Worksheet: Landau's o, ω, θ, O, Ω, Θ

(1) Assume that the following expressions give the processing time $T(n)$ spent by an algorithm for solving a problem of size *n*. Give the dominant term (having the steepest increase in *n*) and specify the lowest Landau O complexity of each algorithm.

Expression	Dominant Term	O(…)
$5 + 0.1n + 0.01n^2$		
$8 + 0.1n + 0.2n \log_2 n + \frac{n^2}{25}$		
$2n + n^{0.5}$		
$0.3 \log_8 n + \log_2(\log_2 n)$		
$0.0003 \log_4 n + \log_2(\log_2 n)$		
$n \log_3(n) + n \log_4(n)$		
$n^2 + \log_n^4 n + 5n$		
$n^2 \log n + n \log_2^2(n)$		

(2) A sorting method with complexity $\Theta(n \log n)$ uses .792 millisecond to sort 1000 data, 1.721 milliseconds to sort 2000 data, and 4.760 milliseconds to sort 5000 data. Assume that the true complexity is $f(n) = a + bn + cn \log n$ with fixed, but unknown coefficients *a*, *b*, and *c*. Calculate the time it takes to process 10,000 data.

(3) Use the formal definition of Landau's O to prove that

$$T(n) = a + bn + cn^2 + dn^3 \in O(n^3),$$

with positive coefficients $a, b, c,$ and d.

(4) Use the formal definition of Landau's O to prove that

$$T(n) = \log(n) \in O(\sqrt{n}).$$