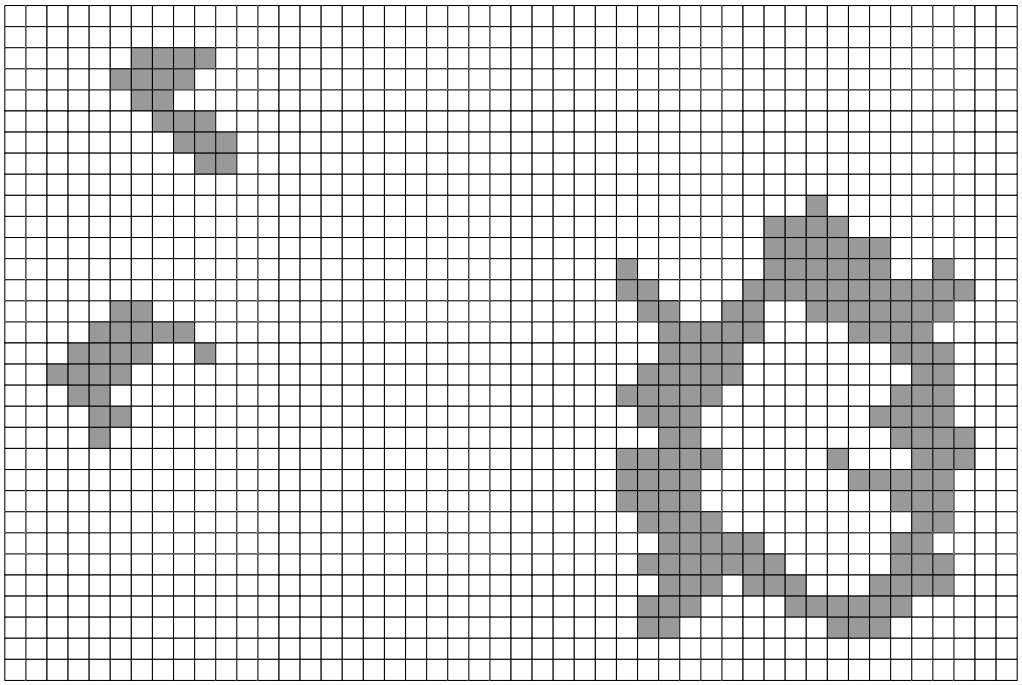
Homework 10

due April 28, 2025

Problem 1:

You are given a square plot of size 1000 by 1000. Some of the squares are colored, forming "islands". Two colored squares belong to the same island if they are horizontal, vertical, or diagonal neighbor. For example, the plot below has three islands.



Find an efficient algorithm that given such a plot returns the number of islands.

Problem 2:

The Erdös number is a humorous parameter defined for most published Mathematicians (or for all if we allow the number to be infinity). Mine is 3. Paul Erdös was a very prolific Mathematician who visited Mathematics departments and published good papers with his host(s). Erdös has number 0. All of his co-authors have number 1. The co-authors of co-authors have number 2 (unless they also published directly with Erdös, in which case they get Erdös number 1). The co-authors of the co-authors of the co-authors get Erdös number 3, etc.

You are given access to the database of the Mathematical Reviews, which contains data on all serious Mathematical papers. In particular, you are given author names and titles that you can extract as a CSV file. Explain how you can set up a web-site that quickly calculates the Erdös number short of creating a table with all precalculated Erdös numbers.

Problem 3:

Give an algorithm that finds out whether all the nodes in an undirected graph can be colored with one of two colors such that two adjacent nodes have different colors.

Problem 4:

You are given a large, unordered collection of integers. You repeatedly are given a number t and you need to find two elements x, y of the collection such that $x + y = t$. Describe a data structure that allows you to answer this question in time linear in the size of the collection.