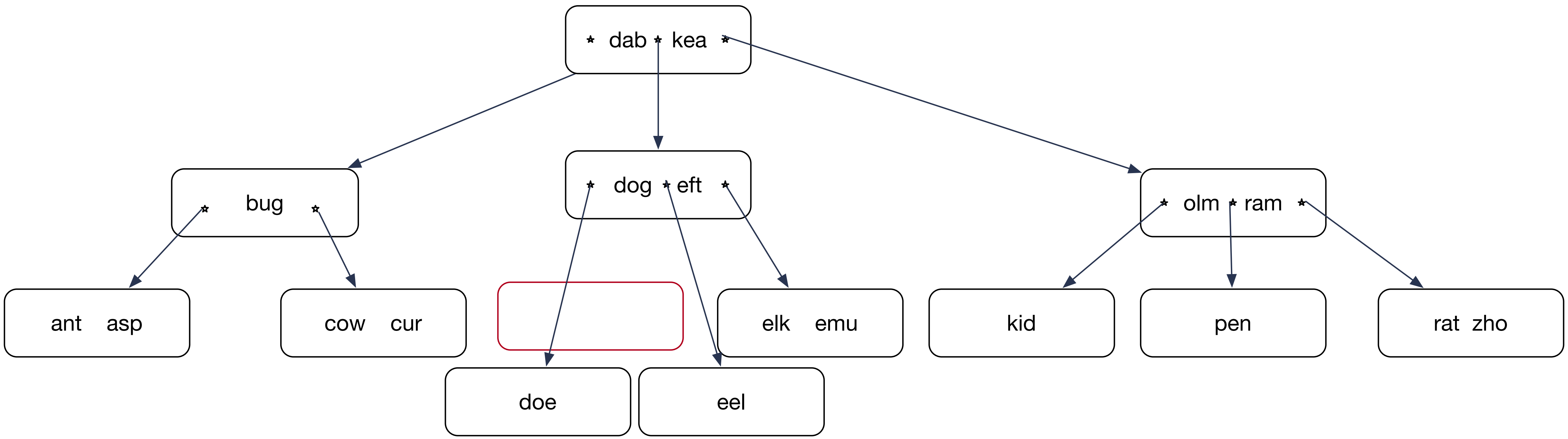
B-trees



dab kea
bug
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olm ram
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cow cur
elk emu
kid
pen
rat zho
doe
eel

B-trees

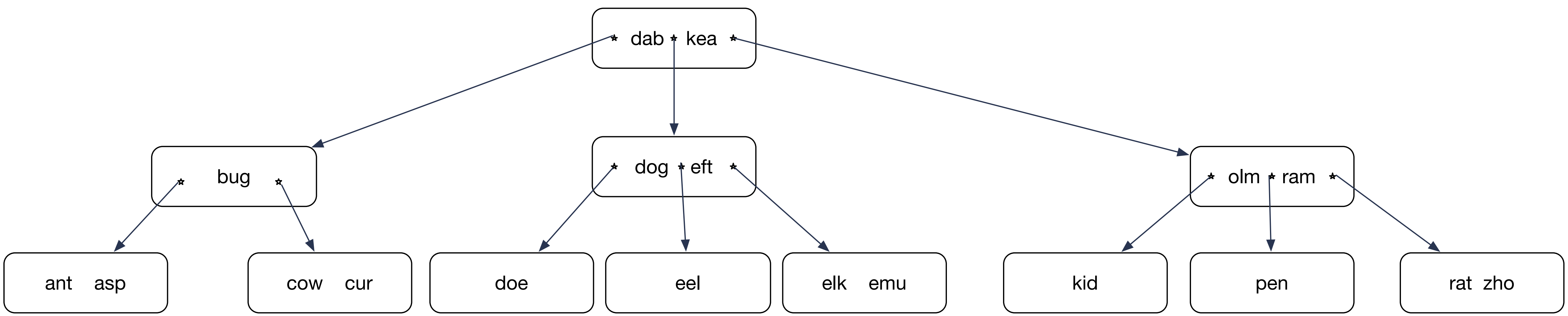
- Insert: Lock all nodes from root on down so that only one process can operate on the nodes
- Tree only grows a new level by splitting the root

B-Trees

- Using only splits leads to skinny trees
  - Better to make use of potential room in adjacent nodes
  - Insert “ewe”.
    - Node elk-emu only has one true neighbor.
      - Node kid does not count, it is a cousin, not a sibling

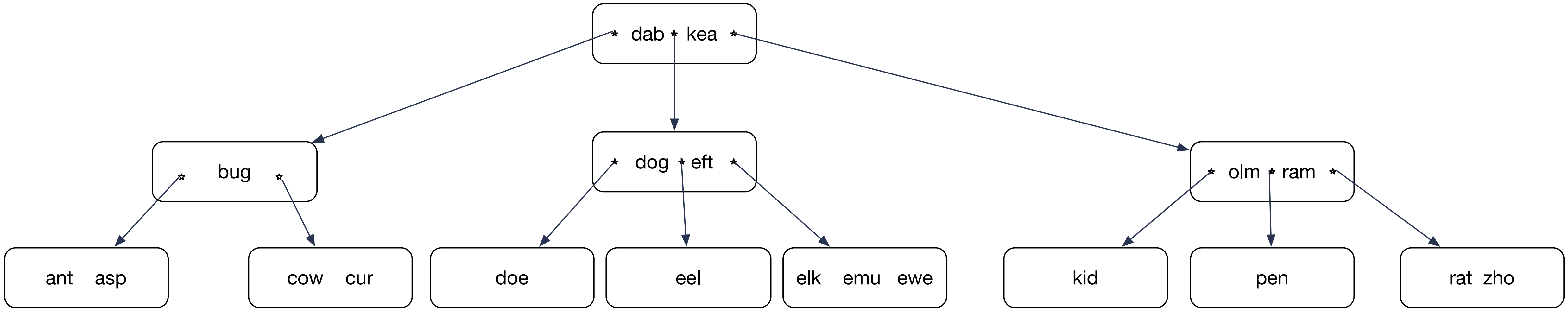
B-tree

- Insert ewe into



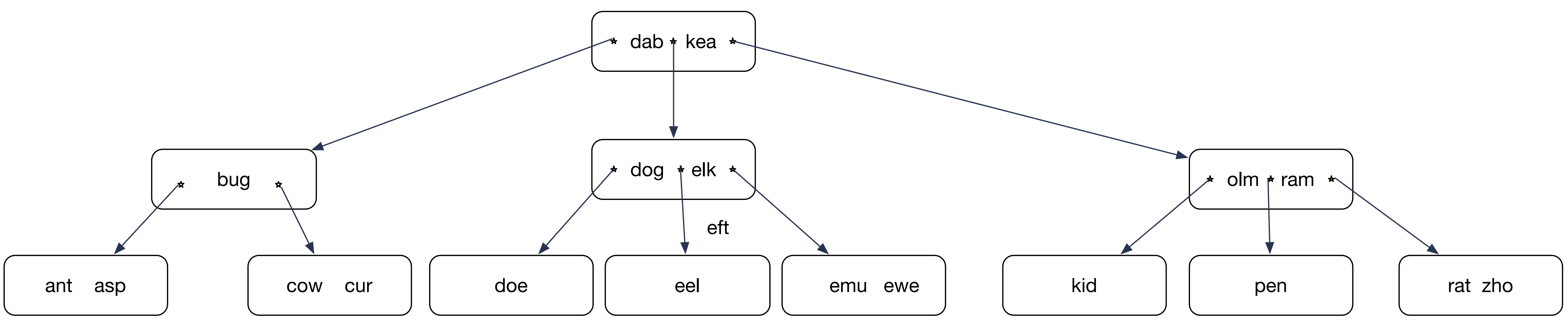
B-tree

- Insert ewe



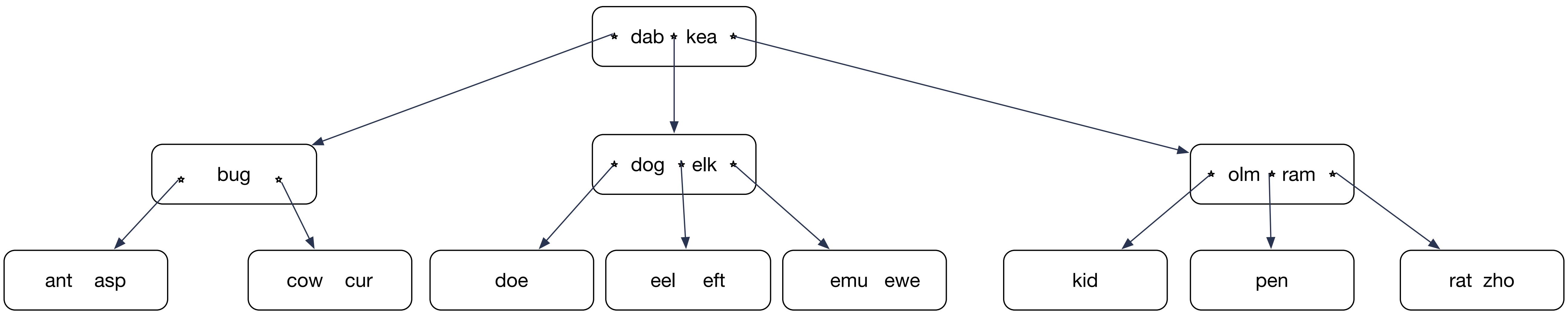
B-tree

- Promote elk. elk is guaranteed to come right after eft.
- Demote eft



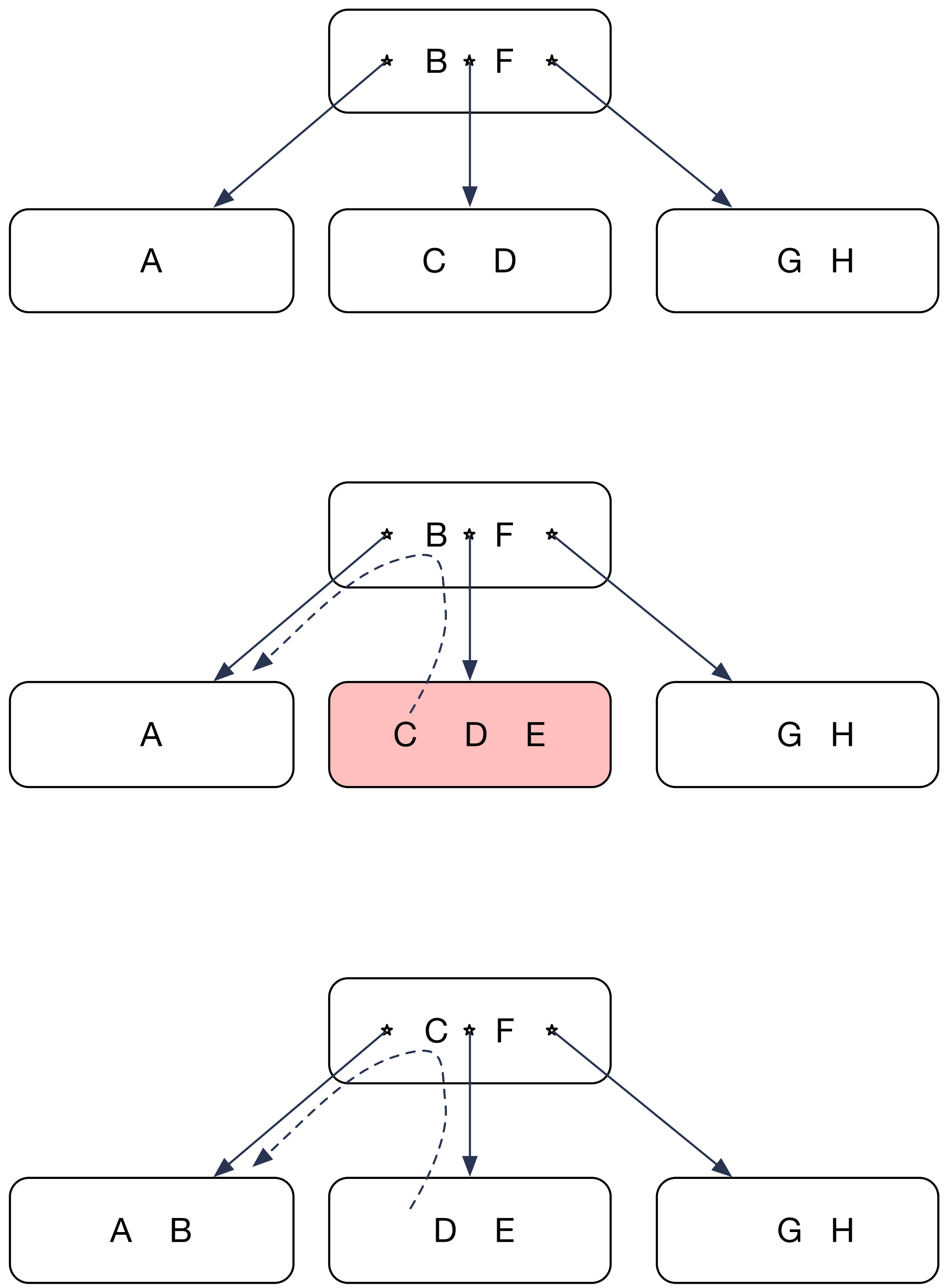
B-tree

- Insert eft into the leaf node

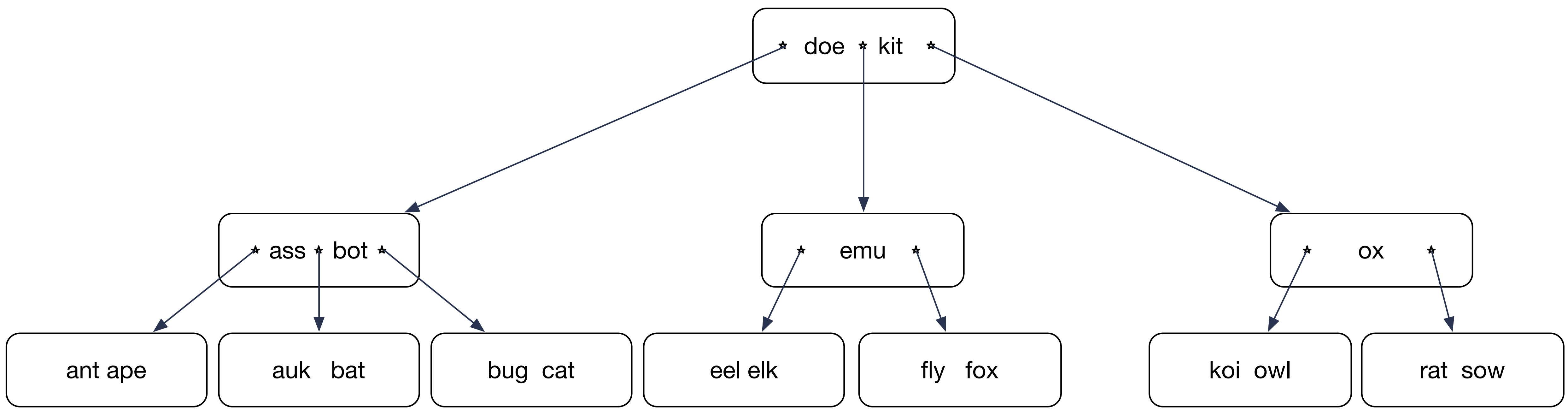


B-tree

- Left rotate
  - Overflowing node has a sibling to the left with space
  - Move left-most key up
  - Lower left-most key

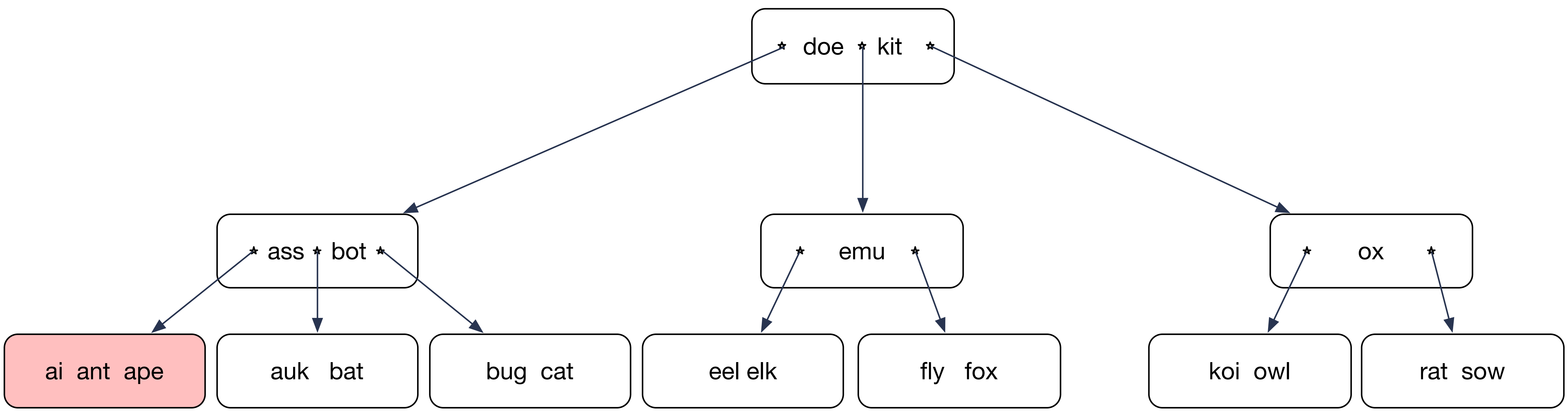


B-tree



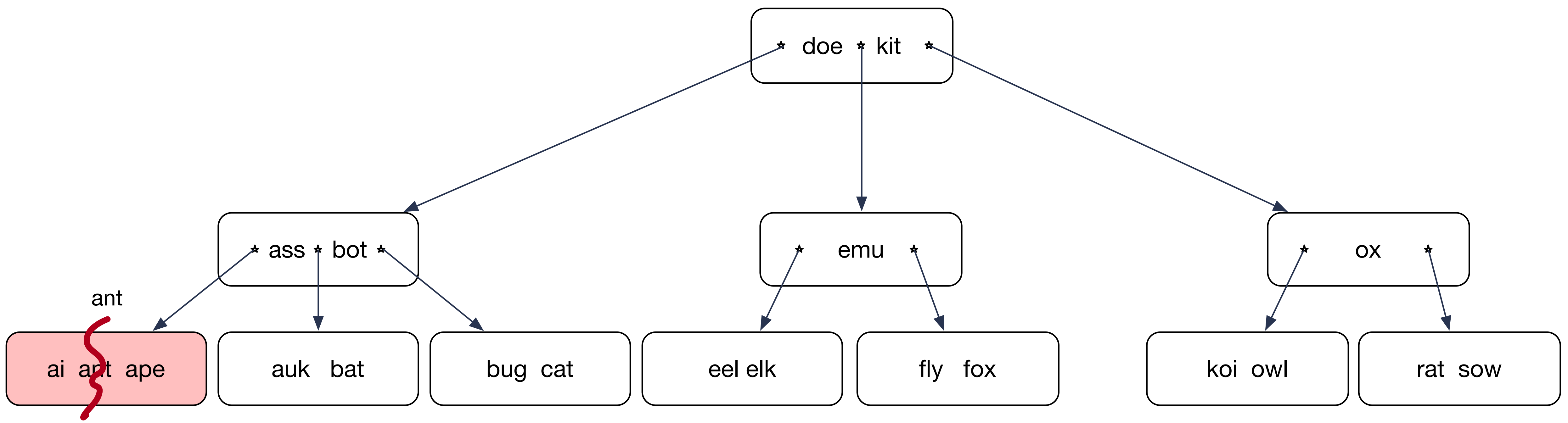
Now insert “ai”

B-tree



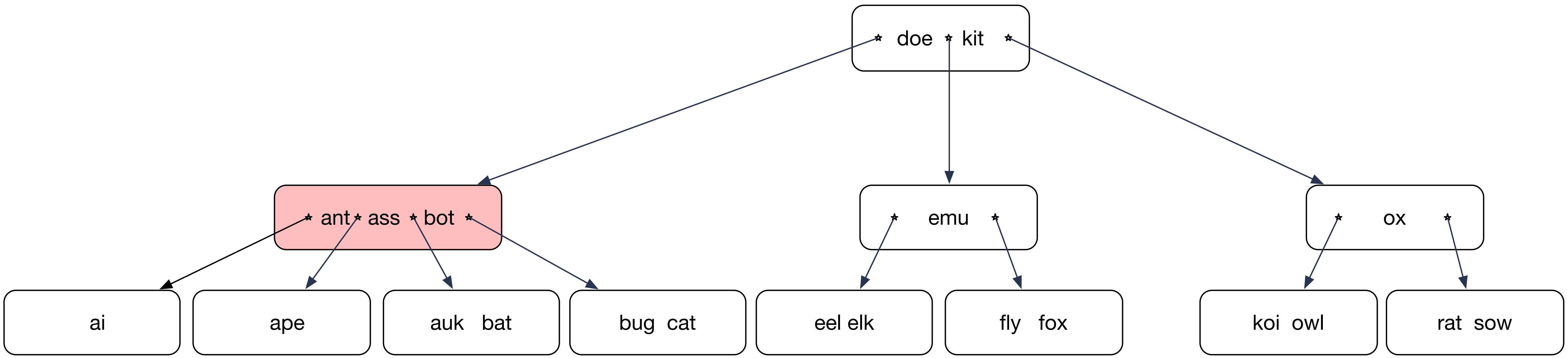
Insert creates an overflowing node
Only one neighboring sibling, but that one is full
Split!

