Nested For Loops

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Nesting

- We can use nested loops
 - E.g. to solve mathematical puzzles or diophantine equations
 - Where we just try out all possibilities
 - Nota bene: Diophantes was preceded by Indian mathematicians

A diophantine equation is a polynomial equation with variables in the integers

• E.g.
$$x^2 - y^2 = z^3$$

- Has obviously trivial solutions
 - x any number, y = x, z = 0
- We exclude those by restricting ourselves to x > 0, y > 0, z > 0

- This still leaves us with an infinite number of number combinations
 - So, we <u>arbitrarily</u> limit ourselves to numbers < 1000
 - First attempt:

- Question:
 - How often are we executing the if-statement?
 - 999 times per y
 - 999 × 999 per x
 - 999 × 999 × 999 total
 - which is a lot (almost 1000 million)

- We can reduce this by observing that for a solution
 - 0 < x < 1000
 - 0 < x < y
 - 0 < z < x
- Now:

- Number of executions of the if-statement is now:
 - For each *x*
 - $(x-1) \times (x-1)$

Total number of times $\sum_{i=1}^{999} (x-1)^2$ is 331,835,499 or about a third of the previous value

- But still takes noticeable time
 - And we get a bunch of solutions with absolutely no insight

• A classic example:

SEND +MORE

MONEY

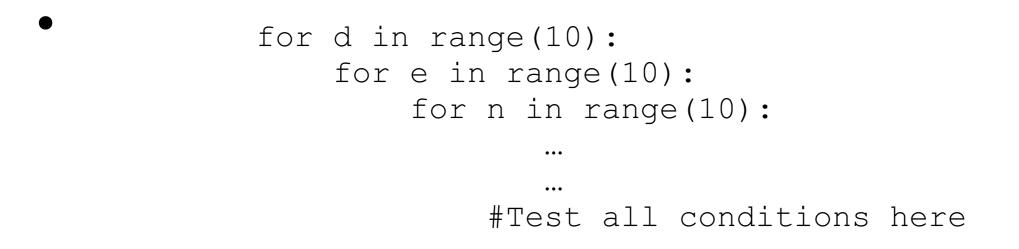
- D, E, M, N, O, R, S, Y are digits
 - In fact, they are different digits
 - And S and M are non-zero

- We can of course deduce solution
 - E.g. *M* has to be one
 - Therefore, S = 9

SEND +MORE

MONEY

• Our "brute force" algorithm could just start out with



- This just guarantees a lot of unnecessary tests
 - Easier to exclude cases as soon as possible
 - That all variables have to be different is a good candidate for early tests

- Now we need to deal with arithmetic!
 - Adding involves carries

- Carry out of a + b is (a + b)//10
- Resulting digit is (a + b) % 10

```
for d in range(10):
for e in range(10):
    if not e == d:
        for m in range (1, 10):
            if not m==d and not m==e:
                 for n in range(10):
                     if not n==m and not n==e and not n==d:
                         for o in range(10):
                             if not o==n and not o==m and not o==e and not o==d:
                                  for r in range (10):
                                      if not r==n and not r==o and not r==m and not r==e and not r==d:
                                          for s in range (1, 10):
                                              if s != n and s != r and s != o and s != m and s != e and s != d:
                                                  v = (d+e) %10
                                                  if y!=s and y!= r and y!=o and y!=r and y!=s and y!= d and y!=e and y!=m:
                                                       c1 = (d+e) / / 10
                                                       if e == (n+r+c1)%10:
                                                           c_{2} = (n+r+c_{1})/(10)
                                                           if n == (e+o+c2) \$10:
                                                               c3 = (e+o+c2)//10
                                                               if (s+m+c3)%10 == o and (s+m+c3)//10 == m:
                                                                   print(' ', s, e, n, d)
                                                                   print('+', m, o, r, e)
                                                                   print(m, o, n, e, y)
```