Rod Cutting Problem

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Problem

- Given a long rod of length *n*

  - Cut it into sub-rods

  - Each sub-rod of length *i* has worth *w*[*i*]

Dynamic Programming

- Need to identify similar sub-problems

  - Definition of sub-problems

  - Division into sub-problems

  - Solution to sub-problems

  - Merging solutions of sub-problems

Dynamic Programming

- Similar sub-problems:

  - After any cut belonging to an optimal set of cuts:

    - Left and right sub-rods are cut optimally

- Therefore:

  - How to cut a rod of length $n < m$

Dynamic Programming

- How to put the solutions together:

  - Go through all possibilities for the last cut

Dynamic Programming

- This gives us:

  - Best way to cut a rod of length n

    - Best of:

      - Rod of length k

      - Cutting a rod of length n-k

Dynamic Programming

- Memoized version of

```
best_cut(n):
    return max([best_cut[n-k]+v[k] for k in arg(v)])
```

Dynamic Programming

- Can we do better:

  - Sure, we can cut down on possibilities:

    - Order possible cuts by size

    - $k \in \arg(v)$

    - Define best $\mathrm{cut}(n, k_0)$ to be best cut for rod of length n with sub-cuts of length $\leq k_0$.

Dynamic Programming

- best cut$(n, k_0) = \max(\{\text{best cut}(n - r \cdot k_0, k_0 - 1) + rv[k_0]\} \mid r = 0,1,2,\ldots)$