B-tree



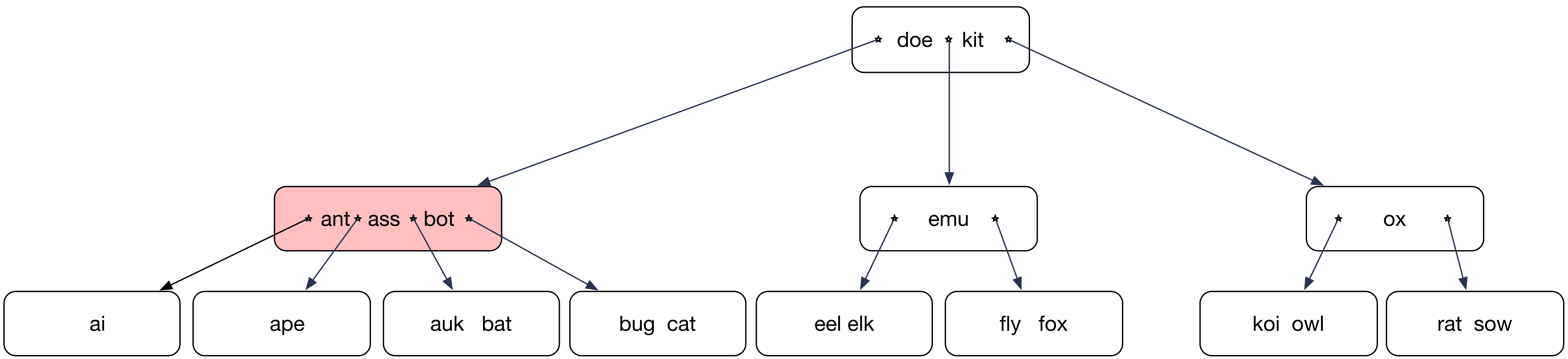
Middle key moves up

B-tree



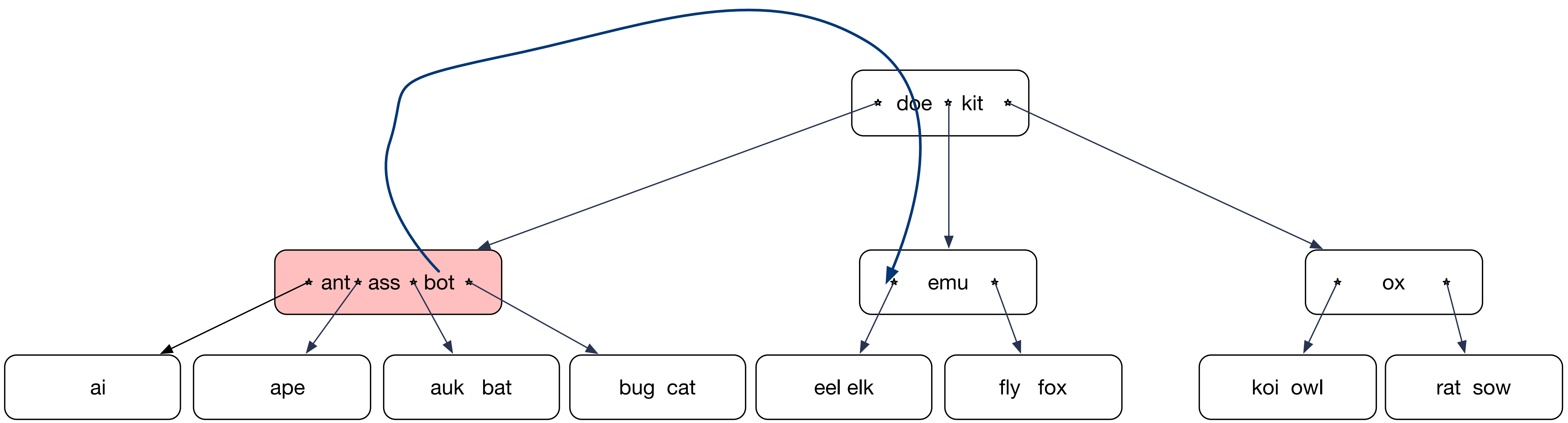
Unfortunately, this gives another overflow
But this node has a right sibling not at full capacity

B-tree



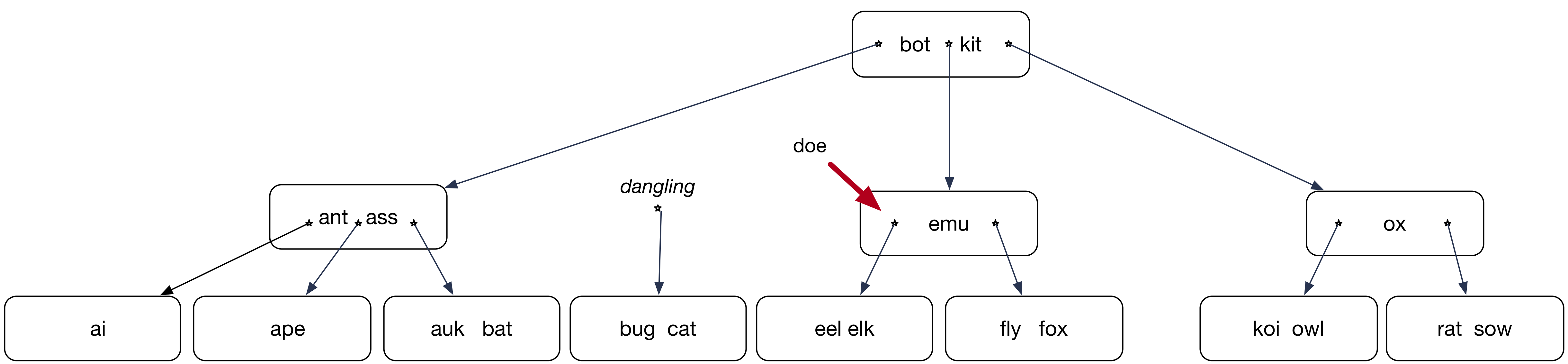
Right rotate:
Move “bot” up
Move “doe” down
Reattach nodes

B-tree



Move “bot” up
Move “doe” down
Reattach the dangling node

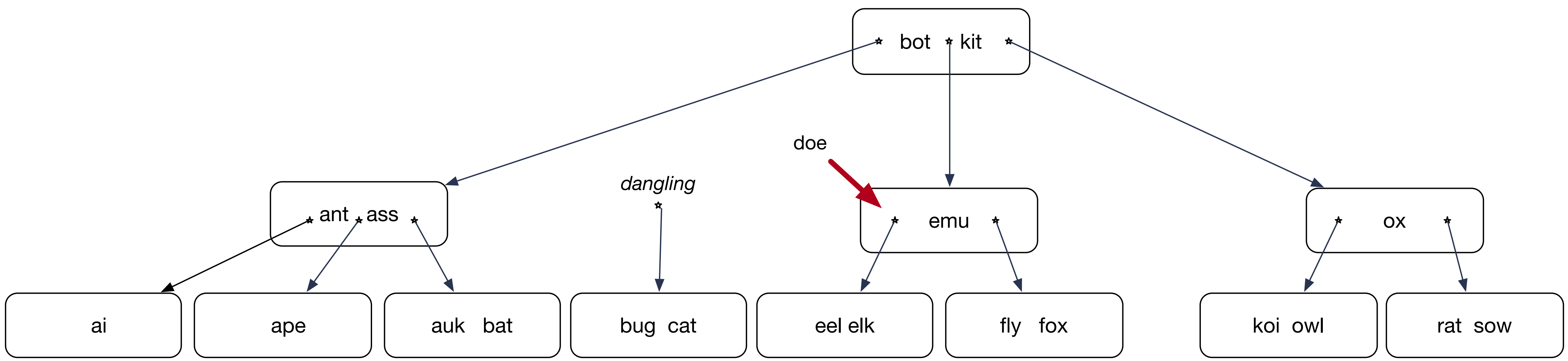
B-tree



“bot” had moved up and replaced doe

The “emu” node needs to receive one key and one pointer

B-tree


bot kit
doe
dangling
ant ass
emu
ox
ai
ape
auk bat
bug cat
eel elk
fly fox
koi owl
rat sow

B-tree

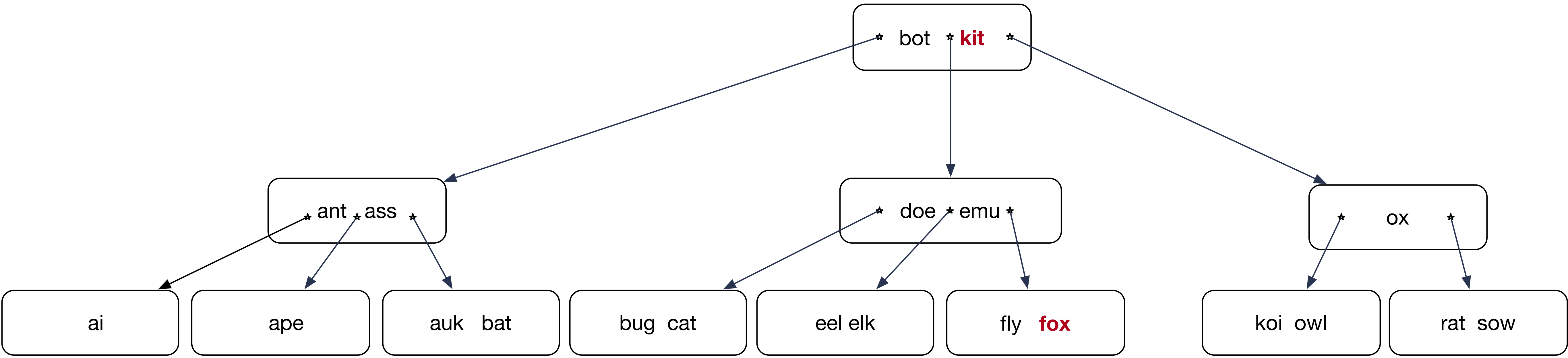
- Deletes
  - Usually restructuring not done because there is no need
  - Underflowing nodes will fill up with new inserts

B-tree

- Implementing deletion anyway:
  - Can only remove keys from leaves
  - If a delete causes an underflow, try a rotate into the underflowing node
  - If this is not possible, then merge with a sibling
    - A merge is the opposite of a split
  - This can create an underflow in the parent node
    - Again, first try rotate, then do a merge

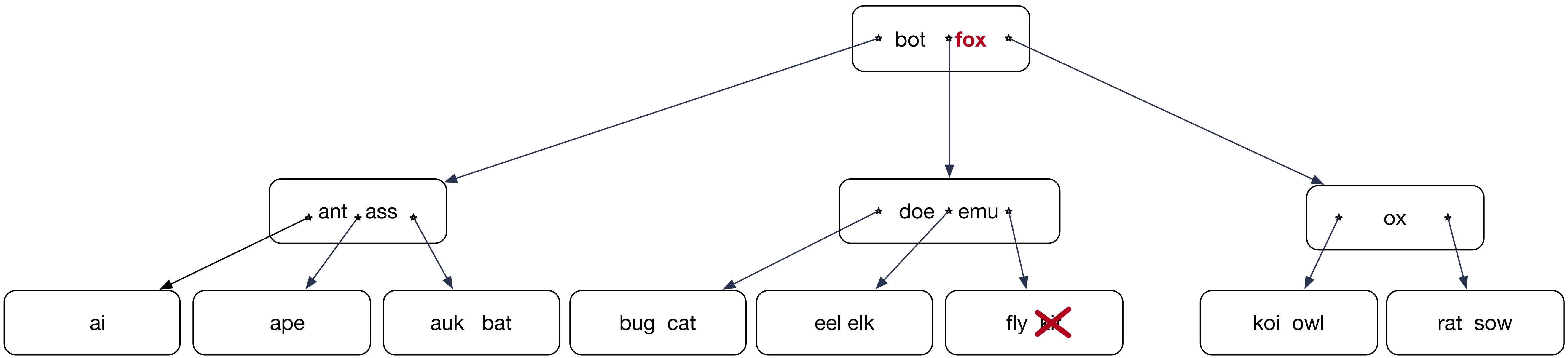
B-tree

Delete “kit”



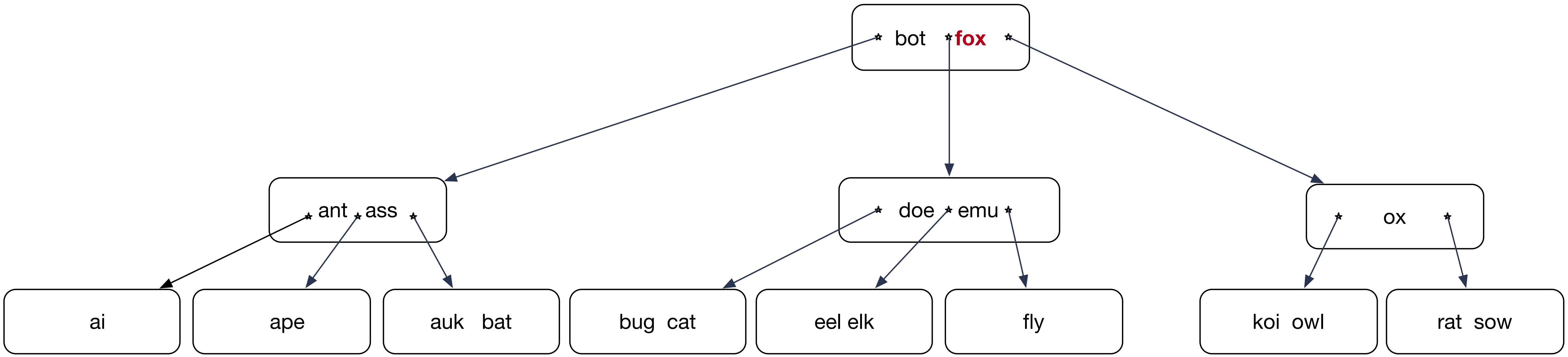
Delete “kit”
“kit” is in an interior node.
Exchange it with the key in the leave immediately before
“fox”

B-tree



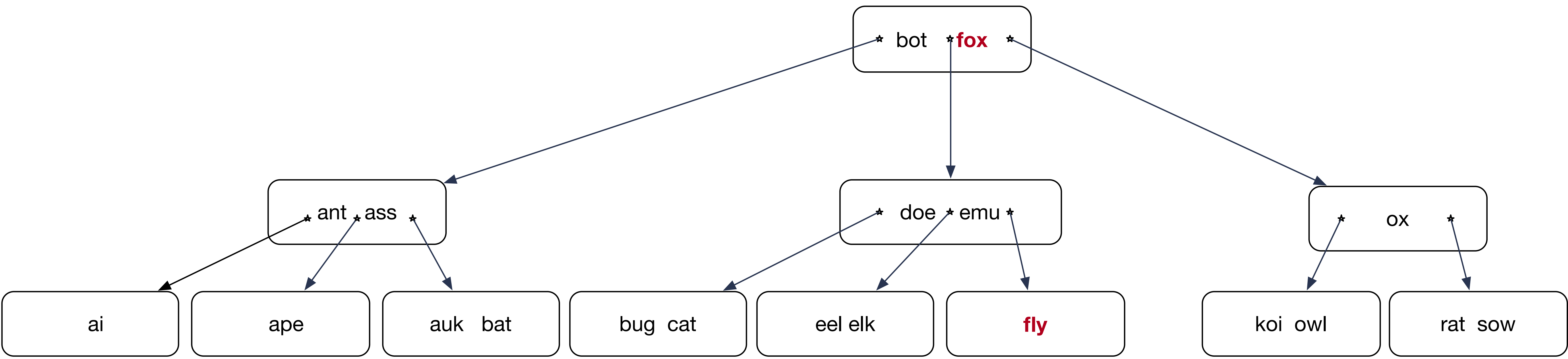
After interchanging “fox” and “kit”, can delete “kit”

B-tree



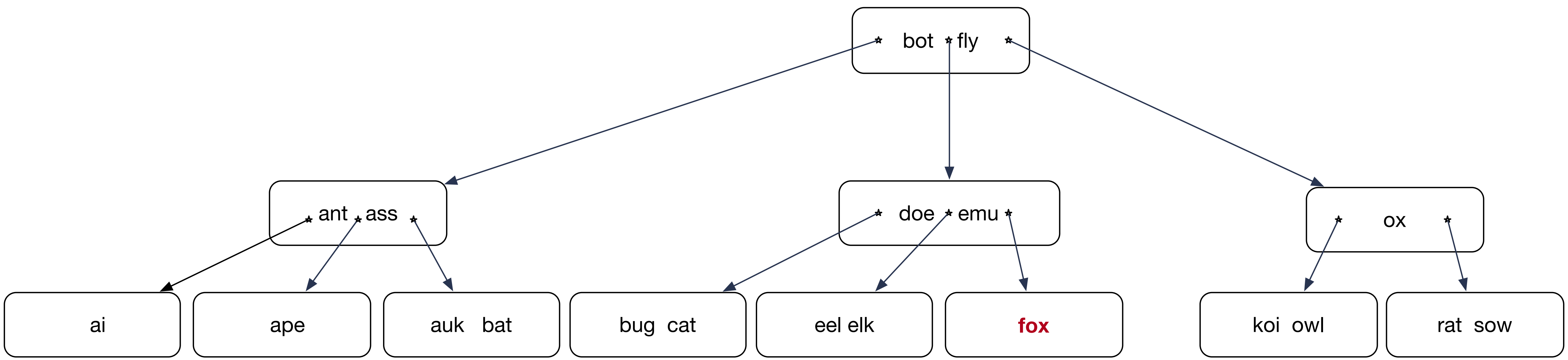
Now delete “fox”

B-tree



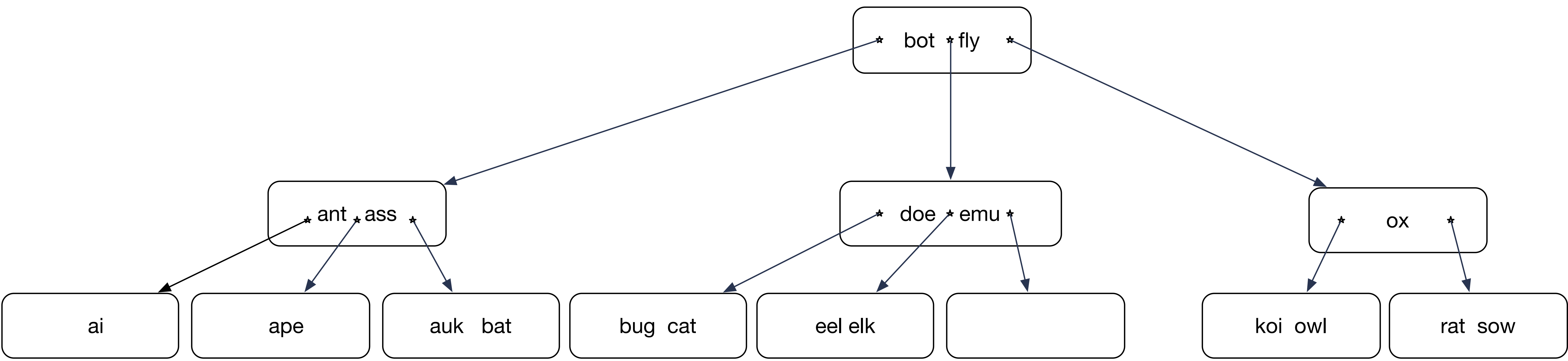
Step 1: Find the key. If it is not in a leaf
Step 2: Determine the key just before it, necessarily in a leaf
Step 3: Interchange the two keys

B-tree



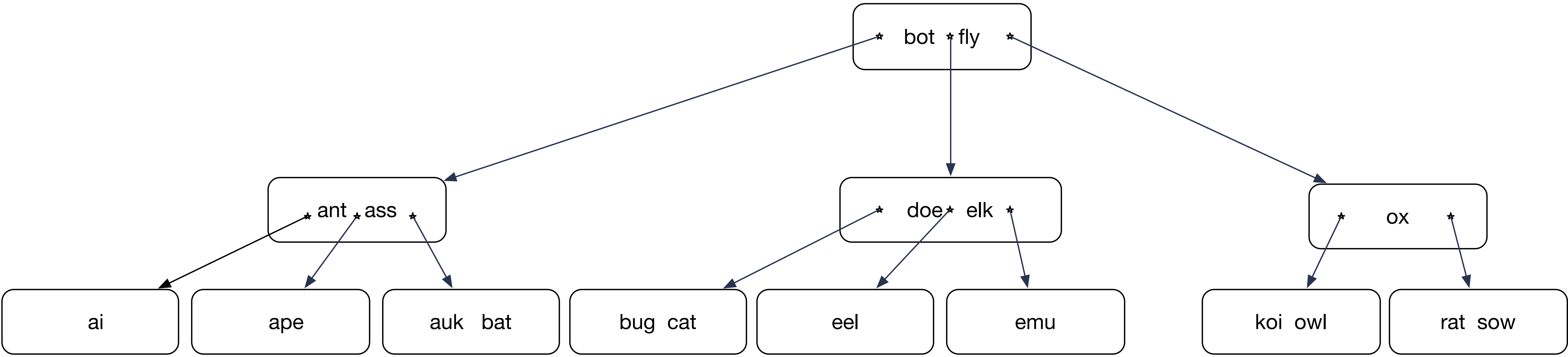
Step 4: Remove the key now from a leaf

B-tree



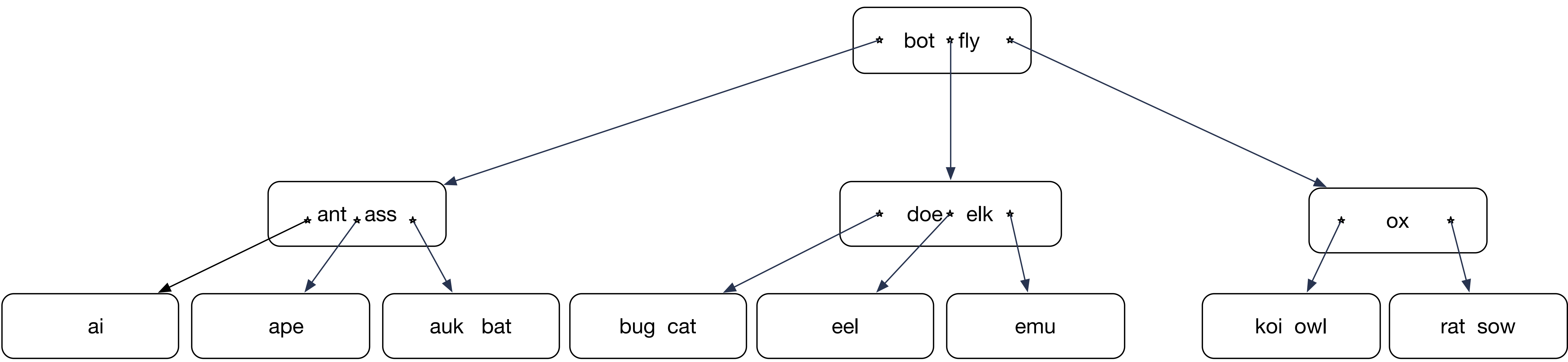
This causes an underflow
Remedy the underflow by right rotating from the sibling

