Homework 3

(1) Use limits in order to compare the asymptotic growth of the following pair of functions (given as expressions in the variable *n*). Show all your work. Use o, Θ, and Ω to express the relationship. You can check your derivations using Mathematica, Maple, or Matlab.

(1) $\log(n)^2,\ \sqrt{n}$

(2) $\dfrac{n^2+5}{n+4},\ n$

(3) $e^n,\ 3^n$

(4) $n^n, 2^n$

(5) $n^2, 2^n$

(6) $n\log(n)^2,\ n\log(n^2)$

(2) Use induction to show that the recurrence $a_i = 2a_{i-1} + 1$ with $a_0 = 0,\ a_1 = 1$ is solved by $a_i = 2^i - 1$.

(3) Use induction to show that the recurrence $a_i = 2/5a_{i-1} + 3/5a_{i-2}, a_0 = 0, a_1 = 1$ is solved by $a_i = -\dfrac{(-3)^i - 5^i}{8 \cdot 5^{i-1}}$.

(4) Use the substitution method to show that $T(n) = T(n-1) + n + 1$ implies that $T(n) \leq Cn^2$ as long as $C \geq 1$ and $C \geq T(1)$.