Homework: Algorithms — Order Statistics

If you use the median of median trick for order statistics on group of size 7, the argument shows that at least

$$4(\lceil \frac{1}{2} \lceil \frac{n}{7} \rceil \rceil - 2) \geq \frac{4}{14}n - 8 = \frac{2}{7}n - 8$$

elements are larger than the pivot and at least that many smaller than the pivot. Therefore, SELECT is called on at most $\frac{5}{7}n + 8$ elements. We can assume that any input smaller than 500 elements requires $O(1)$ time. This gives us the recurrence for the runtime of SELECT on n elements input as

$$T(n) \leq \begin{cases} O(1) & \text{if } n < 500 \\ T(\lceil \frac{n}{7} \rceil) + T(\frac{5}{7}n + 8) + an & \text{otherwise} \end{cases}.$$

In this equation, a represents the costs of grouping the array into groups of seven.

Problem: Show that there exists a constant $c > 0$ such that $T(n) \leq cn$.