B-tree



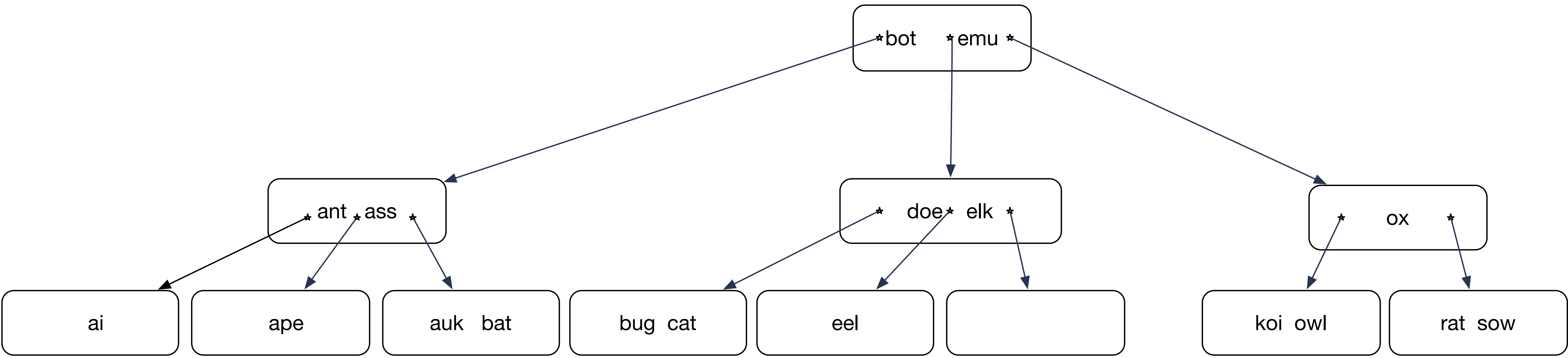
Everything is now in order

B-tree



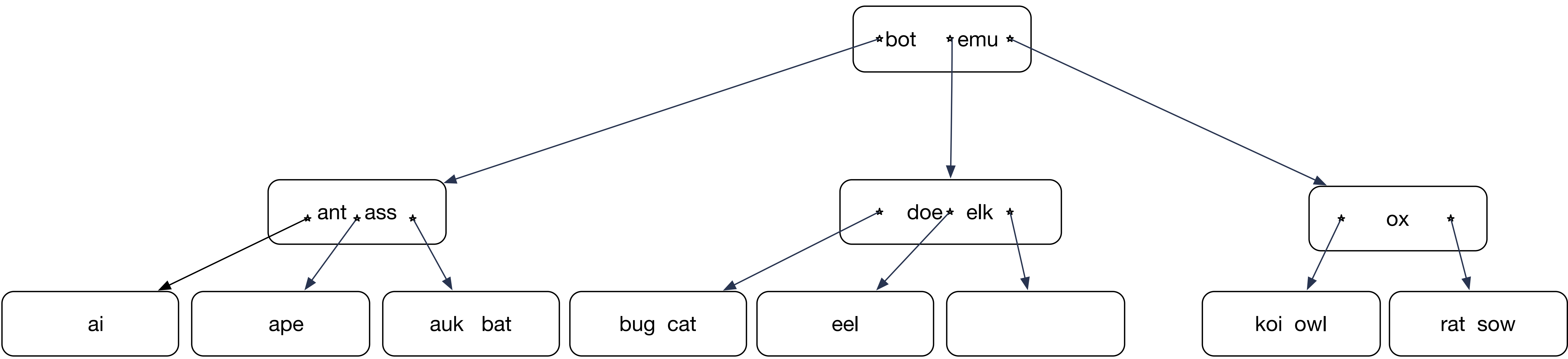
Now delete fly

B-tree



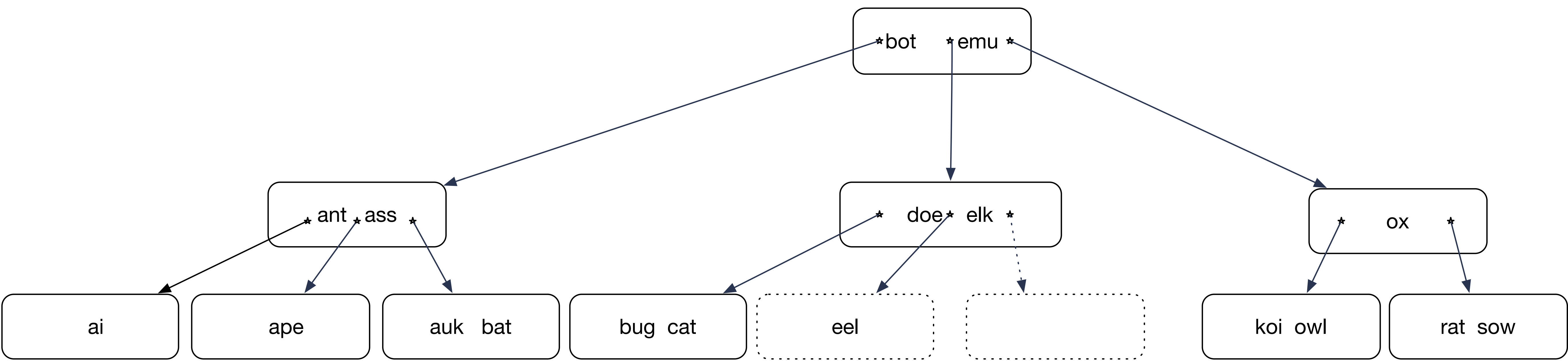
Switch “fly” with “emu”
remove “fly” from the leaf
Again: underflow

B-tree



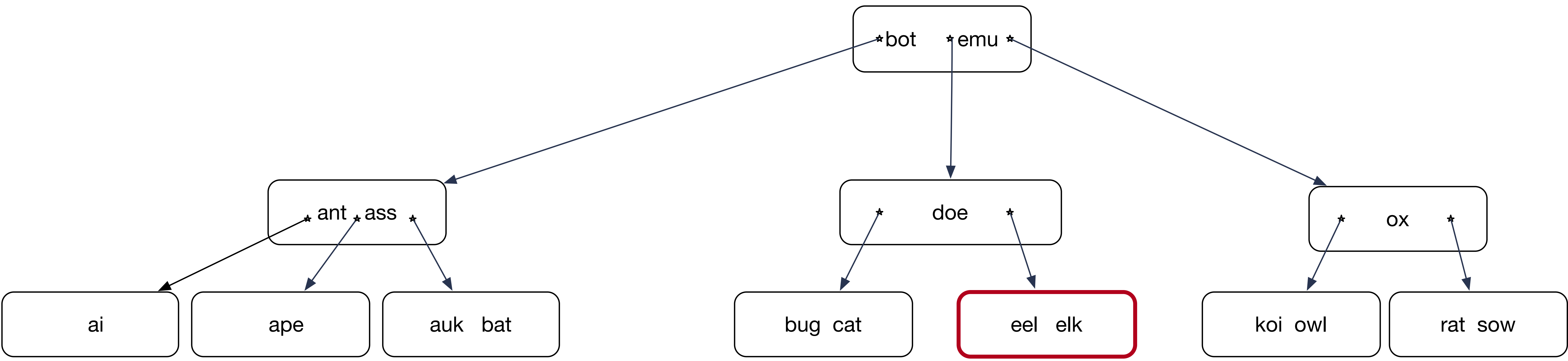
Cannot left-rotate: There is no left sibling
Cannot right-rotate: The right sibling has only one key
Need to merge: Combine the two nodes by bringing down “elk”

B-tree



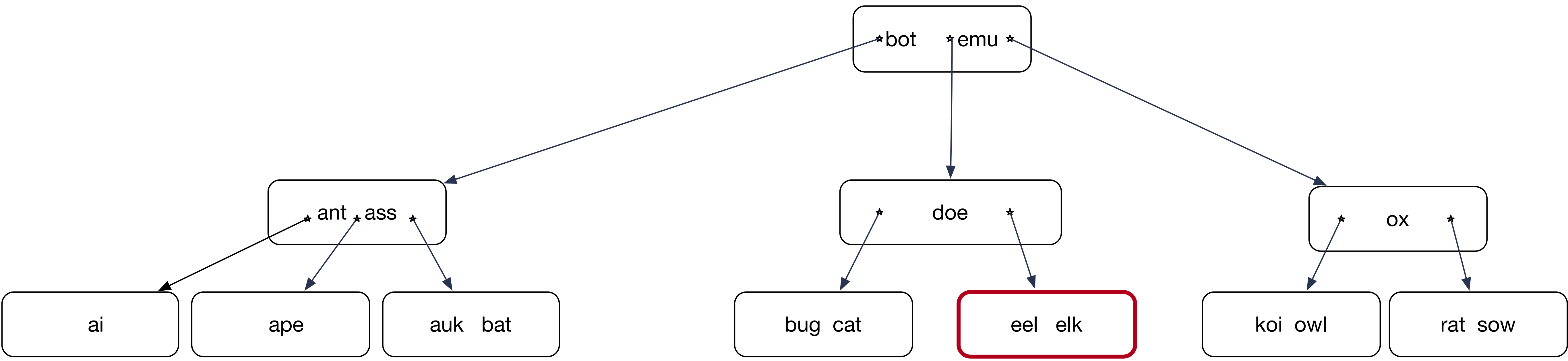
We can merge the two nodes because the number of keys combined is less than 2 *k*

B-tree



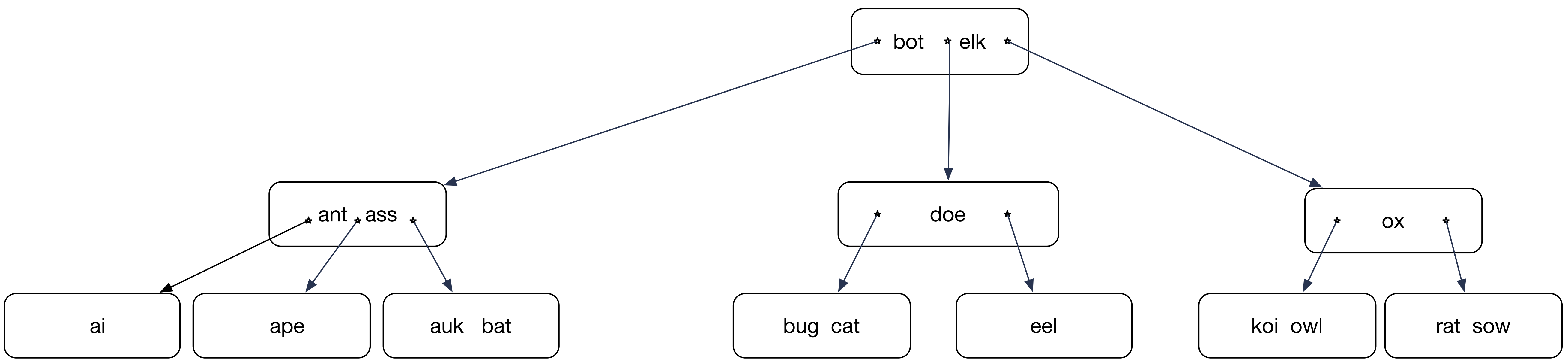
bot emu
ant ass
doe
ox
ai
ape
auk bat
bug cat
eel elk
koi owl
rat sow

B-tree



Delete “emu”

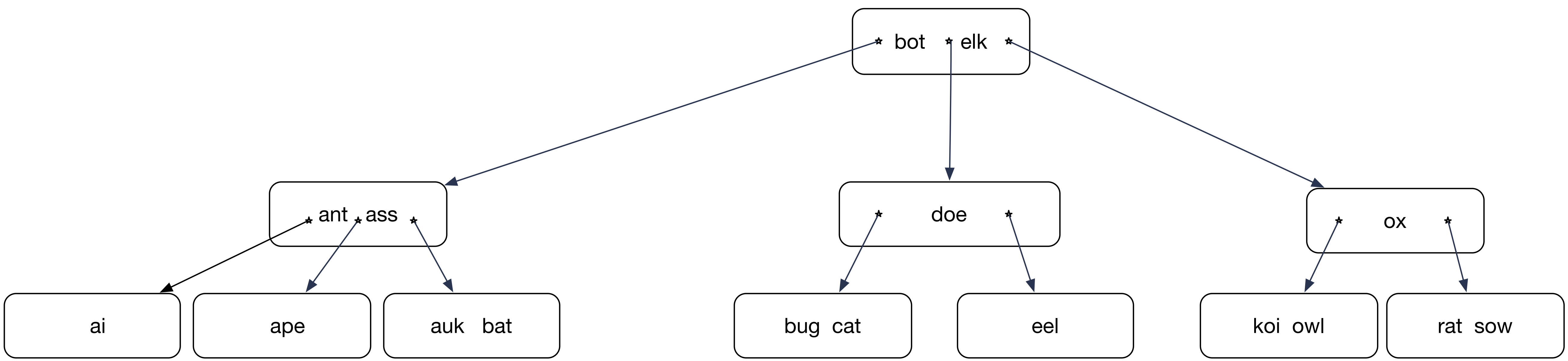
B-tree



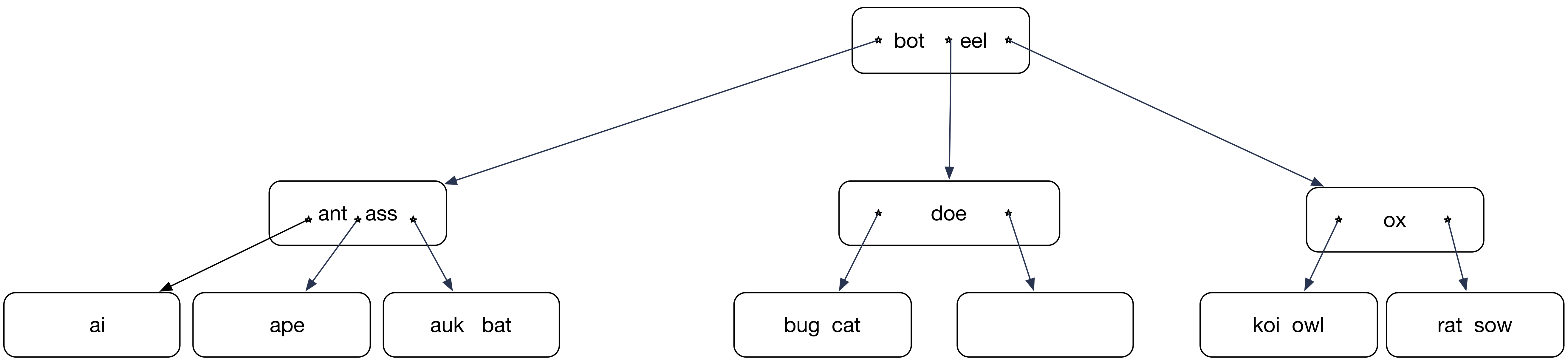
Switch predecessor, then delete from node

B-tree

Now delete “elk”

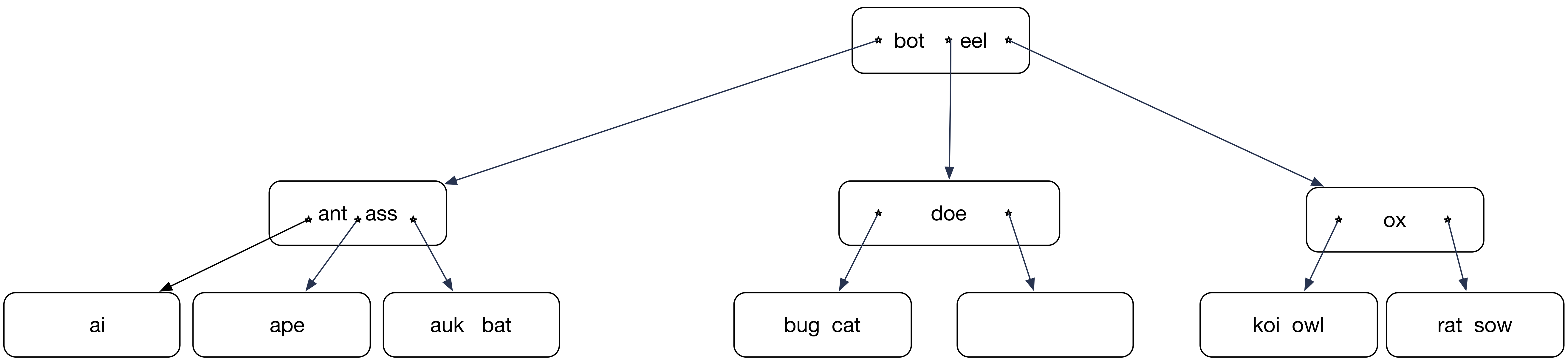


B-tree



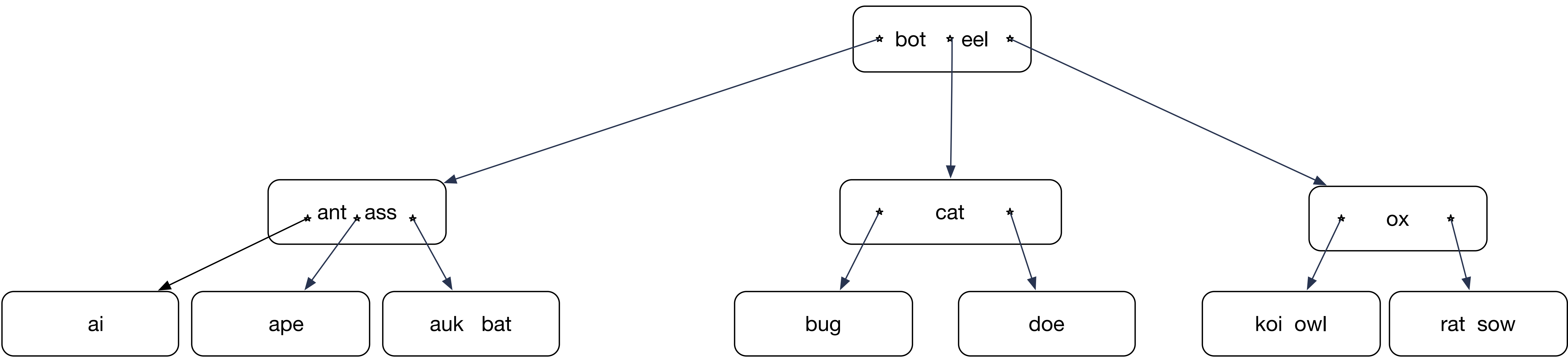
Results in an underflow

B-tree



Results in an underflow
But can rotate a key into the underflowing node

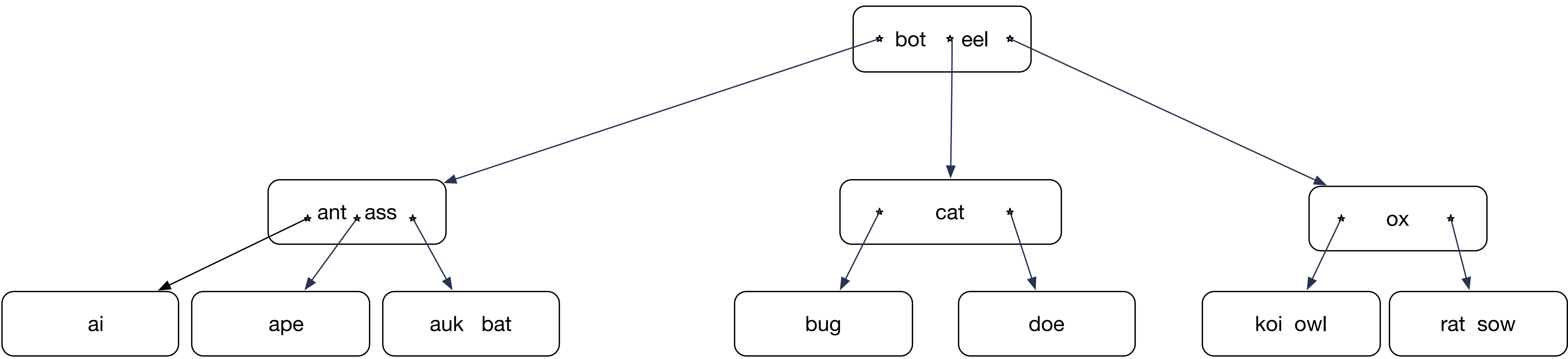
B-tree



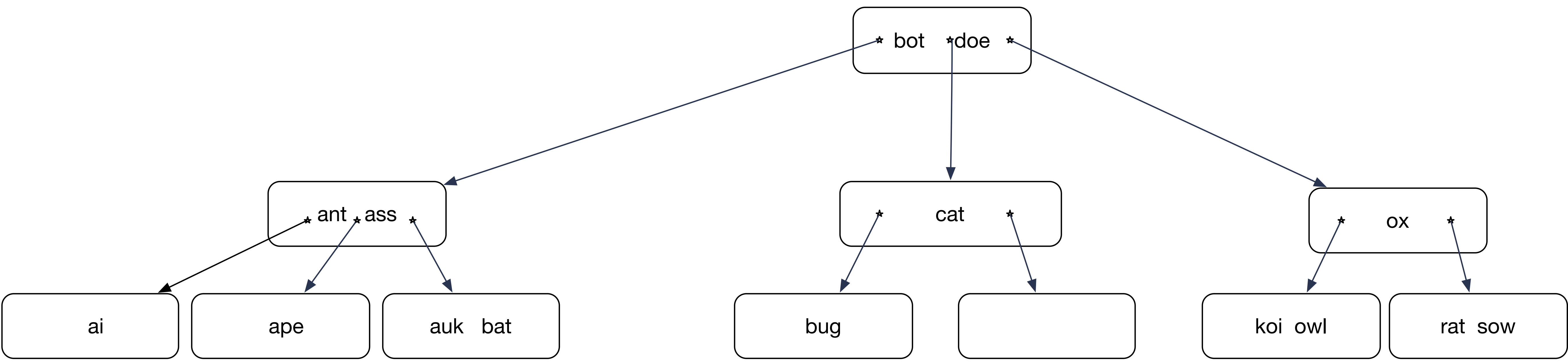
Result after left-rotation

B-tree

“Now delete “eel”

