Selected Worksheet answers

1. Since $\log n < n$ for $n \geq 1$, we have $10n + \log n \leq 11n$. The definition is fulfilled with $n_0 = 1$ and $c = 11$.

2. If there were an $n_0 \in \mathbb{N}$ and a $c > 0$ such that
$$(1) \qquad \forall n > n_0 \ : \ 0 < n^2 + 5 < cn,$$
then $\forall n > n_0 \ : n + 5/n < c$. If we pick
$$m = \max(n_0 + 1, \lceil c \rceil + 1),$$
then $m > n_0$ and $m + 5/m > m > c$, and so
$$m^2 + 5 > cm.$$
However, this is in contradiction to (1).

9. $\displaystyle \lim_{n \to \infty} \frac{n^a}{n^b} = \lim_{n \to \infty} n^{a-b} = 0$, since $a - b < 0$.