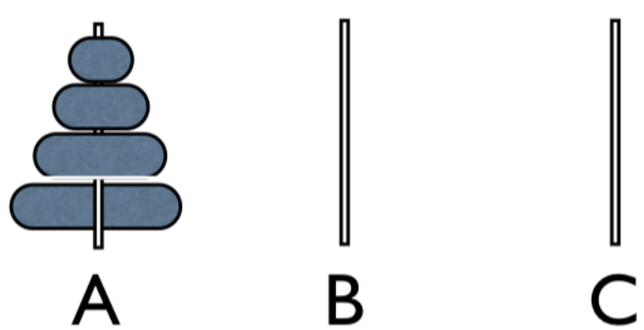
Towers of Hanoi

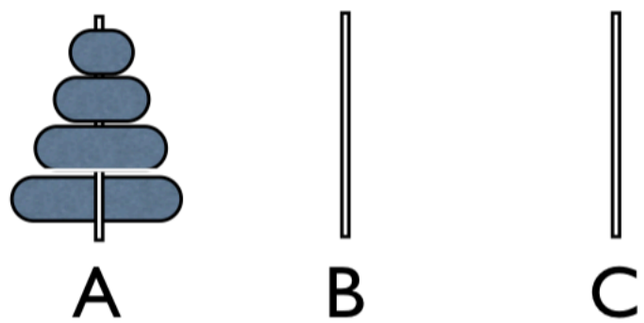
Problem

- *n* disks of *n* different parameters are on Peg A.
- Need to move them to Peg C subject to
  - Can only one disk at a time
  - Can only place smaller disk on bigger ones



Algorithm

- Recursive Solution
  - One disk: Just move the disk (1 move)
  - General case: Move top *n*-1 disks from A to C. Move remaining disk to B. Move *n*-1 disks from C to A



Evaluation

- If $T(n)$ is the number of moves for *n* disks, then
  - $T(1) = 1 \qquad T(n+1) = 2T(n) + 1$

Solving the recurrence

$$
\begin{aligned}
T(n) &= 2T(n-1) + 1 \\
&= 2(2T(n-2) + 1) + 1 = 4T(n-2) + 2 + 1 \\
&= 2^3 T(n-3) + 4 + 2 + 1 \\
&= 2^4 T(n-4) + 2^3 + 2^2 + 1 \\
&= \vdots \\
&= 2^{n-1} + 2^{n-2} + \ldots 2^2 + 2^1 + 2^0 \\
&= 2^n - 1
\end{aligned}
$$