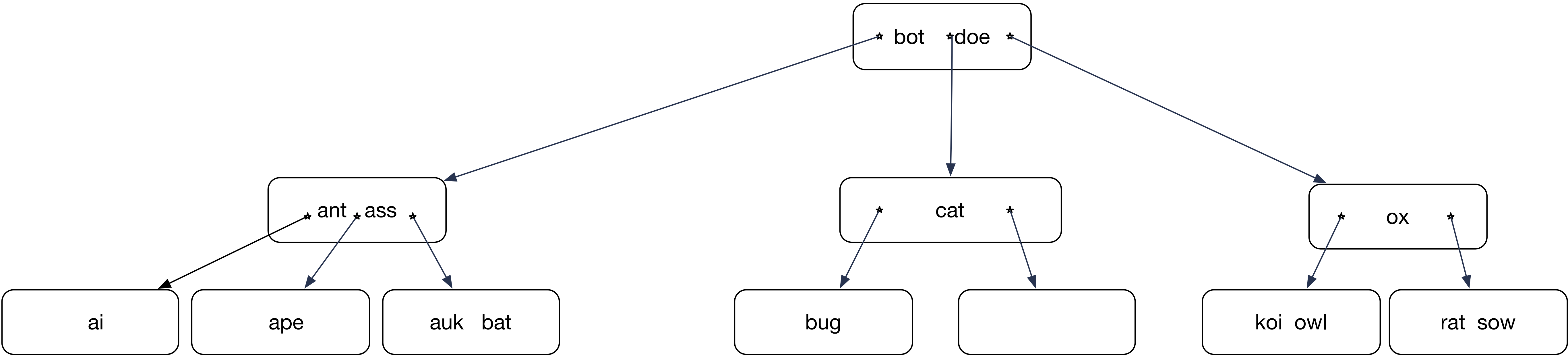


B-tree



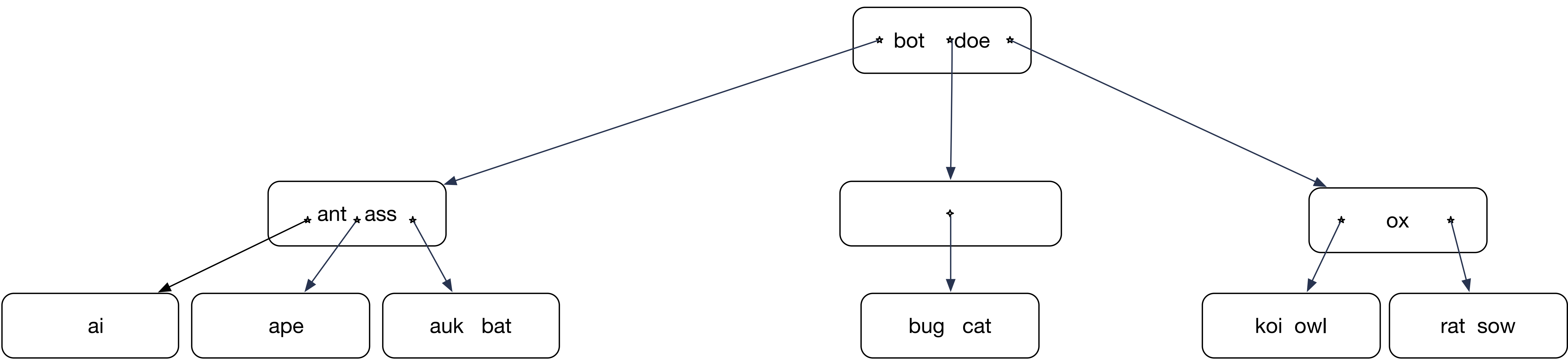
Interchange “eel” with its predecessor
Delete “eel” from leaf:
Underflow

B-tree



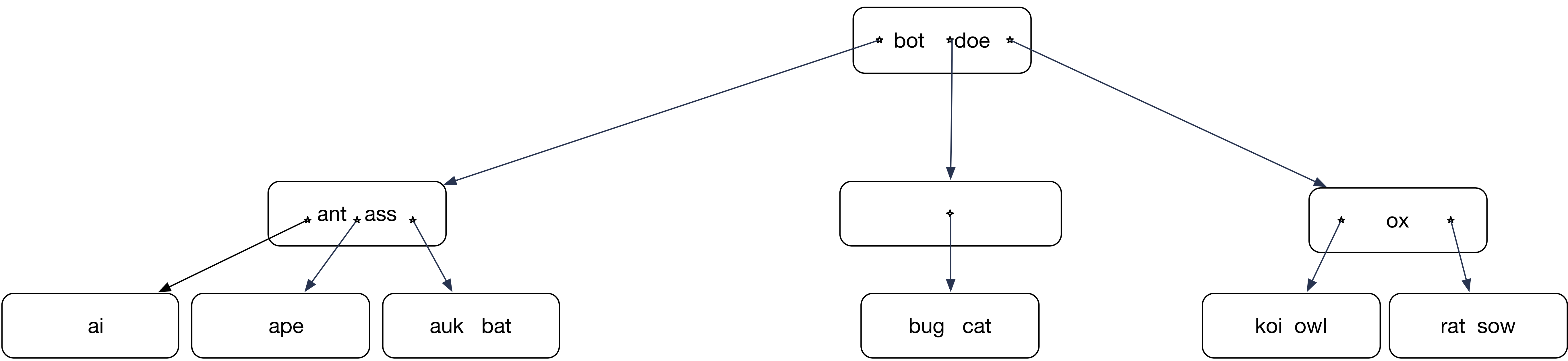
Need to merge

B-tree



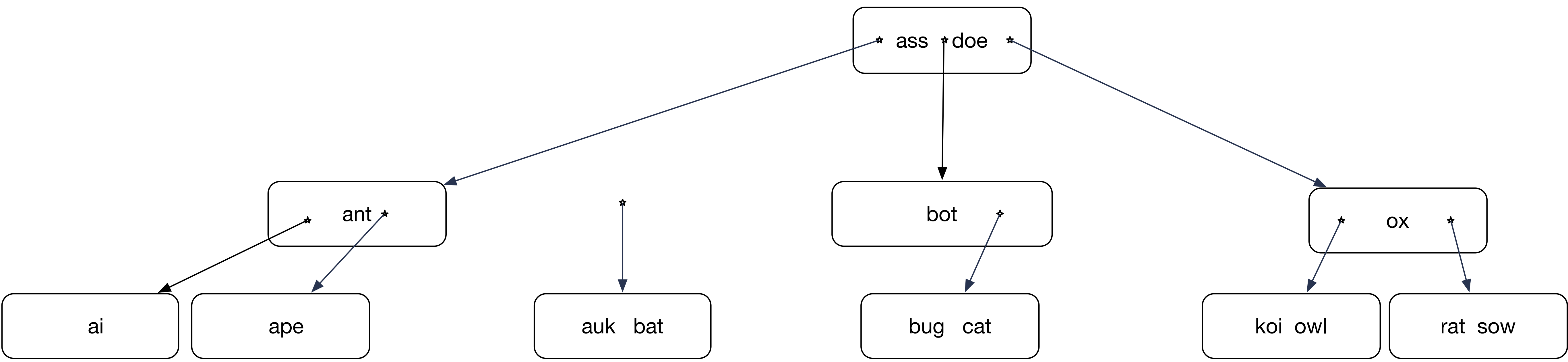
Merge results in another underflow
Use right rotate
(though merge with right sibling
is possible)

B-tree



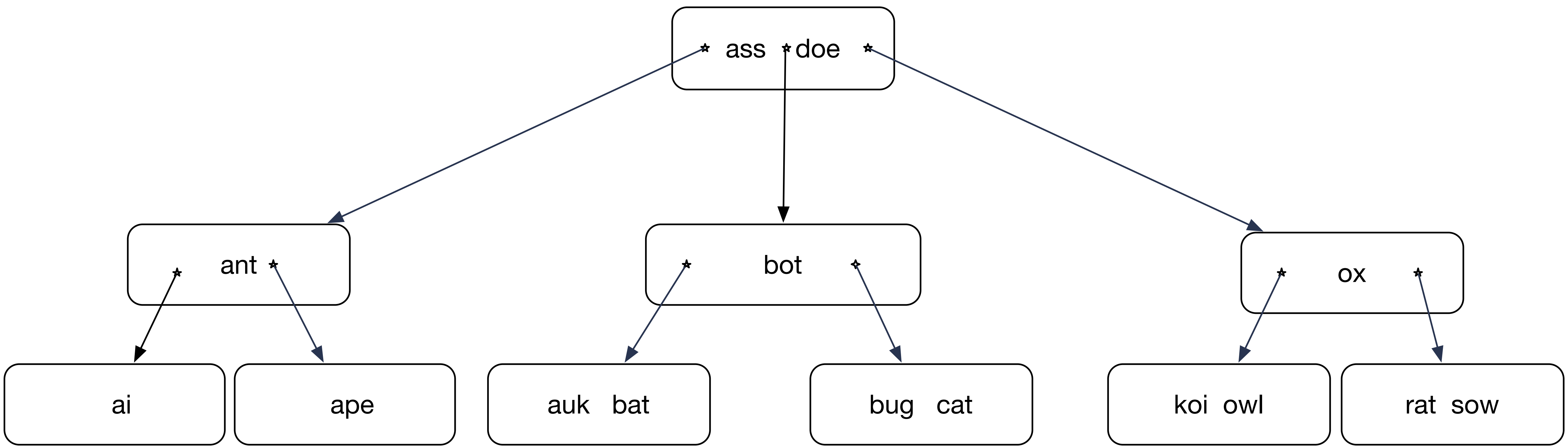
“ass” goes up, “bot” goes down
One node is reattached

B-tree

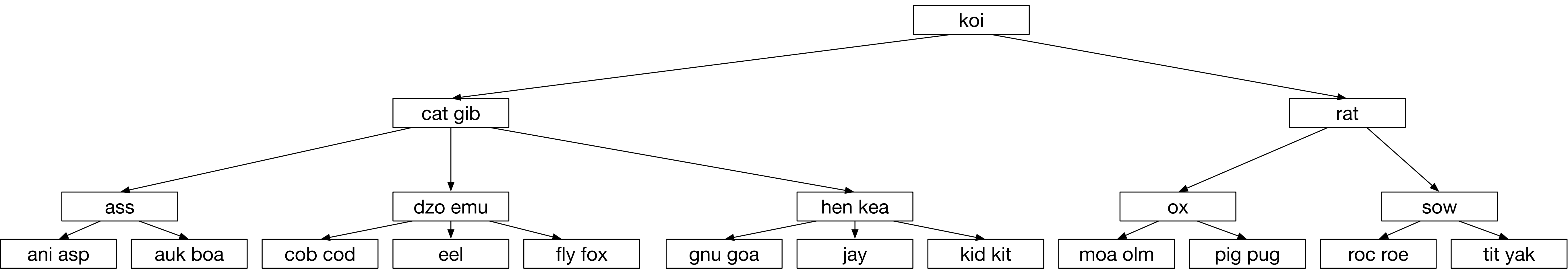


Reattach node

B-tree

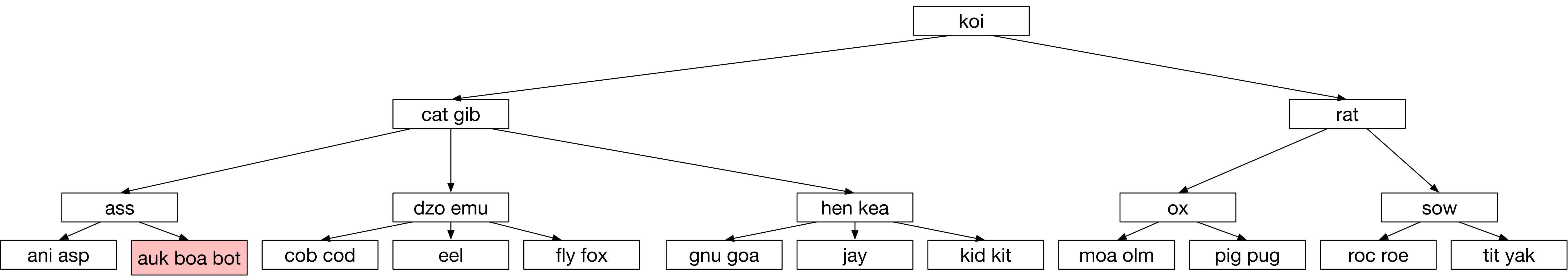


Activities



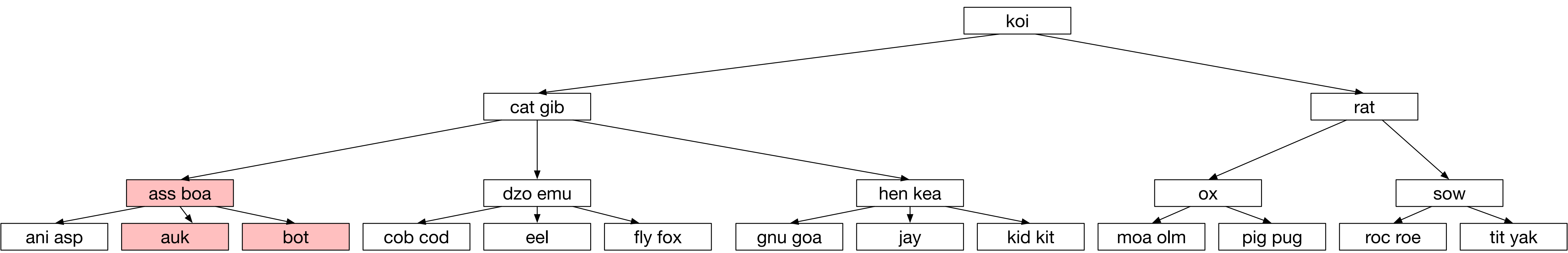
- Insert bot

Activities



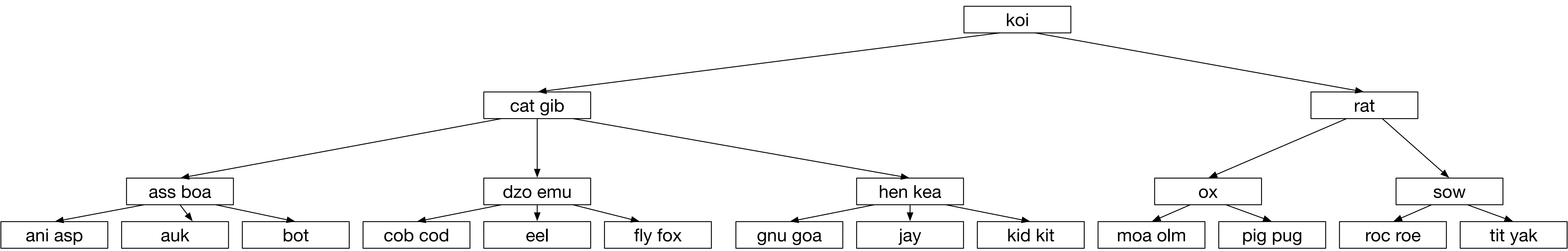
- Inserting bot in the leaf
  - Leads to an overflow

Activities



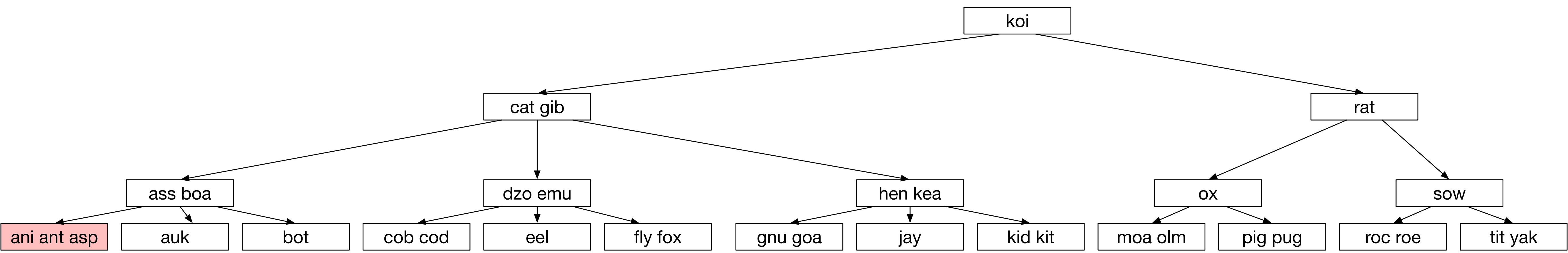
- Since rotates are not possible:
- Split: Middle node goes up, left and right become their own nodes

Activities



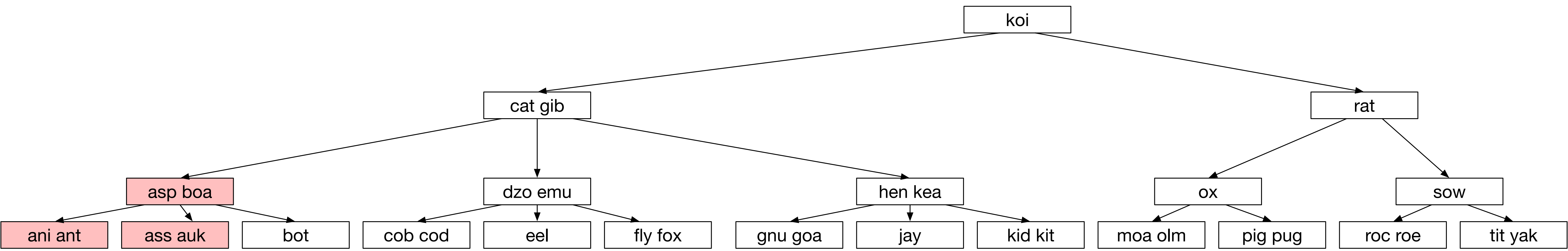
- Insert ant

Activities



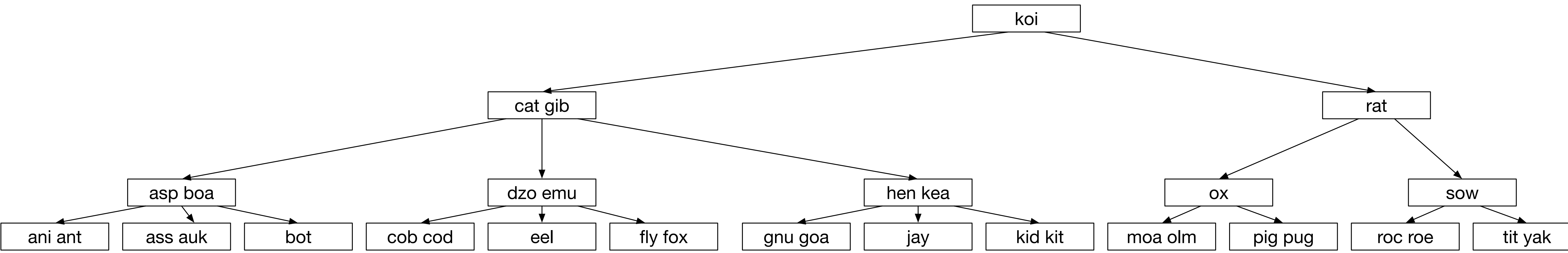
- Insert ant into leaf: Leaf is now at overflow

