Activities — for and while loops

(1) Use a Python while loop to find the smallest integer that fulfills the following sets of congruences:

$x \equiv 1 \pmod 2,\ x \equiv 2 \pmod 3,\ x \equiv 4 \pmod 5, x \equiv 6 \pmod 7.$

Make sure that you do not forget to modify your loop variable `x`. And of course, use the \% operator. For example, the first condition means that `x%2` is `1`.

(2) Using two nested Python for-loops and guessing that the solution is not larger than the product of the moduli (13×11), find a solution of the system of linear congruences

$3 \cdot x + 4 \cdot y \equiv 5 \pmod{13},\ 2 \cdot x - 3 \cdot y \equiv 4 \pmod{11}.$

(3) Calculate the following sums. The result is given for checking. Use **both** for and while loops.

$$\sum_{i=0}^{1000} \frac{1}{i^2+2} = 2.07657$$

$$\sum_{i=2}^{100} \frac{i+1}{i-1} = 109.355$$

(4) A person takes out a credit of one lakh rupees. Each month, the person pays 0.8\% interest on the loan. Thus, the first month, the interest payment is 800 ₹. The person pays every month 1000 ₹ for the loan. After the first month, the loan amount is now

100000 ₹ + 800 ₹- 1000 ₹ = 99800 ₹.

Write a product that displays the month, the interest paid, the amount of loan still outstanding until the outstanding loan becomes zero. Here is the beginning of the output:

```
1 800.0 99800.0
2 798.4 99598.4
3 796.7872 99395.1872
4 795.1614976000001 99190.3486976
```

(5) We want to solve the equation

$$-x^6 + x^5 - x^3 + x^2 - x + 0.3 = 0.$$

If we plug in 0 for the expression on the left, we obtain 0.3. If we plug in 1, we obtain -0.7. We decide that there should be a solution between 0 and 1. We divide the interval between 0 and 1 in the middle and have a new point 0.5. If we evaluate the expression at 0.5, we get -0.059375. Therefore, there should be a solution between 0 and 0.5. Dividing the interval in half, we get the new midpoint 0.25. Since the value of the expression is 0.0976074, we decide that a solution should be between 0.25 and 0.5. We now evaluate at the midpoint, which is 0.375, and obtain 0.0175255. Since this is a positive number, we deduce that a solution lies between 0.375 and 0.5. The midpoint is 0.4375 and the evaluation of the expression there gives -0.020818. Therefore, a solution is between 0.375 and 0.4375. Continue this process with a Python program until the interval between the lower and the upper estimate is less than 0.0001 long. Notice that if the evaluation at the midpoint is positive, than the lower limit of the interval becomes the midpoint and otherwise the upper limit becomes the midpoint.