Laboratory 4: Loops and Functions

1. Write a function that calculates the expression $\sqrt{\frac{x^2+2}{x^2+1}} - 1$. Remember that you need to **return** the value of the expression, not print it out.

2. Write a function with a single argument n that we assume to be an integer. If n is zero, then the function returns 1, if n is negative, the function returns 0, and otherwise it returns $\sum_{\nu=0}^{n} \frac{1+\nu}{1+\nu^2}$.

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```

3. Write a function of *n* and *m* that prints out the grid on the right with *n* columns and *m* rows.

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*********
**** ****
***   ***
**     **
*       *
**     **
***   ***
**** ****
*********
```

4. Write a function that prints out *m* asterisks followed by 2*n*+1 spaces, followed by *m* asterisks. Then use this function repeatedly to print out the pattern on the left.

5. Write a function that finds the fraction $\frac{a}{b}$ of two integers with up to three digits that comes closest to π. Similarly, for $\sqrt{2}$. (Hint: All integers of up to three digits are in `range(1, 1000)`. Use two nested for loops to try out all possibilities, comparing each fraction with the target and remembering the best one and its goodness of fit seen so far.

6. Create a function of a sum, the annual interest rate, and a number of years that calculates the value of the sum after the stated number of years receiving annual interest payments. Create another function that accumulates interests every month with 1/12 of the rate. Write a program that for a sum of 10000 and interest rate between 2% and 5% shows the accumulated amount after 20 years.

7. Write a function that calculates $\sum_{k=0}^{100} \frac{(-1)^k}{(2k+1)!} x^{2k+1}$ as a function of *x*. Then compare the value of this function with the sine-function for $x \in \{-10, -9, \ldots, 9, 10\}$. Remember that `math.factorial(x)` calculates x!. You need to say `import math` on the first line of your script before you can use it.