Activities



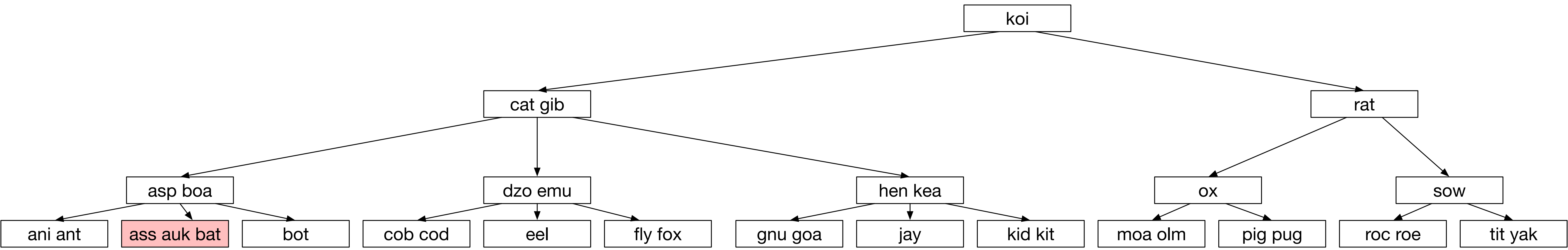
- Right rotate: asp goes up, ass goes down

Activities



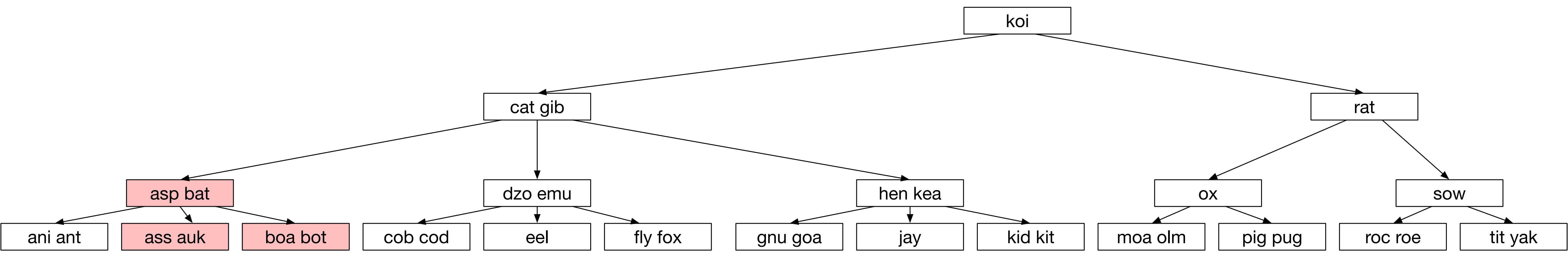
- Insert bat

Activities



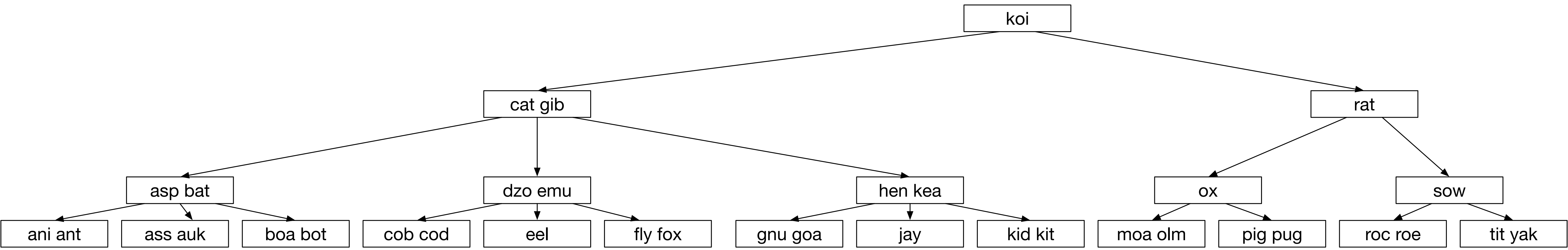
- Insert into leaf: overflow

Activities



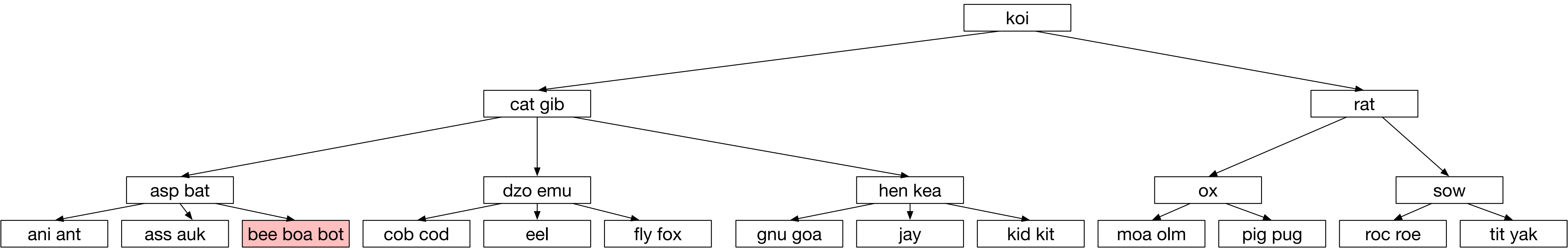
- Right rotate: bat goes up, boa goes down

Activities



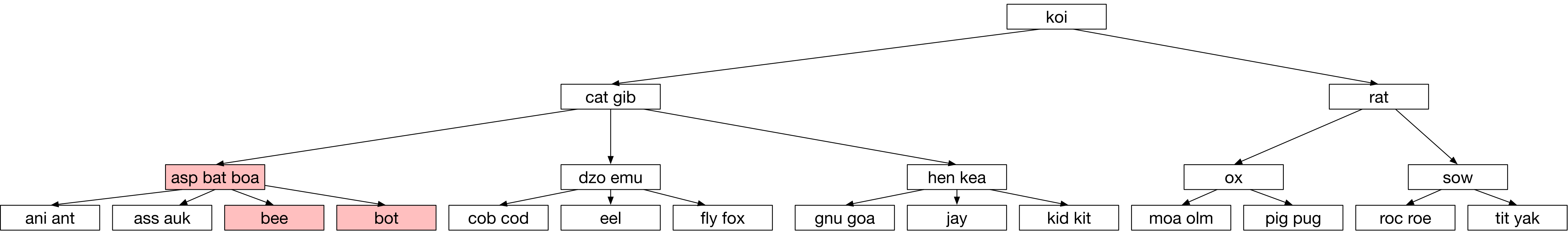
- Insert bee

Activities



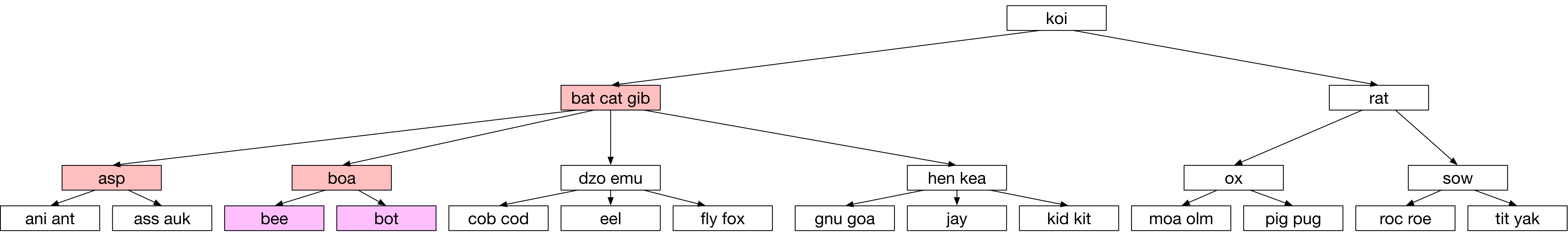
- Insert into leaf

Activities



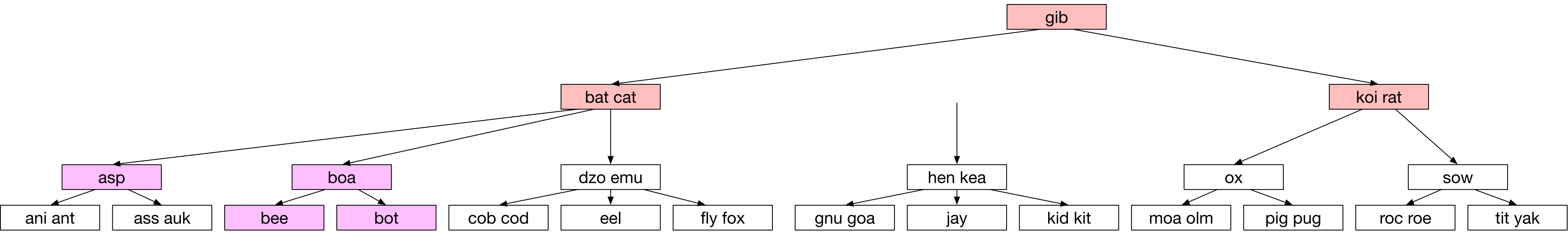
- Need to split: boa goes up, bee and bot get their own nodes
- Now there is an overflow at a higher level
- Rotate is not possible

Activities



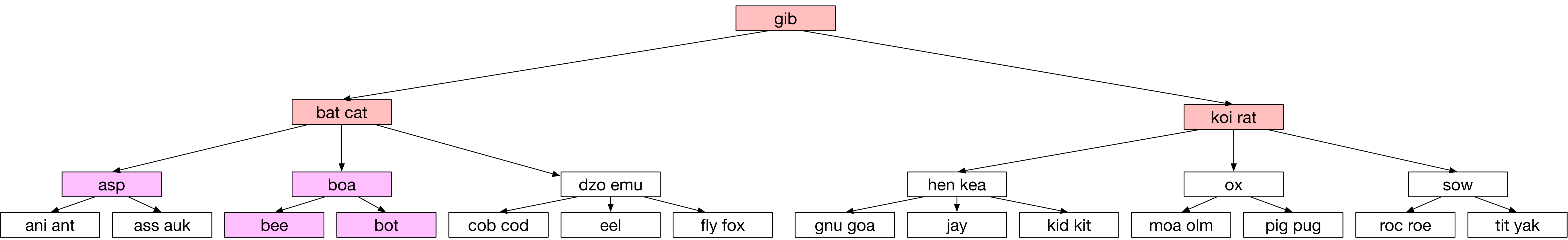
- Now we have yet another overflow
- Need to do a right rotate: gib goes up and koi goes down

Activities



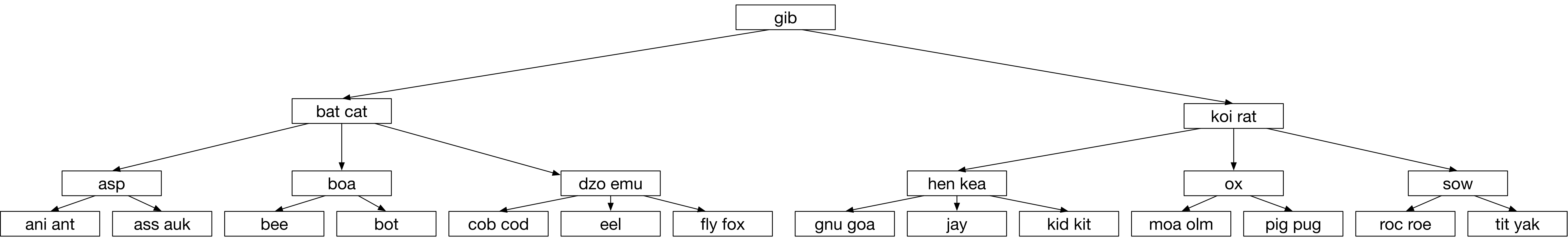
- Reattach the dangling pointer

Activities



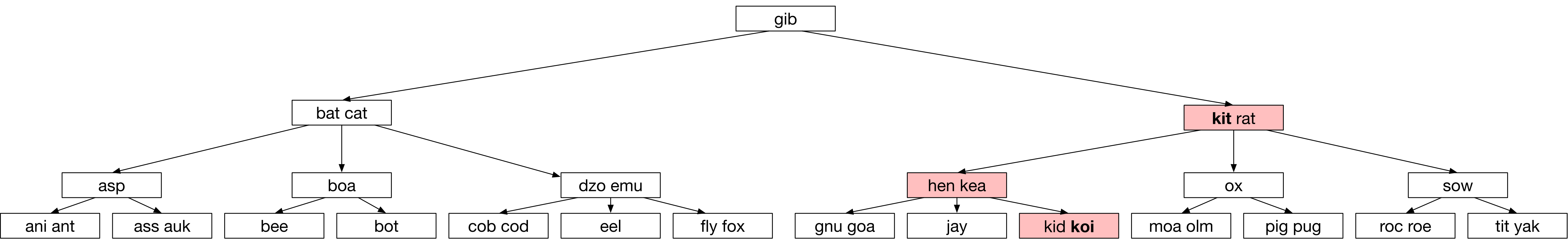
- Final result

Activities



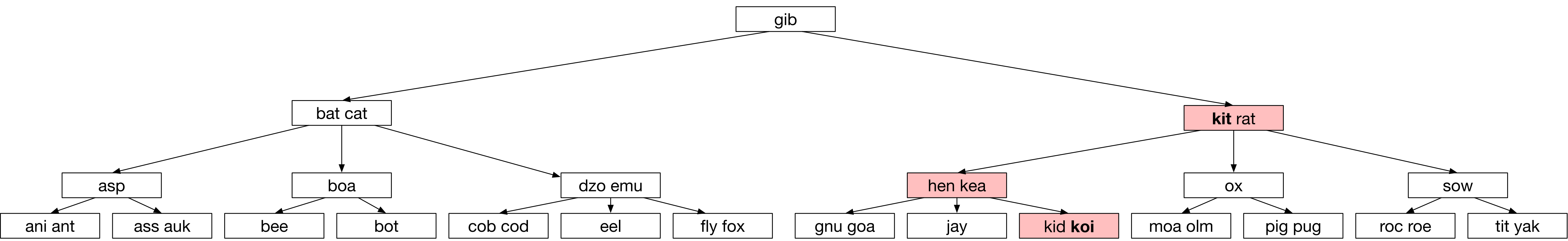
- Delete koi

Activities



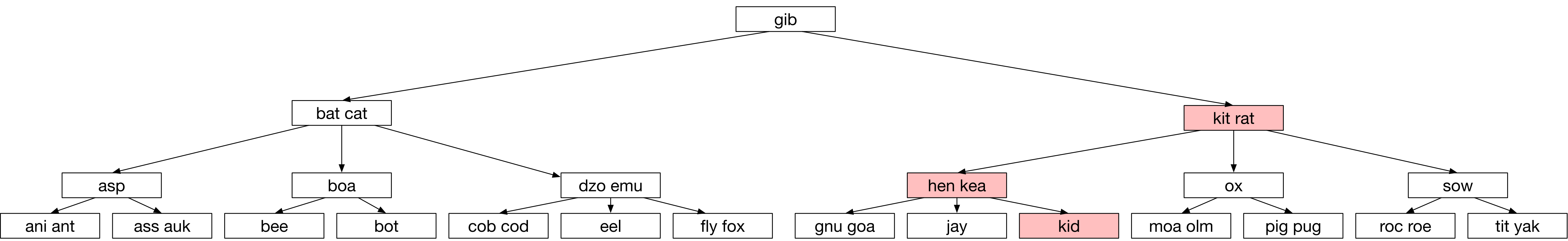
- Switch with predecessor

Activities



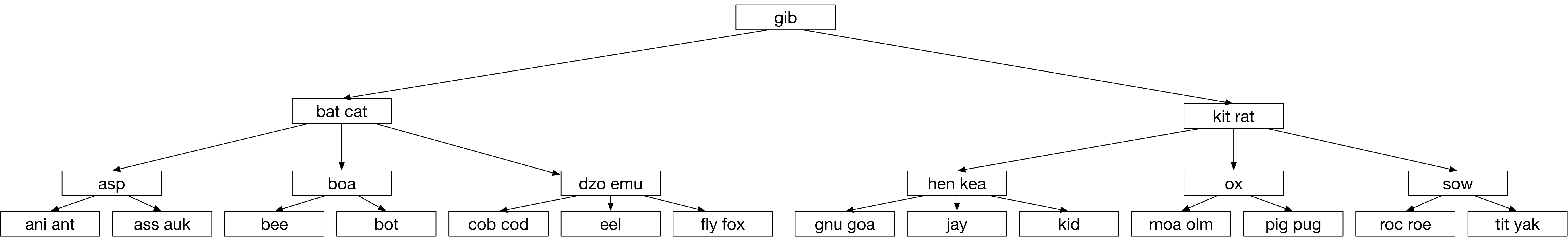
- Delete koi from leave

Activities



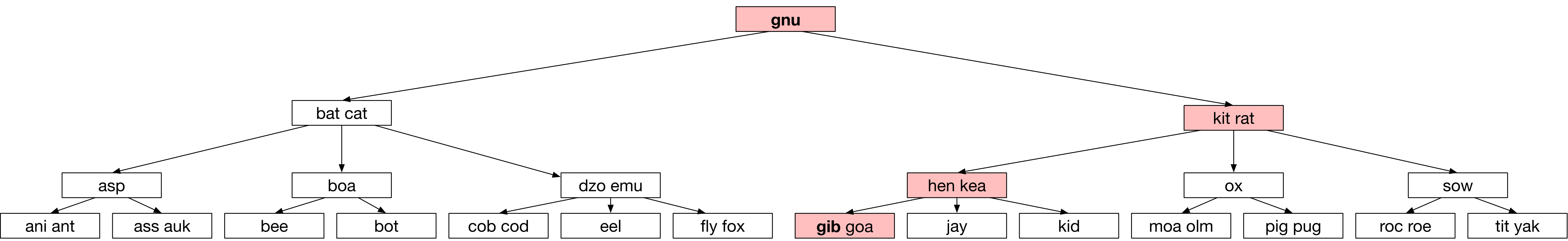
- Done

Activities



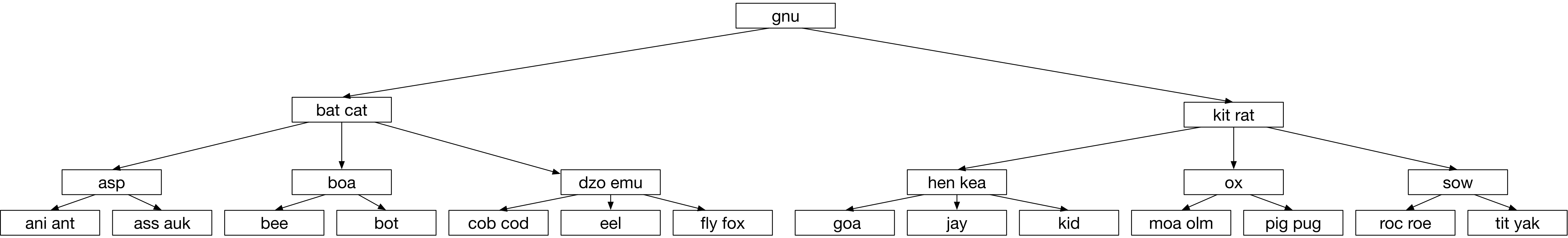
- Delete gib using successor

Activities



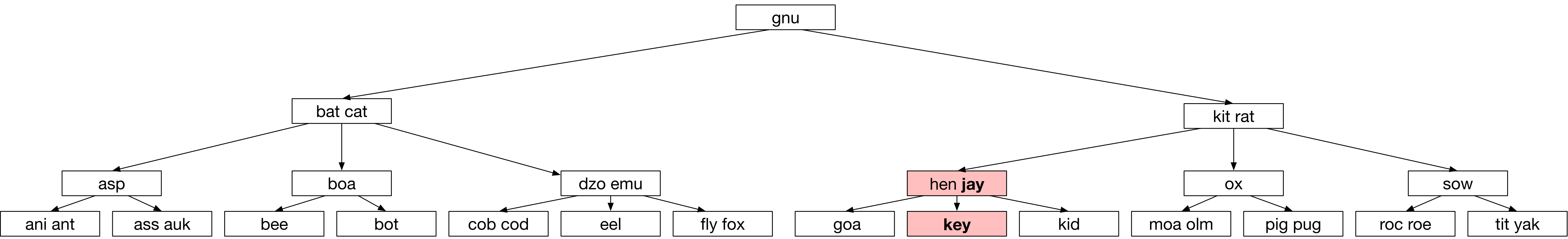
- After swap, delete gib from leaf

Activities



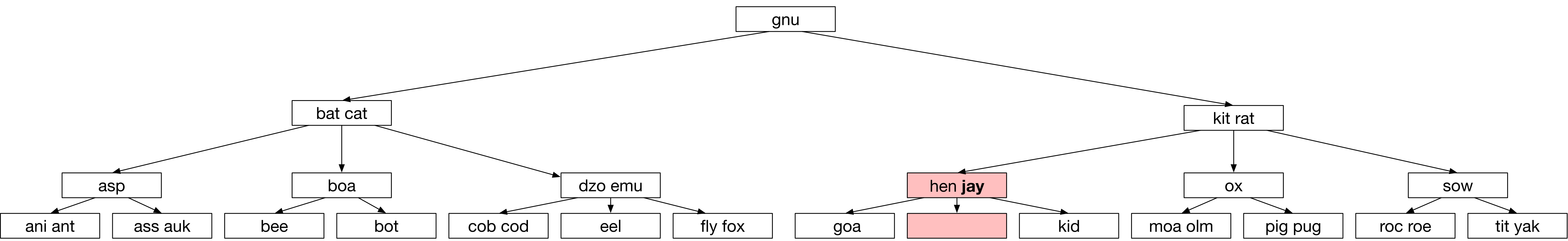
- Delete kea with predecessor

Activities



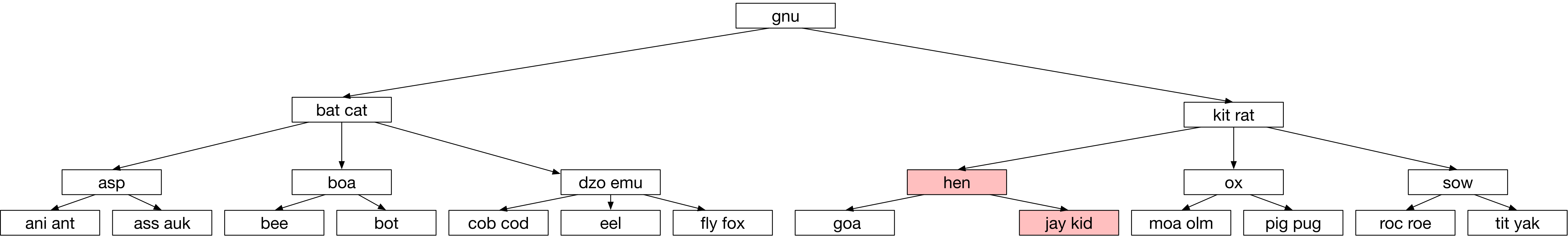
- Delete from leaf

Activities



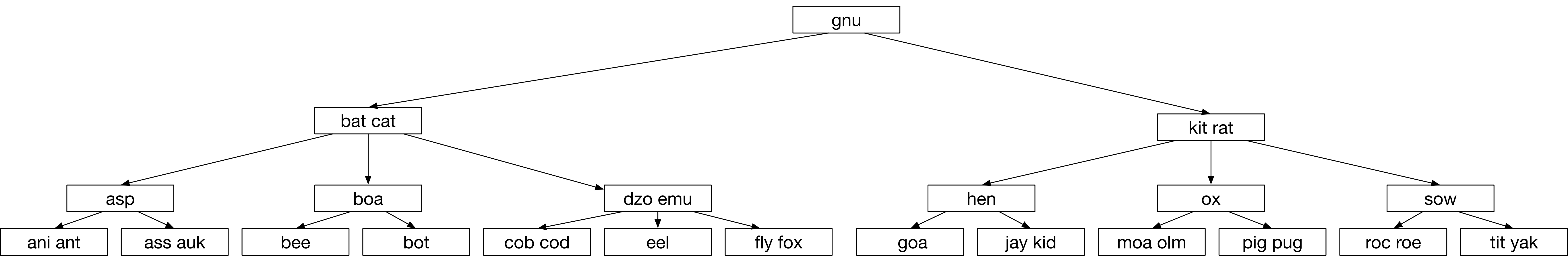
- Underflow:
- Rotate is not possible
- Merge: Two candidates, merge right

