

# Module 8

Week 4 Monday

# Simulating population growth

Given an initial population, a rate of growth, and a maximum capacity how to model population growth?

We need a growth formula:

$$\text{growth} = \text{rate} * (1 - \text{population} / \text{capacity}) * \text{population}$$

We can run the simulation 500 times and observe how the population changes over time.

# Simulating population growth

```
def number(initial, rate, capacity):  
    num = initial  
    for i in range(500):  
        growth = rate*(1-num/capacity)*num  
        num += growth  
        print(i, num)
```

We can call this function with different values: `number(10, .2, 100)`

Copy and paste the population numbers into a spreadsheet and plot a graph of the growth. (`i` on the x-axis and `num` on the y-axis).

# Lotka-Volterra Models

- In the great war, there was no fishing in the Adriatic Sea
- When fisherman could fish again, they expected abundant catches
- However, this turned out to be false
  
- Search for an explanation led to the Lotka Volterra model for two species

# Lotka Volterra

- Predator – Prey Model:
  - $x$  – number of prey
  - $y$  – number of predators
- Population develops:
  - For prey:
    - few predators: gain population (no threats)
    - many predators: loss of population (get eaten)
  - For predator:
    - few prey: loss of population (starvation)
    - much prey: gain of population (feeding frenzy)

# Lotka Volterra

$$x_{n+1} = ax_n - bx_n y_n$$

$$y_{n+1} = cy_n + dx_n y_n$$

- $x$  – prey at time  $n$
- $y$  – predators at time  $n$
- $a$  natural growth rate of prey
- $b$  predation rate
- $c$  efficiency of turning prey into predators
- $d$  natural death rate of predator

# Implementing Lotka Volterra

- Define a function with parameters, including initial values  $x_0$  and  $y_0$
- Use a for loop in order to model 500 time periods

```
def lotka_volterra(x0, y0, a, b, c, d):  
    x = x0  
    y = y0  
    for i in range(500):  
        ...  
        print(i, x, y)
```

# Implementing Lotka Volterra

- Updating x and y according to the formula:
  - Need to use old value of x until both new values are set

```
def lotka_volterra(x0, y0, a, b, c, d):  
    x = x0  
    y = y0  
    for i in range(500):  
        xnew = a*x-b*x*y  
        ynew = c*y+d*x*y  
        x = xnew  
        y = ynew  
        print(i, x, y)
```

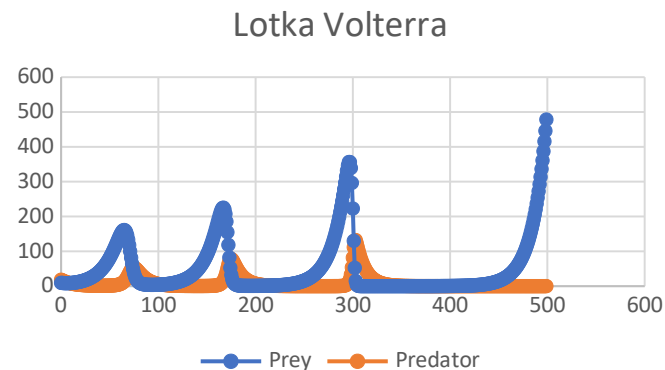


# Running Lotka Volterra

- Parameters are difficult to pick.
  - These ones work well

```
lotka_volterra(x0=10, y0=20, a=1.073, b=0.006, c=0.9, d=0.0021)
```

- Obtain numbers and paste into a spreadsheet
- Then make a table and see how the solution develops



# Running Lotka Volterra

- Periodic, but not stable
  - Population sizes are increasing
- Problem with Modeling
  - This is a cute problem
  - Suffers from the problem of the atto-tiger
    - quintillionth of a tiger
    - The predator population can become very small
- But still first successful population model that explained reality

