

Regular Expressions Worksheet:

(1) Given the following two languages over $\Sigma = \{0,1,2\}$, $L_1 = \{\epsilon,0,1\}$ and $L_2 = \{2\}$, determine :

$$L_1 + L_2$$

$$L_1 \cdot L_2$$

$$L_1^0$$

$$L_2 \cdot L_1$$

$$L_2^2$$

$$L_1^2$$

$$L_1^3$$

$$L_1^n$$

(2) Describe the following regular expressions as sets.

$$\mathbf{1 * 0 *}$$

$$\mathbf{(0 + 1)^+}$$

$$\mathbf{01*0}$$

$$\mathbf{(101)^*}$$

Solutions:

$$L_1 + L_2 = \{\epsilon, 0, 1, 2\}$$

$$L_1 \cdot L_2 = \{2, 02, 12\}$$

$$L_1^0 = \{\epsilon\}$$

$$L_2 \cdot L_1 = \{2, 20, 21\}$$

$$L_2^2 = \{22\}$$

$$L_1^2 = \{\epsilon, 0, 1, 00, 01, 10, 11\}$$

$$L_1^3 = \{\epsilon, 0, 1, 00, 01, 10, 11, 000, 001, 010, 011, 100, 101, 110, 111\}$$

$$L_1^n = \{\text{all strings in } \{0, 1\} \text{ of length up to } n\}. \text{ You see this by induction.}$$

1*0* The set of all finite strings that starts out with zero or more ones and finishes with zero or more zeroes

(0 + 1)⁺ The set of all finite strings with letters 0 or 1, the empty string not included

01*0 = {00, 010, 0110, 01110, ...} The set of all finite strings that start out with a 0, followed by none or more letters 1, followed by a final 0

(101)* = { ϵ , 101, 101101, 101101101, ...}