Activities



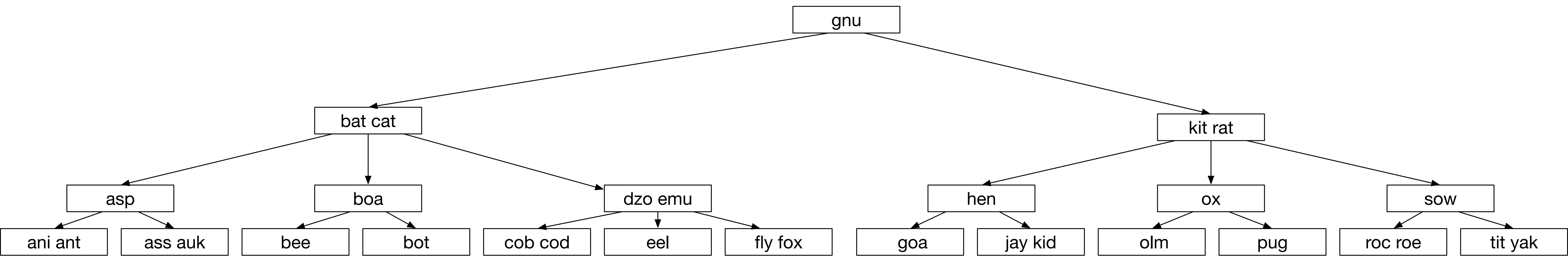
gnu
bat cat
kit rat
asp
boa
dzo emu
hen
ox
sow
ani ant
ass auk
bee
bot
cob cod
eel
fly fox
goa
jay kid
moa olm
pig pug
roc roe
tit yak

Activities



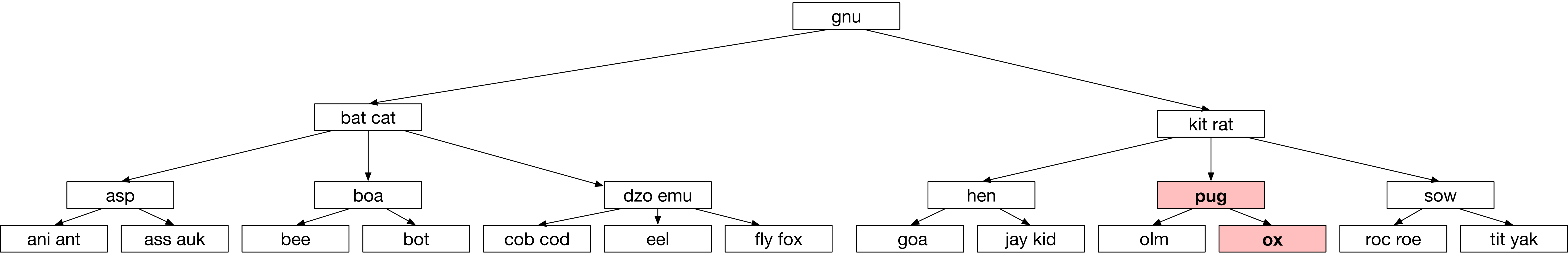
- Delete moa and pig

Activities



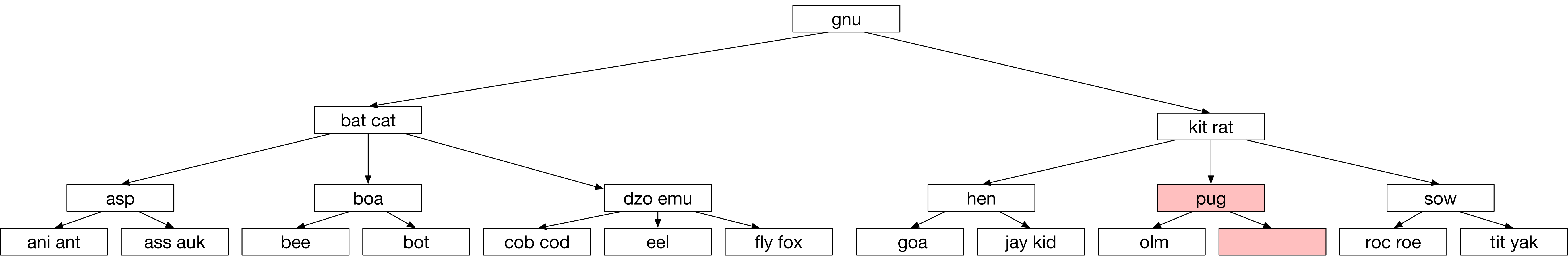
- Delete ox with successor

Activities



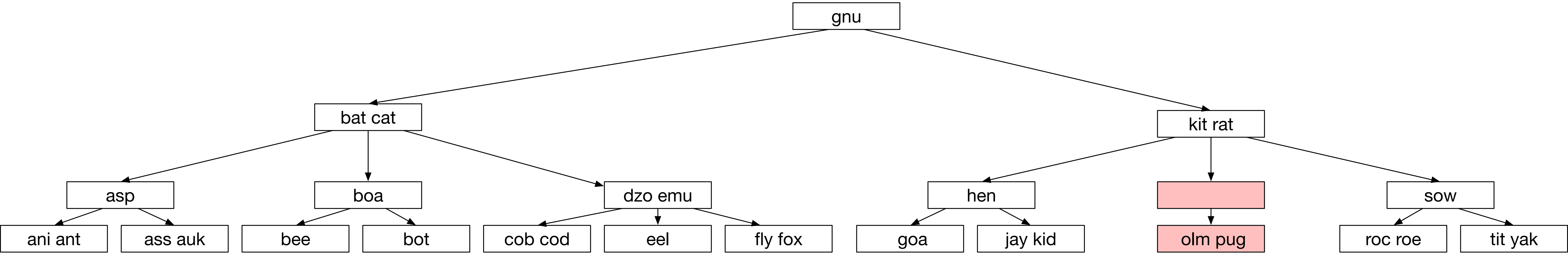
- Now delete from leaf

Activities



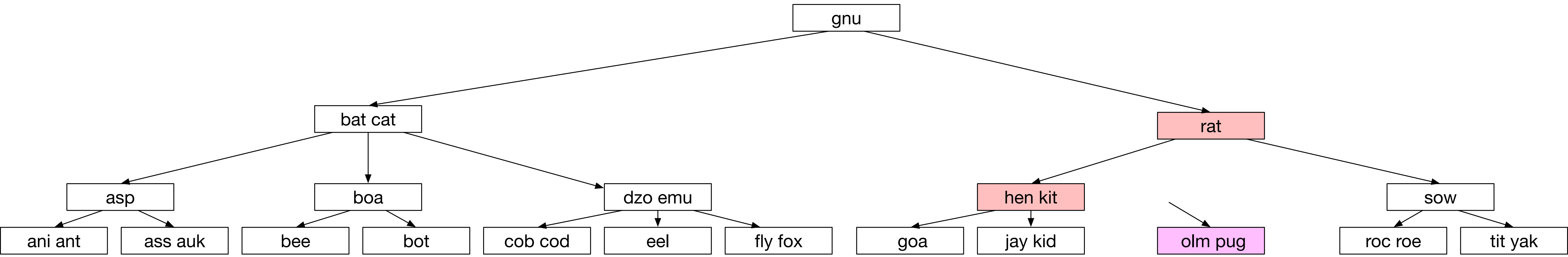
- Underflow: need to do a merge

Activities



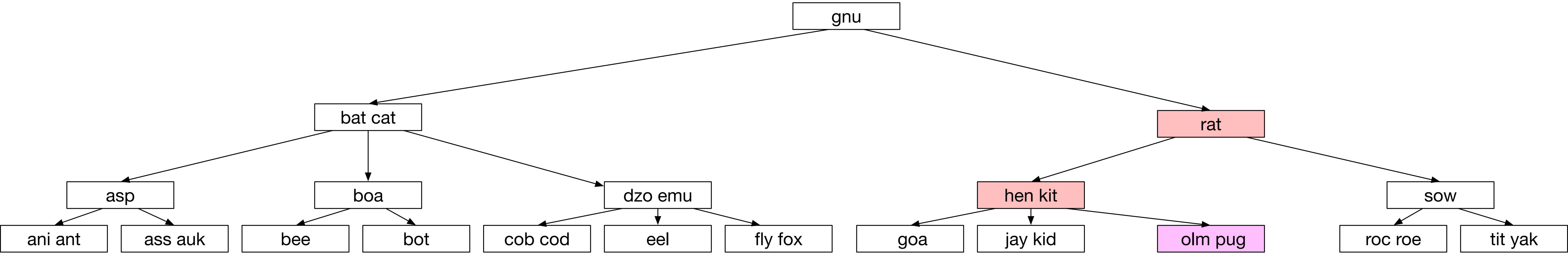
- Now the leaf is fine, but the parent has an underflow
- Cannot rotate, so we need to do another merge
- This time we merge left

Activities



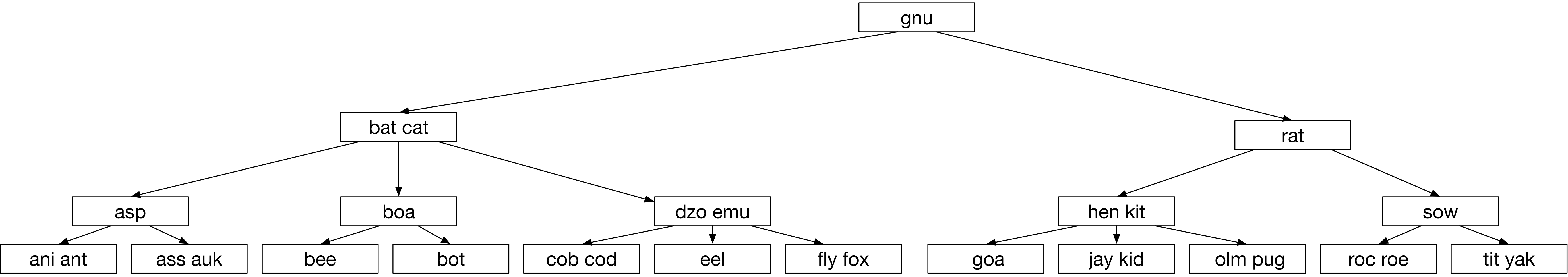
- Need to re-attach dangling node pointer

Activities



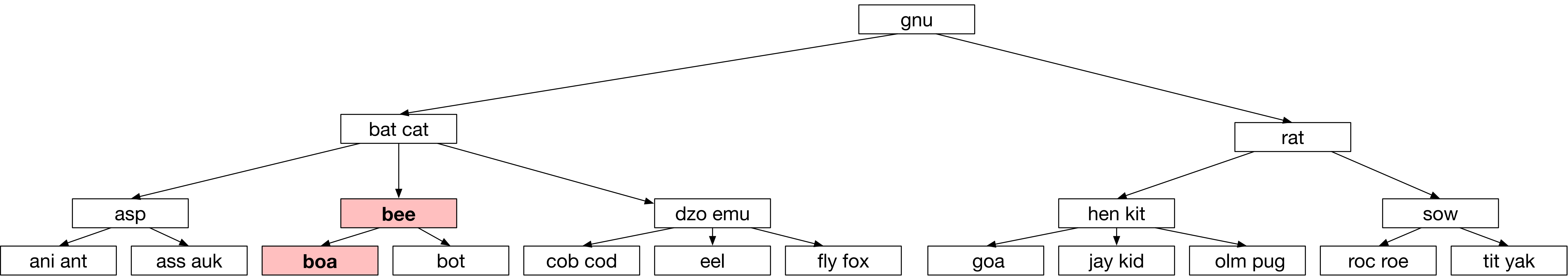
gnu
bat cat
rat
asp
boa
dzo emu
hen kit
sow
ani ant
ass auk
bee
bot
cob cod
eel
fly fox
goa
jay kid
olm pug
roc roe
tit yak

Activities



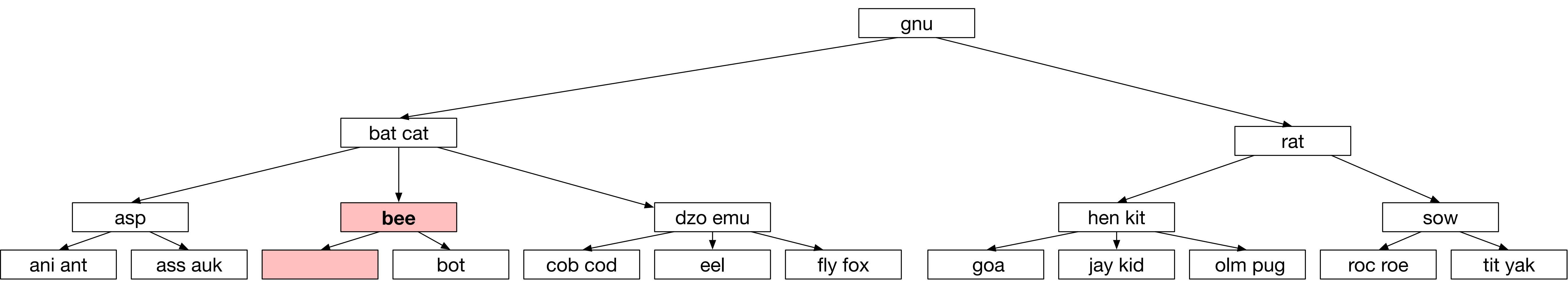
- Delete boa with predecessor

Activities



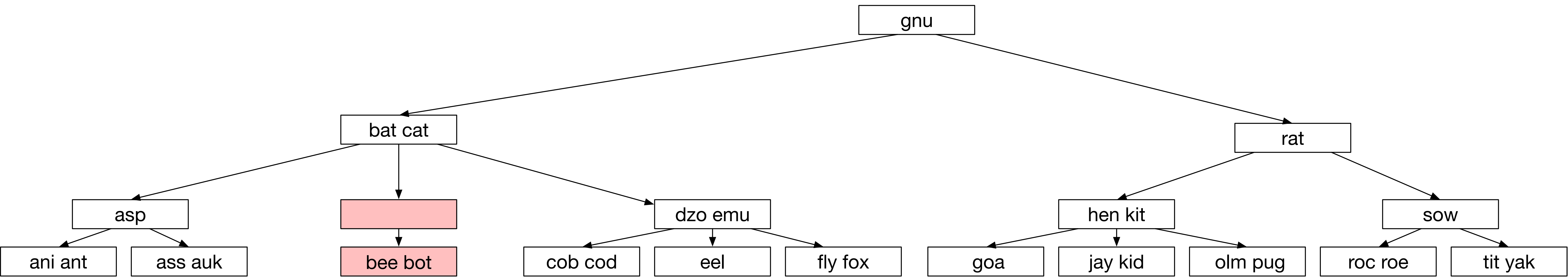
- Swap

Activities



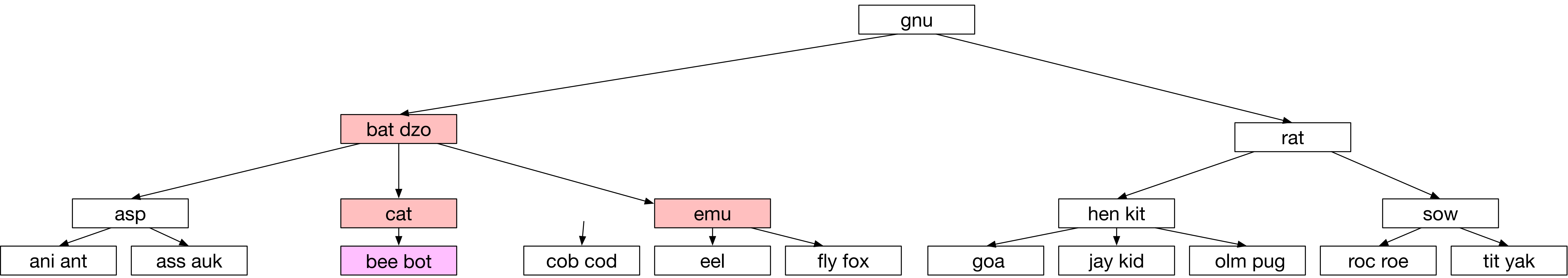
- Delete
- Deal with underflow
- Only merge is possible

Activities



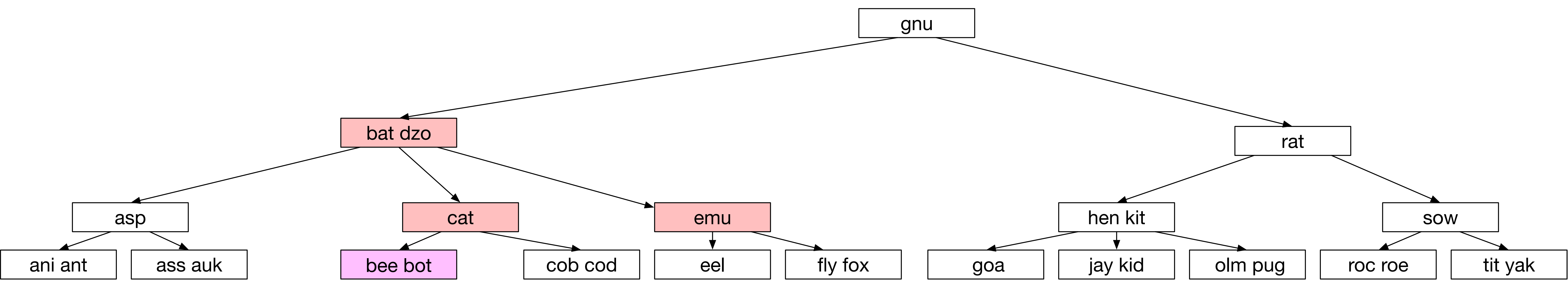
- This leaves the parent at an underflow
- Left sibling is at minimum capacity
- Right sibling is not
- Rotate Left: dzo goes up, cat goes down

Activities



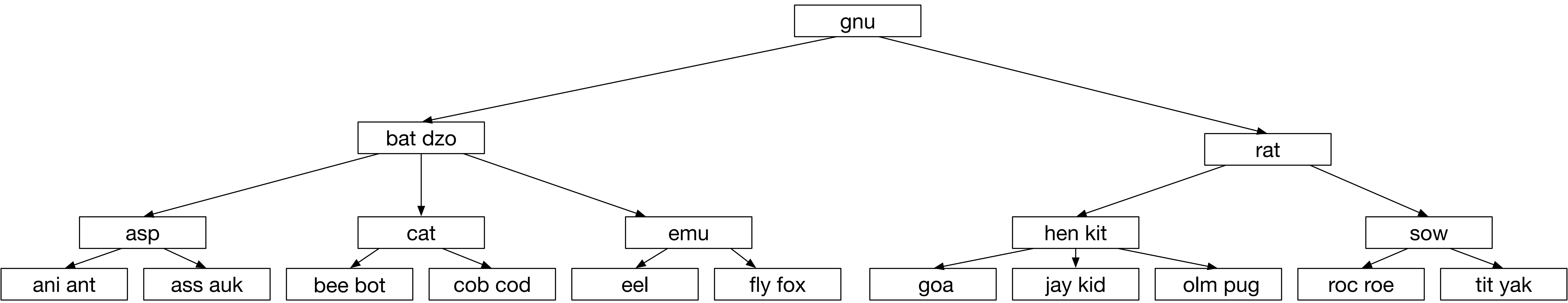
- Need to re-attach dangling node pointer

Activities



- And pretty-print

Activities

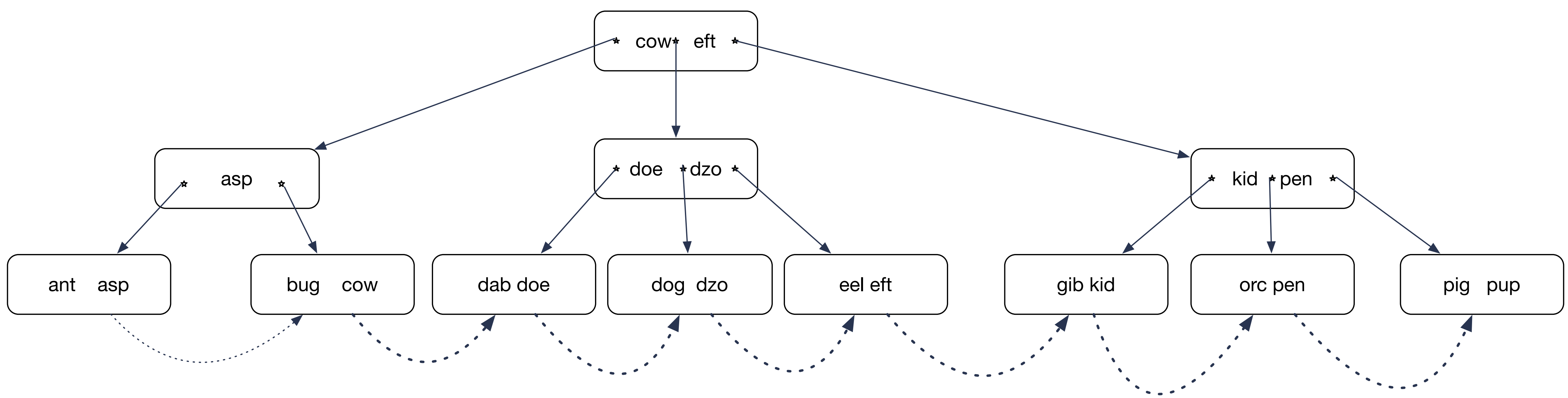


- The End

In real life

- Use B+ tree for better access with block storage
    - Data pointers / data are only in the leaf nodes
    - Interior nodes only have keys as signals
    - Link leaf nodes for faster range queries.

