Adjustable flat layouts for Two-Failure Tolerant Storage Systems

1 Knapsack Problem

Given the following table for the 0-1 knapsack problem with nine items, with values and weights being (15, 6), (14, 6), (12, 5), (11, 5), (10, 5), (7, 4), (6, 4), (3, 2), and (1, 1), find the optimal way to fill the knapsack.

	A	B	C	D	E	F	G	H	I
1	0	0	0	0	0	0	0	0	1
2	0	0	0	0	0	0	0	3	3
3	0	0	0	0	0	0	0	3	4
4	0	0	0	0	0	7	7	7	7
5	0	0	12	12	12	12	12	12	12
6	15	15	15	15	15	15	15	15	15
7	15	15	15	15	15	15	15	15	16
8	15	15	15	15	15	15	15	18	18
9	15	15	15	15	15	19	19	19	19
10	15	15	15	23	23	23	23	23	23
11	15	15	27	27	27	27	27	27	27
12	15	29	29	29	29	29	29	29	29
13	15	29	29	29	29	29	29	30	30
14	15	29	29	29	29	30	30	32	32
15	15	29	29	29	33	34	34	34	34
16	15	29	29	38	38	38	38	38	38
17	15	29	41	41	41	41	41	41	41
18	15	29	41	41	41	41	41	41	42
19	15	29	41	41	41	41	41	44	44
20	15	29	41	41	41	45	45	45	45
21	15	29	41	41	48	48	48	48	48
22	15	29	41	52	52	52	52	52	52
23	15	29	41	52	52	52	52	52	53
24	15	29	41	52	52	52	52	55	55
25	15	29	41	52	52	55	55	55	56

2 Order Statistics

We want to calculate the i^{th} element in a set S. Assume that we have a way to find an element x and then partition around x:

$$S = L \cup \{x\} \cup U$$

such that all elements in L are smaller than x and all elements in U are larger than x. Furthermore, L and U have at least $|S|/3$ elements. Finally, assume that the runtime of this partition algorithm is in $O(n)$.

Find a linear time algorithm that finds the i^{th} largest element in S and prove its runtime.

3 Linear Hashing

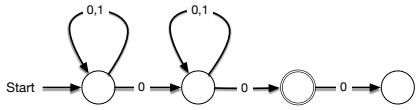
A linear hash file A with 13 buckets is backed up into another linear hash files, B, with 27 buckets. Thus, the same set of records is present in both A and B. Now, Bucket 0 of A fails. Which buckets in B do we need to access in order to find all the records in the lost bucket.

4 Complexity

Is $f(n) = \sqrt{n \log n}$ in $\Omega(n)$?

5 NFA

Give a regular expressions for the binary strings accepted by the following NFA.



6 Dynamic Programming

A marker is located at a random vertex of an $n \times n$ grid. At each time interval, the marker moves with equal probability up, down, left, or right. When the marker moves off the grid, the system stops. Use dynamic programming to calculate the probability that the marker does not leave the grid in j moves.

7 Greedy Algorithm

Given post stamps of denominations 1, 4, 9, 10, what stamps would the greedy algorithm choose for a postage of 36. What would a better way be to make postage.

8 Huffman Encoding

Give a Huffman encoding for five symbols with frequencies 0.30, 0.25, 0.20, 0.15 0.10.

9 Graphs

Show that for three vertices x, y, and z in a graph

$$\delta(x, y) + \delta(y, z) \geq \delta(x, z).$$