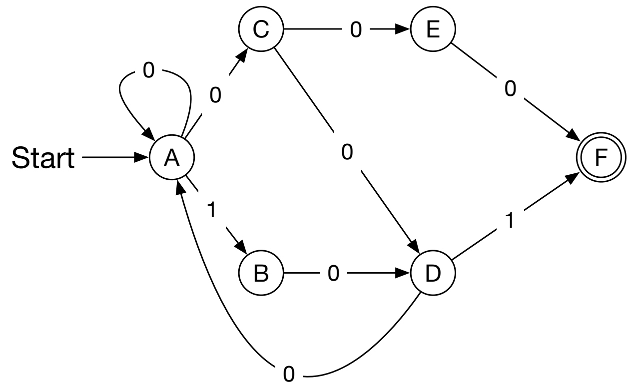
Solutions : Midterm — Algorithms

1. NFA to DFA

If we are in A, on 0, we can go to A or C. If we are in C, we can go to E or D on 0. If we are in E, we can only go to F on 0. Therefore, from $\{A, C, E\}$, we go to $\{A, C, D, E, F\}$.

If we are in A, on 1 we have to go to B. If we are in C or in D, there is no transition on 1. Therefore, from $\{A, C, E\}$ on 1 we go to $\{B\}$.



2. Asymptotic Notation:

Compare $\log(n)n$ and $n^{3/2}$:

$$\lim_{n\to\infty} \frac{\log(n)n}{n^{3/2}} = \lim_{n\to\infty} \frac{\log(n)}{n^{1/2}} =^{\text{l'hopital}} \lim_{n\to\infty} \frac{n^{-1}}{\frac{1}{2}n^{-1/2}} = \lim_{n\to\infty} \frac{2}{\sqrt{n}} = 0 \ .$$

Therefore, $\log(n)n = o(n^{3/2})$.

3. Divide and Conquer Algorithm

(a) This is the number of pairs of two different elements without order, or $\binom{n}{2}$. Alternatively, it is

$$\sum_{i=1}^{n-1}(n-i) = (\sum_{i=1}^{n-1} n) - (\sum_{i=1}^{n-1} i) = n(n-1) - \frac{n(n-1)}{2} = \frac{n(n-1)}{2}.$$

(b) We do constant work in order to divide the array and calculate the three values. Therefore, the recurrence is

$$T(n) = 2T(n/2) + c.$$

As $\log_2(2) = 1$, we compare c with n. We are therefore in case 1 of the Master Theorem and have $T(n) = \Theta(n)$.

4. Order Statistics

(a) There is a total of $3l$ comparisons within the triplets and then there are $l-1$ comparisons for the maximum of the triplet maxima and $l-1$ comparisons for the triplet minima. This gives a total of $5l-2 \ = \frac{5}{3}n-2$. If n is even, this is worse than $\frac{3}{2}n-2$ comparisons for pairs, and if n is odd, this is worse than $3\frac{n-1}{2} = \frac{3}{2}n-\frac{3}{2}$ comparisons.

(b) Now we have on average $\frac{1}{3}\cdot 2+\frac{2}{3}\cdot 3=\frac{8}{3}$ comparisons per triplet, giving us

$$\frac{8}{3}l+2(l-1)=\frac{14}{3}l-2=\frac{14}{9}n-2$$

comparisons. Even now, it is not a good idea.

5. Knapsack

We start at the lower right square. If the value in a square is equal to that of its left neighbor, then we could have obtained the current value by not including the current item. Otherwise, the value needs to be the value on the left moved up by as many rows as the item has weight. We can check for errors by adding the value of the item to the knapsack value in that cell. We now mark the cells.

As we can see, we do not include J, we include I, we include H, we do not include G, we include F, we skip over E, D, C, and we include A and B. We check: The combined weight of $\{A,B,C,F,H,I\}$ is $7+6+5+3+2+1=24$ and the combined value is $13+11+10+6+4+2=46$.

	ϕ	A	A-B	A-C	A-D	A-E	A-F	A-G	A-H	A-I	A-J
0:	0	0	0	0	0	0	0	0	0	0	0
1:	0	0	0	0	0	0	0	0	0	2	2
2:	0	0	0	0	0	0	0	0	4	4	4
3:	0	0	0	0	0	0	6	6	6	6	6
4:	0	0	0	0	0	7	7	7	7	8	8
5:	0	0	0	10	10	10	10	10	10	10	10

	ϕ	A	A-B	A-C	A-D	A-E	A-F	A-G	A-H	A-I	A-J
6:	0	0	11	11	11	11	11	11	11	12	12
7:	0	13	13	13	13	13	13	13	14	14	14
8:	0	13	13	13	13	13	16	16	16	16	16
9:	0	13	13	13	13	17	17	17	17	18	18
10:	0	13	13	13	19	19	19	19	20	20	20
11:	0	13	13	21	21	21	21	21	21	22	22
12:	0	13	13	23	23	23	23	23	23	23	23
13:	0	13	24	24	24	24	25	25	25	25	25
14:	0	13	24	24	24	26	27	27	27	27	27
15:	0	13	24	24	24	28	29	29	29	29	29
16:	0	13	24	24	30	30	30	30	31	31	31
17:	0	13	24	24	32	32	32	32	33	33	33
18:	0	13	24	34	34	34	34	34	34	35	35
19:	0	13	24	34	34	34	36	36	36	36	36
20:	0	13	24	34	34	37	38	38	38	38	38
21:	0	13	24	34	34	39	40	40	40	40	40
22:	0	13	24	34	34	41	41	41	42	42	42
23:	0	13	24	34	43	43	43	43	44	44	44
24:	0	13	24	34	43	43	45	45	45	46	46