B+ Tree



cow eft
asp
doe dzo
kid pen
ant asp
bug cow
dab doe
dog dzo
eel eft
gib kid
orc pen
pig pup

B+ Tree

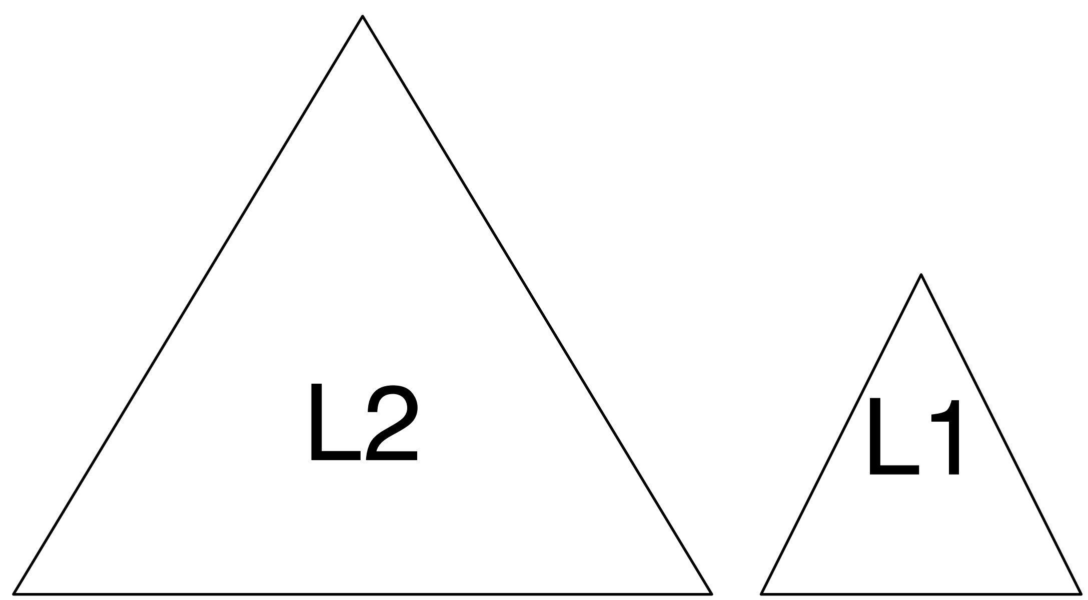
- Real life B+ trees:
  - Interior nodes have many more keys (e.g. 100)
  - Leaf nodes have as much data as they can keep
  - Need few levels:
    - Fast lookup

Combating Fringe Behavior

- Restructuring often happens with (almost) consecutive inserts
    - “Waves of misery” in running databases
- Use a way to buffer inserts

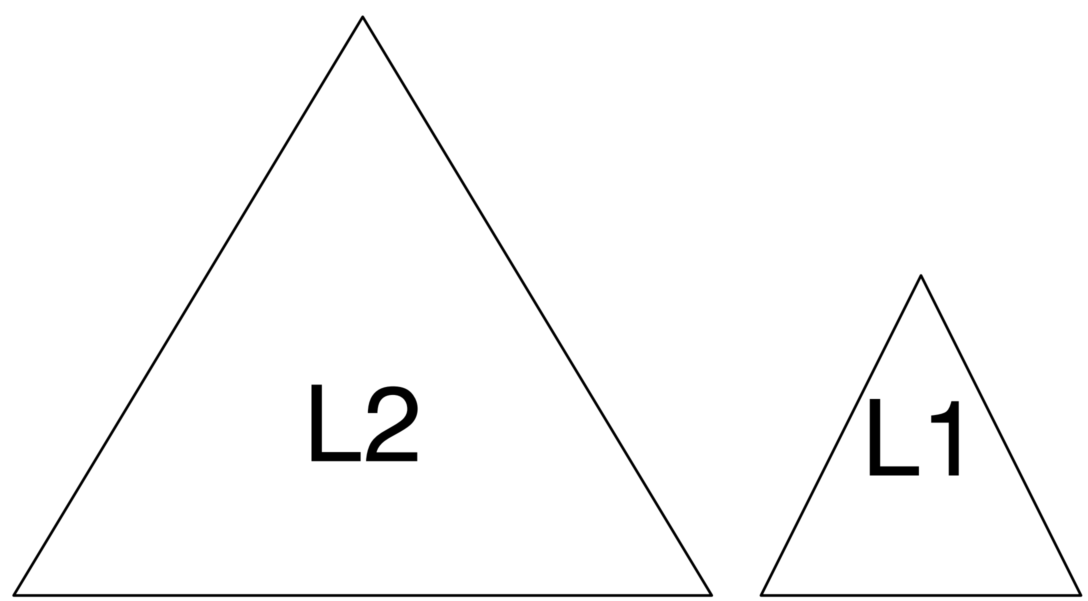
Combating Fringe Behavior

- Log-structured Merge Trees
  - Two related B-trees
    - Can optimize merge of L1 into L2
      - Allow merges to run in parallel with CRUD



Combating Fringe Behavior

- Log-structured Merge Trees
  - Keep L1-tree in main memory
  - Keep L2-tree in storage
  - Use Bloom filters in order to check whether keys are in either



Hashing

- Central idea of hashing:
  - Calculate the location of the record from the key
  - Hash functions:
    - Can be made indistinguishable from random function
      - SH3, MD5, …
      - Often simpler
        - ID modulo slots

Hashing

- Can lead to collisions:
  - Two different keys map into the same address
  - Two ways to resolve:
    - **Open Addressing**
      - Have a rule for a secondary address, etc.
    - **Chaining**
      - Can store more than one datum at an address

Hashing

- Open addressing example:
  - Linear probing: Try the next slot

Hashing Example

```
def hash(a_string):
    accu = 0
    i = 1
    for letter in a_string:
        accu += ord(letter)*i
        i+=1
    return accu % 8
```

Insert “fly”

0	
1	
2	“fly”, 2
3	
4	
5	
6	
7	

Hashing Example

```
def hash(a_string):
    accu = 0
    i = 1
    for letter in a_string:
        accu += ord(letter)*i
        i+=1
    return accu % 8
```

0	
1	
2	“fly”, 2
3	“gnu”, 2
4	
5	
6	
7	

Insert “gnu”

```
hash(“gnu”) —> 2
```

Since spot 2 is taken, move to the next spot

Hashing Example

```
def hash(a_string):
    accu = 0
    i = 1
    for letter in a_string:
        accu += ord(letter)*i
        i+=1
    return accu % 8
```

0	
1	
2	“fly”, 2
3	“gnu”, 2
4	“hog”, 3
5	
6	
7	

Insert “hog”

`hash(“hog”) -> 3`

Since spot is taken, move to the next

Hashing Example


```
def hash(a_string):
    accu = 0
    i = 1
    for letter in a_string:
        accu += ord(letter)*i
        i+=1
    return accu % 8
```

0	
1	
2	“fly”, 2
3	“gnu”, 2
4	“hog”, 3
5	
6	
7	“pig”, 7

Looking for “gnu”

```
hash(“gnu”) -> 2
```

Try out location 2. Occupied, but not by “gnu”

Hashing Example


```
def hash(a_string):
    accu = 0
    i = 1
    for letter in a_string:
        accu += ord(letter)*i
        i+=1
    return accu % 8
```

0	
1	
2	“fly”, 2
3	“gnu”, 2
4	“hog”, 3
5	
6	
7	“pig”, 7

Looking for “gnu”

`hash(“gnu”) —> 2`

Try out location 3. Find “gnu”

Hashing Example


```
def hash(a_string):
    accu = 0
    i = 1
    for letter in a_string:
        accu += ord(letter)*i
        i+=1
    return accu % 8
```

Looking for “ram”

```
hash(“ram”) -> 3
```

0	
1	
2	“fly”, 2
3	“gnu”, 2
4	“hog”, 3
5	
6	
7	“pig”, 7

Look at location 3: someone else is there
Look at location 4: someone else is there
Look at location 5: nobody is there, so if it were in the dictionary, it would be there

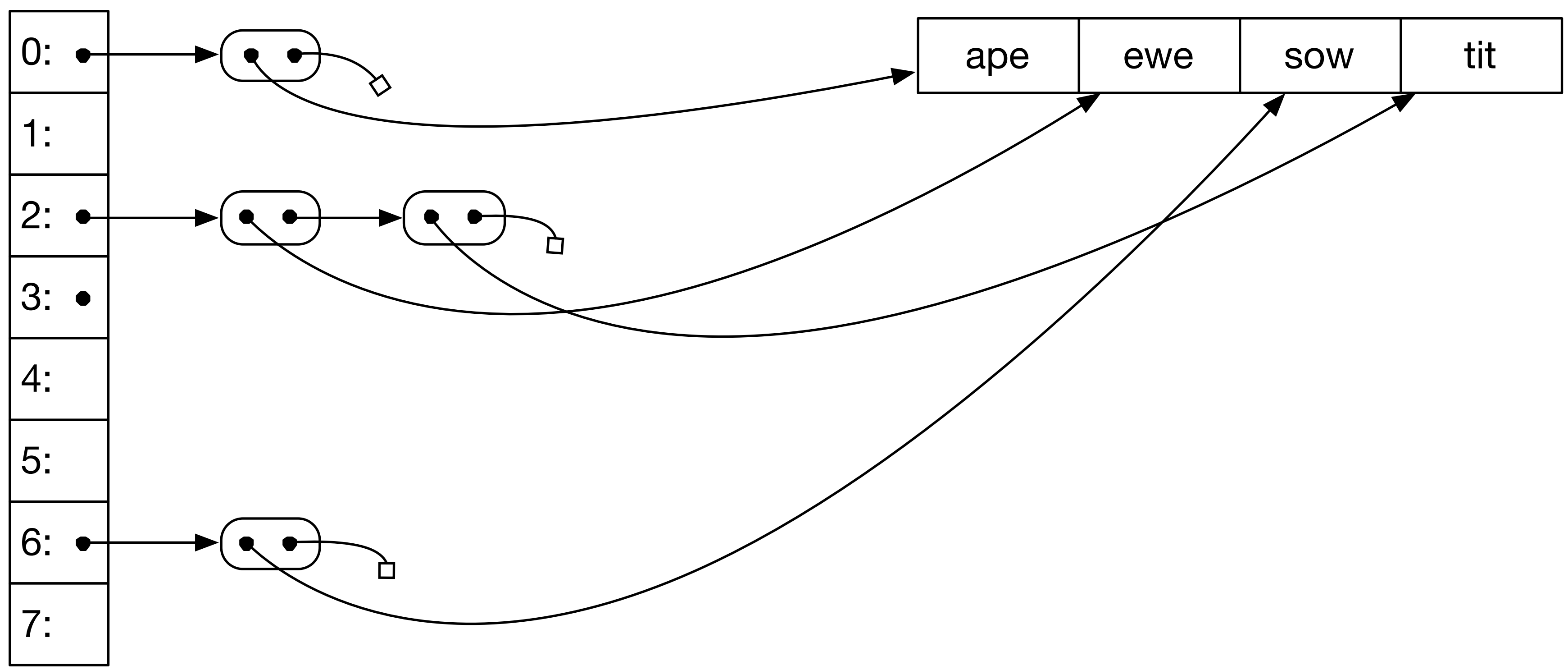
Hashing

- Linear probing leads to convoys:
  - Occupied cells tend to coalesce
- Quadratic probing is better, but might perform worse with long cache lines
- Large number of better versions are used:
  - Passbits
  - Cuckoo hashing
    - Uses two hash functions
  - Robin Hood hashing …

Hashing

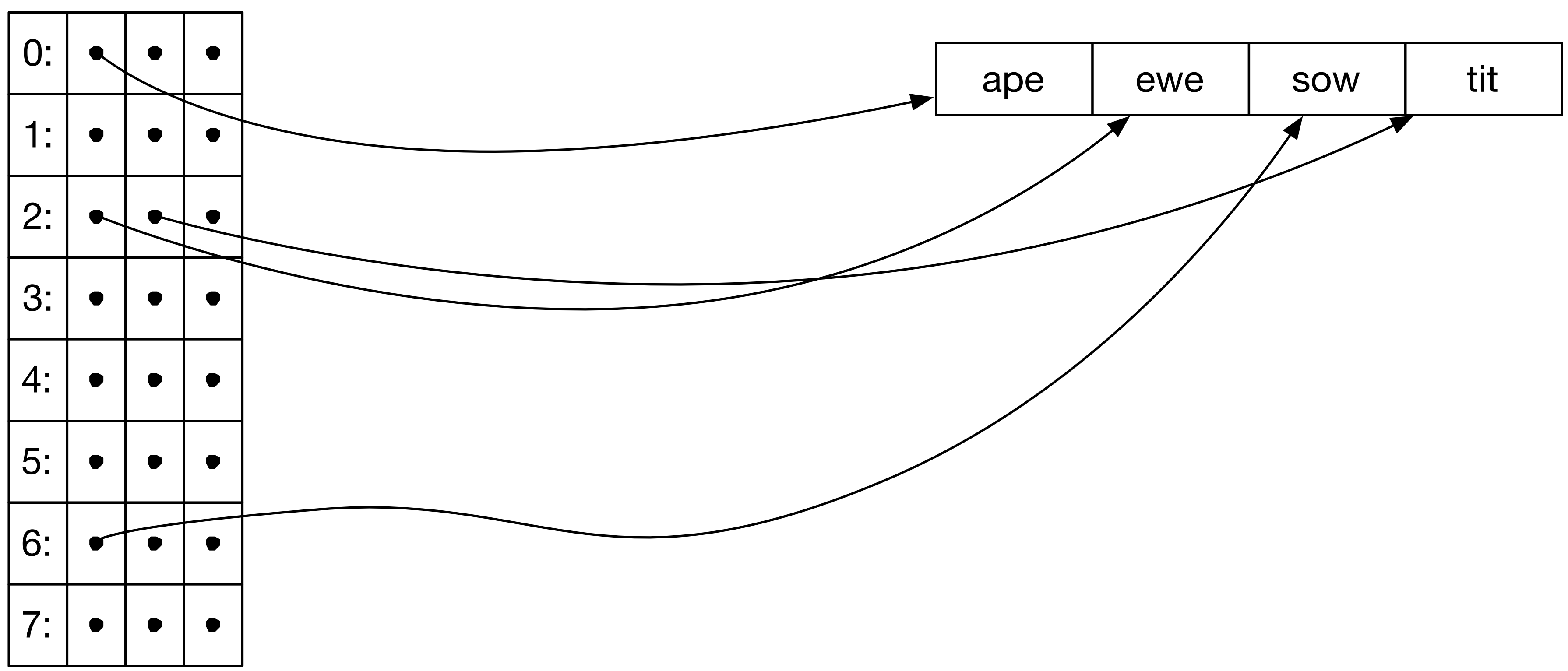
- Chaining
  - Keep data mapped to a location in a “bucket”
    - Can implement the bucket in several ways
      - Linked List

Hashing



Chaining Example with linked lists

Hashing Example



Chaining Example with an array of pointers (with overflow pointer if necessary)

Hashing Example

0:	ape	null	null
1:	null	null	null
2:	ewe	tit	null
3:	null	null	null
4:	null	null	null
5:	null	null	null
6:	sow	null	null
7:	null	null	null

Chaining with fixed buckets
Each bucket has two slots and a pointer to an overflow bucket

Hashing

- Extensible Hashing:
  - Load factor α = Space Used / Space Provided
  - Load factor determines performance
  - Idea of extensible hashing:
    - Gracefully add more capacity to a growing hash table

Linear Hashing

Linear Hashing

- Extensible Hashing:
  - Uses a lot of metadata to reflect history of splitting
    - But only splits buckets when they are needed
  - Linear Hashing
    - Splits buckets in a predefined order
    - Minimal meta-data
    - Sounds like a horrible idea, but …

Linear Hashing

- Assume a hash function that creates a large string of bits
  - We start using these bits as we extend the address space
  - Start out with a single bucket, Bucket 0
  - All items are located in Bucket 0

Bucket 0:

19, 28, 33

Items with keys 19, 28, 33

Linear Hashing

- Eventually, this bucket will overflow
  - E.g. if the load factor is more than 2
  - Bucket 0 splits
  - All items in Bucket 0 are rehashed:
    - Use the last bit in order to determine whether the item goes into Bucket 0 or Bucket 1
    - Address is $h_1(c) = c \pmod 2$

Linear Hashing

- After the split, the hash table has two buckets:

Bucket 0:	Bucket1:
28	19, 33

- After more insertions, the load factor again exceeds 2

Bucket 0:	Bucket1:
28, 40	11, 19, 33

Linear Hashing

- Again, the bucket splits.
    - But it has to be Bucket 0

Bucket 0:	Bucket1:	Bucket 2:
28, 40	11, 19, 33	

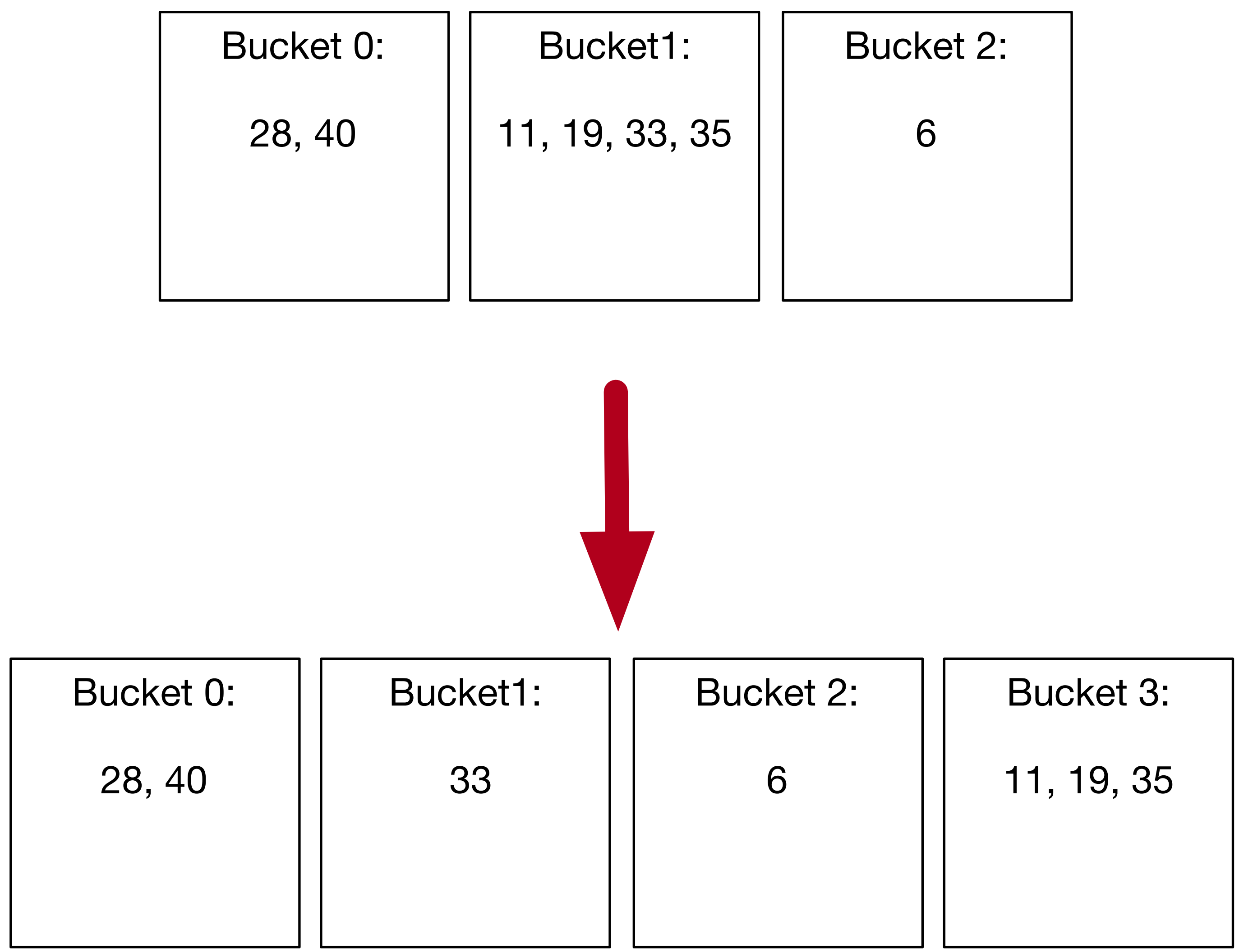
- For the rehashing, we now use two bits, i.e.

