Midterm Preparation

1. Divide and Conquer and the Master Theorem

1. Strassen's algorithm calculates the product of an $n \times n$ matrix by multiplying **seven** $n/2 \times n/2$ matrices plus a number of additions for each matrix coefficient. If $M(n)$ is the number of multiplications, what is the recursion formula of the corresponding divide and conquer algorithm. Then use the master theorem in order to find the Θ of $M(n)$.

2. Binary search for a key c on a sorted array A calculates the midpoint of A, compares c with the midpoint and then searches either to the left or to the right of the midpoint. What is the **exact** formula for the number of steps $T(n)$ in the recursion if c is not in the array. You need to use ceiling and/or floor.

3. Apply the Master Theorem to the following recursion formulae:

   (a) $T(n) = 3T(n/3) + \sqrt{n}$

   (b) $T(n) = 2T(n/10) + \sqrt{n}$

   (c) $T(n) = 3T(n/3) + n/3$

   (d) $T(n) = 2T(n/4) + \sqrt{n} \log n$

2. Order Statistics

4. Find the worst case run-times for the following methods to list the largest i numbers in a list of n elements.

   (a) Sort the numbers with merge-sort and then list the i largest numbers.

   (b) Use an order statistics to find the i-th largest number and then partition around the i.

5. Assume we want to find the second-largest element in an array A. Can we speed this up by comparing $A[0]$ with $A[1]$, $A[2]$ with $A[3]$, ... , collect the winners in a list with $\lceil \frac{n}{2} \rceil$ elements and look for the second largest element there? What is the recursion formula for this algorithm?

3. Order Statistics

6. Compare the asymptotic complexity of the following pairs of functions. Use Big-Theta, Big-O, or Big-Omega to express the relationship.

(a) $n \log n, n(\log n)^2$

(b) $n^n, 2^n$

(c) $2^{2^n}, (2^2)^n$

4.   Finite Automata and Regular Expressions

7. Identify the regular expressions in the following lists that contain the string "00111000".

(a) $0*(01)*(0+1)*$

(b) $0*1*0*$

(c) $(01 + 10 + 00)*$

8. Change the following NDA with epsilon moves to an FDA.