$$h_2(c) = c \pmod 4$$

    - But only for those items in Bucket 0

Linear Hashing

- After some more insertions, Bucket 1 will split



Linear Hashing

- The state of a linear hash table is described by the number N of buckets
  - The level l is the number of bits that are being used to calculate the hash
  - The split pointer s points to the next bucket to be split
  - The relationship is

$$N = 2^l + s$$

    - This is unique, since always $s < 2^l$

Linear Hashing

- Addressing function
  - The address of an item with key c is calculated by

```
def address(c):
   a = hash(c) % 2**l
   if a < s:
      a = hash(c) % 2**(l+1)
   return a
```

  - This reflects the fact that we use more bits for buckets that are already split

Linear Hashtable Evolution

$N = 1 = 2^0 + 0$

```
Number of buckets: 1
Split pointer: 0
Level: 0
```

```
def address(c):
   a = hash(c) % 2**l
   if a < s:
     a = hash(c) % 2**(l+1)
   return a
```

Bucket 0:

19, 28, 33

Linear Hashtable Evolution

$N = 2 = 2^1 + 0$

```
Number of buckets: 2
Split pointer: 0
Level: 1
```

```
def address(c):
   a = hash(c) % 2**l
   if a < s:
      a = hash(c) % 2**(l+1)
   return a
```

Bucket 0:	Bucket1:
28	19, 33

```
Add items with hashes 40 and 11
This gives an overflow and we split Bucket 0
```

Linear Hashtable Evolution

$N = 3 = 2^1 + 1$

```
Number of buckets: 3
Split pointer: 1
Level: 1
```

```
def address(c):
   a = hash(c) % 2**l
   if a < s:
      a = hash(c) % 2**(l+1)
   return a
```

Bucket 0:	Bucket1:
28, 40	11, 19, 33

```
split Bucket 0
Create Bucket 2
Use new hash function on items in Bucket 0
```

Bucket 0:	Bucket1:	Bucket 2:
28, 40	11, 19, 33	

```
No items were moved
```

Linear Hashtable Evolution

$N = 3 = 2^1 + 1$

```
Number of buckets: 3
Split pointer: 1
Level: 1
```

```
def address(c):
   a = hash(c) % 2**l
   if a < s:
      a = hash(c) % 2**(l+1)
   return a
```

Bucket 0:	Bucket1:	Bucket 2:
28, 40	11, 19, 33	

```
Add items 6, 35
```

Bucket 0:	Bucket1:	Bucket 2:
28, 40	11, 19, 33, 35	6

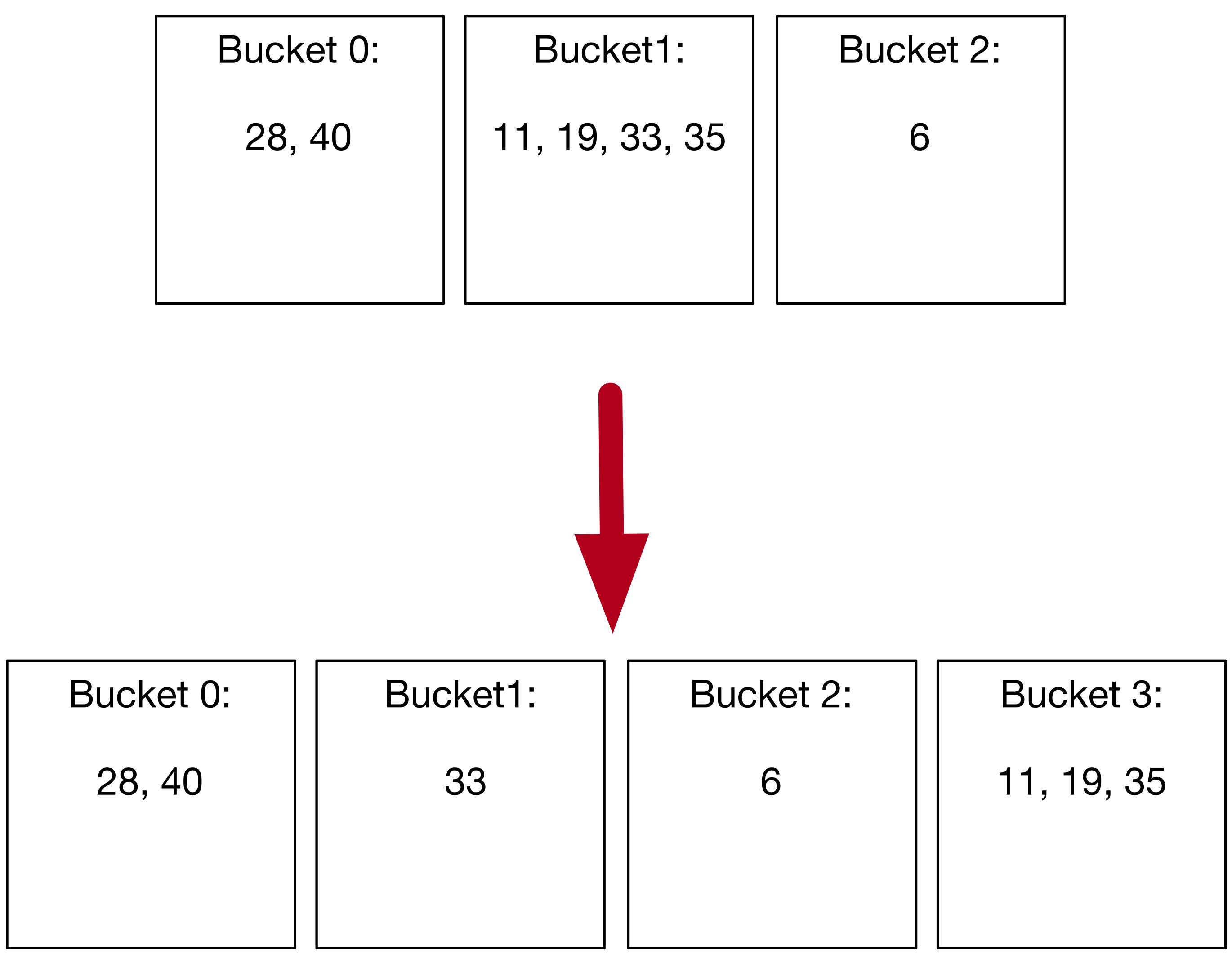
```
Because of overflow, we split
Bucket 1
```

Linear Hashtable Evolution

$$N = 4 = 2^2 + 0$$

```
Number of buckets: 4
Split pointer: 0
Level: 2
```

```
def address(c):
   a = hash(c) % 2**l
   if a < s:
      a = hash(c) % 2**(l+1)
   return a
```



Linear Hashtable Evolution

$N = 4 = 2^2 + 0$

```
Number of buckets: 4
Split pointer: 0
Level: 2
```

```
def address(c):
    a = hash(c) % 2**l
    if a < s:
        a = hash(c) % 2**(l+1)
    return a
```

Bucket 0:	Bucket1:	Bucket 2:	Bucket 3:
28, 40	33	6	11, 19, 35

Now add keys 8, 49

Bucket 0:	Bucket1:	Bucket 2:	Bucket 3:
28, 40, 8	33, 49	6	11, 19, 35

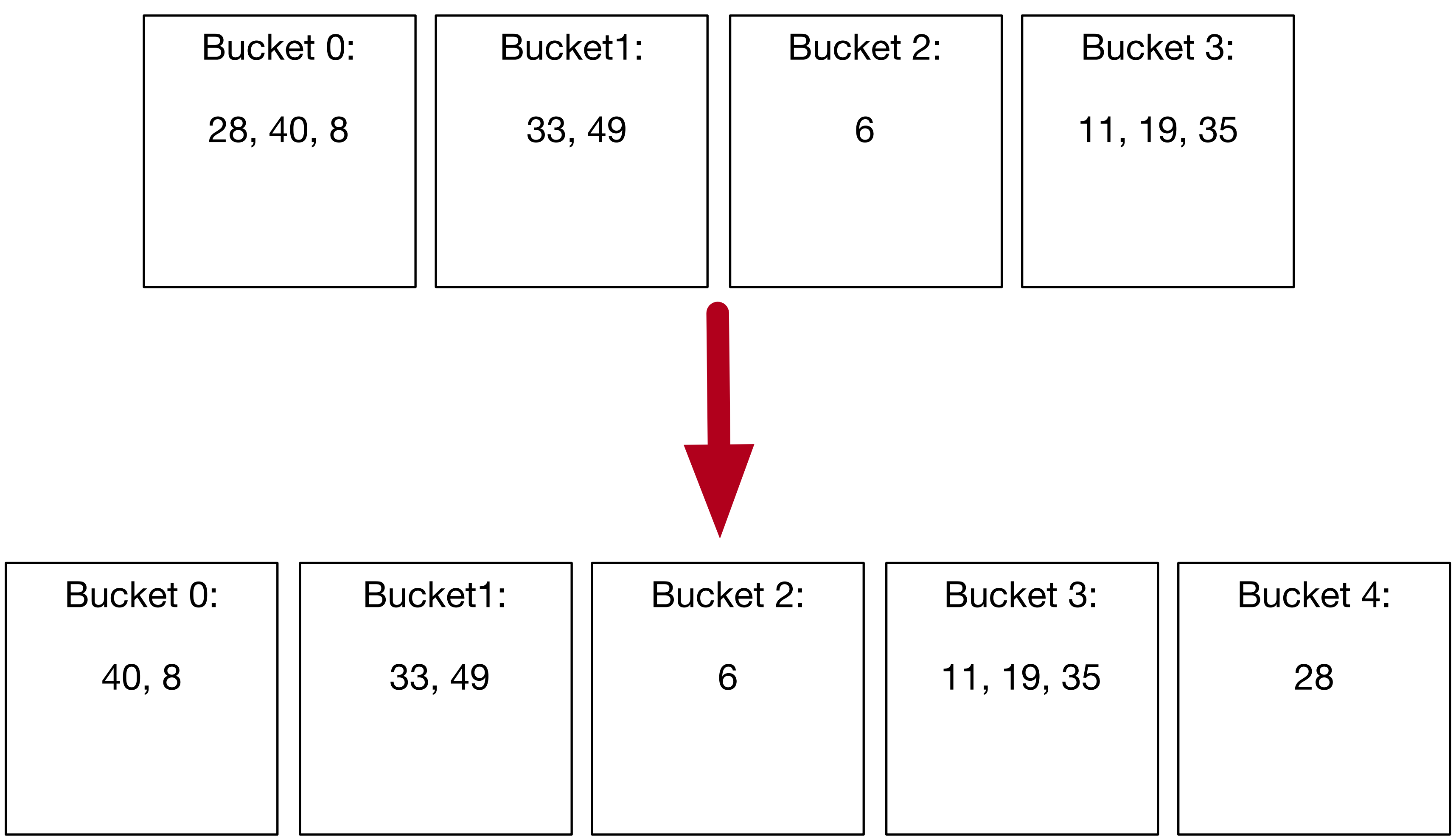
Creates an overflow!
Need to split!

Linear Hashtable Evolution

$$N = 5 = 2^2 + 1$$

```
Number of buckets: 1
Split pointer: 1
Level: 2
```

```
def address(c):
   a = hash(c) % 2**l
   if a < s:
      a = hash(c) % 2**(l+1)
   return a
```




```
Create Bucket 4.
Rehash Bucket 0.
```

Linear Hashtable Evolution

$N = 5 = 2^2 + 1$

```
Number of buckets: 5
Split pointer: 1
Level: 2
```

```
def address(c):
   a = hash(c) % 2**l
   if a < s:
      a = hash(c) % 2**(l+1)
   return a
```

Bucket 0:	Bucket1:	Bucket 2:	Bucket 3:	Bucket 4:
40, 8	33, 49	6	11, 19, 35	28

Add keys 9, 42

Bucket 0:	Bucket1:	Bucket 2:	Bucket 3:	Bucket 4:
40, 8	9, 33, 49	6, 42	11, 19, 35	28

Creates an overflow!
Need to split!

Linear Hashtable Evolution

$$N = 6 = 2^2 + 2$$

```
Number of buckets: 1
Split pointer: 2
Level: 2
```

```
def address(c):
   a = hash(c) % 2**l
   if a < s:
      a = hash(c) % 2**(l+1)
   return a
```

Bucket 0:	Bucket1:	Bucket 2:	Bucket 3:	Bucket 4:
40, 8	9, 33, 49	6, 42	11, 19, 35	28

Split

Bucket 0:	Bucket1:	Bucket 2:	Bucket 3:	Bucket 4:	Bucket 5:
40, 8	9, 33, 49	6, 42	11, 19, 35	28	

No item actually moved, but average load factor is now again under 2.

Linear Hashtable Evolution

$$N = 6 = 2^2 + 2$$

```
Number of buckets: 6
Split pointer: 2
Level: 2
```

```
def address(c):
   a = hash(c) % 2**l
   if a < s:
     a = hash(c) % 2**(l+1)
   return a
```

Bucket 0:	Bucket1:	Bucket 2:	Bucket 3:	Bucket 4:	Bucket 5:
40, 8	9, 33, 49	6, 42	11, 19, 35	28	

`add 5,10`

Bucket 0:	Bucket1:	Bucket 2:	Bucket 3:	Bucket 4:	Bucket 5:
40, 8	9, 33, 49	6, 10, 42	11, 19, 35	28	5

Linear Hashtable Evolution

$$N = 7 = 2^2 + 3$$

```
Number of buckets: 7
Split pointer: 3
Level: 2
```

```
def address(c):
   a = hash(c) % 2**l
   if a < s:
      a = hash(c) % 2**(l+1)
   return a
```

Bucket 0:	Bucket1:	Bucket 2:	Bucket 3:	Bucket 4:	Bucket 5:
40, 8	9, 33, 49	6, 10, 42	11, 19, 35	28	5

Bucket 0:	Bucket1:	Bucket 2:	Bucket 3:	Bucket 4:	Bucket 5:	Bucket 6:
40, 8	9, 33, 49	10, 42	11, 19, 35	28	5	6

Linear Hashtable Evolution

$$N = 7 = 2^2 + 3$$

```
Number of buckets: 7
Split pointer: 3
Level: 2
```

```
def address(c):
   a = hash(c) % 2**l
   if a < s:
      a = hash(c) % 2**(l+1)
   return a
```

Bucket 0:	Bucket1:	Bucket 2:	Bucket 3:	Bucket 4:	Bucket 5:	Bucket 6:
40, 8	9, 33, 49	10, 42	11, 19, 35	28	5	6

`add 92, 74`

Bucket 0:	Bucket1:	Bucket 2:	Bucket 3:	Bucket 4:	Bucket 5:	Bucket 6:
40, 8	9, 33, 49	10, 42, 74	11, 19, 35	28, 92	5	6

Linear Hashtable Evolution

$$N = 8 = 2^3 + 0$$

```
Number of buckets: 8
Split pointer: 0
Level: 3
```

```
def address(c):
   a = hash(c) % 2**l
   if a < s:
      a = hash(c) % 2**(l+1)
   return a
```

Bucket 0:	Bucket1:	Bucket 2:	Bucket 3:	Bucket 4:	Bucket 5:	Bucket 6:
40, 8	9, 33, 49	10, 42, 74	11, 19, 35	28, 92	5	6

Bucket 0:	Bucket1:	Bucket 2:	Bucket 3:	Bucket 4:	Bucket 5:	Bucket 6:	Bucket 7:
40, 8	9, 33, 49	10, 42, 74	11, 19, 35	28, 92	5	6	

Linear Hashtable Evolution

$N = 8 = 2^3 + 0$

```
Number of buckets: 8
Split pointer: 0
Level: 3
```

```
def address(c):
   a = hash(c) % 2**l
   if a < s:
      a = hash(c) % 2**(l+1)
   return a
```

Bucket 0:	Bucket1:	Bucket 2:	Bucket 3:	Bucket 4:	Bucket 5:	Bucket 6:	Bucket 7:
40, 8	9, 33, 49	10, 42, 74	11, 19, 35	28, 92	5	6	

`add 13, 54`

Bucket 0:	Bucket1:	Bucket 2:	Bucket 3:	Bucket 4:	Bucket 5:	Bucket 6:	Bucket 7:
	9, 33, 49	10, 42, 74	11, 19, 35	28, 92	5, 13	6, 54	

Linear Hashtable Evolution

$N = 9 = 2^3 + 1$

```
Number of buckets: 9
Split pointer: 1
Level: 3
```

```
def address(c):
    a = hash(c) % 2**l
    if a < s:
        a = hash(c) % 2**(l+1)
    return a
```

Bucket 0:	Bucket1:	Bucket 2:	Bucket 3:	Bucket 4:	Bucket 5:	Bucket 6:	Bucket 7:
	9, 33, 49	10, 42, 74	11, 19, 35	28, 92	5, 13	6, 54	

Bucket 0:	Bucket1:	Bucket 2:	Bucket 3:	Bucket 4:	Bucket 5:	Bucket 6:	Bucket 7:	Bucket 8:
	9, 33, 49	10, 42, 74	11, 19, 35	28, 92	5, 13	6, 54		40, 8

Linear Hashtable Evolution

$$N = 9 = 2^3 + 1$$

```
Number of buckets: 9
Split pointer: 1
Level: 3
```

```
def address(c):
   a = hash(c) % 2**l
   if a < s:
     a = hash(c) % 2**(l+1)
   return a
```

Bucket 0:	Bucket1:	Bucket 2:	Bucket 3:	Bucket 4:	Bucket 5:	Bucket 6:	Bucket 7:	Bucket 8:
	9, 33, 49	10, 42, 74	11, 19, 35	28, 92	5, 13	6, 54		40, 8

`add 1, 81`

Bucket 0:	Bucket1:	Bucket 2:	Bucket 3:	Bucket 4:	Bucket 5:	Bucket 6:	Bucket 7:	Bucket 8:
	1, 9, 33, 49, 81	10, 42, 74	11, 19, 35	28, 92	5, 13	6, 54		40, 8

Linear Hashtable Evolution

$$N = 10 = 2^3 + 2$$

```
Number of buckets: 10
Split pointer: 2
Level: 3
```

```
def address(c):
   a = hash(c) % 2**l
   if a < s:
     a = hash(c) % 2**(l+1)
   return a
```

Bucket 0:	Bucket1:	Bucket 2:	Bucket 3:	Bucket 4:	Bucket 5:	Bucket 6:	Bucket 7:	Bucket 8:	Bucket 9:
	1, 33, 49, 81	10, 42, 74	11, 19, 35, 67, 99	28, 92	5, 13	6, 54	39	40, 8	9

Bucket 0:	Bucket1:	Bucket 2:	Bucket 3:	Bucket 4:	Bucket 5:	Bucket 6:	Bucket 7:	Bucket 8:	Bucket 9:	Bucket 10:
	1, 33, 49, 81		11, 19, 35, 67, 99	28, 92	5, 13	6, 54	39	40, 8	9	10, 42, 74

Linear Hashing

- Observations:
  - Buckets split in fixed order
    - 0, 0,1, 0, 1, 2, 3, 0, 1, 2, 3, 4, 5, 6, 7, 0, 1, 2, …, 15, 0, …
    - Address calculation is modulo 2^l, i.e. the *l* least significant bits
    - Buckets 0, 1, …, s-1 and 2***l*, 2***l*+1, … *N*-1 are already split, they have on average half the size of the buckets *s*, *s*+1, …, 2***l*.

Linear Hashing

- Observations:
  - An overflowing bucket is not necessarily split immediately
  - Sometimes, a split leaves all keys in the splitting bucket or moves them all to the new bucket
- On average, a bucket will have α items in them