

Homework 4 Solutions

Problem 1:

Subproblem A:

$A, B \rightarrow C$ and $C \rightarrow D$ imply $A, B \rightarrow D$. $C \rightarrow D$ and $D \rightarrow A$ implies $C \rightarrow A$.

This gives us the following non-trivial functional dependencies with singleton right side.

$A, B \rightarrow C$

$A, B \rightarrow D$

$C \rightarrow A$

$C \rightarrow D$

$D \rightarrow A$

The closures are

$\{A\}^+ = \{A\}$,

$\{B\}^+ = \{B\}$,

$\{C\}^+ = \{C, D, A\}$,

$\{D\}^+ = \{D, A\}$,

$\{A, B\}^+ = \{A, B, C, D\}$, so $\{A, B\}$ is a key.

$\{A, C\}^+ = \{A, C, D\}$.

$\{A, D\}^+ = \{A, D\}$.

$\{B, C\}^+ = \{B, C, D, A\}$, so $\{B, C\}$ is a key.

$\{B, D\}^+ = \{B, D, A, C\}$, so $\{B, D\}$ is a key.

$\{C, D\}^+ = \{C, D, A\}$

$\{A, B, C\}^+ = \{A, B, C, D\}$, so $\{A, B, C\}$ is a superkey that is not a key.

$\{A, B, D\}^+ = \{A, B, C, D\}$, so $\{A, B, D\}$ is a superkey that is not a key.

$\{A, C, D\}^+ = \{A, C, D\}$, so $\{A, C, D\}$ is **not** a superkey that is not a key.

$\{B, C, D\}^+ = \{A, B, C, D\}$, so $\{B, C, D\}$ is a superkey that is not a key.

$\{A, B, C, D\}^+ = \{A, B, C, D\}$, so $\{A, B, C, D\}$ is a superkey that is not a key.

Subproblem B:

$A \rightarrow B$ and $B \rightarrow C$ gives $A \rightarrow C$.

$A \rightarrow B$ and $B \rightarrow D$ gives $A \rightarrow D$.

The closures are

$\{A\}^+ = \{A, B, C, D\}$, so $\{A\}$ is the key

$\{B\}^+ = \{B, C, D\}$,

$\{C\}^+ = \{C\}$,
 $\{D\}^+ = \{D\}$,
 $\{A, B\}^+ = \{A, B, C, D\}$, so $\{A, B\}$ is a superkey that is not a key.
 $\{A, C\}^+ = \{A, B, C, D\}$, so $\{A, C\}$ is a superkey that is not a key.
 $\{A, D\}^+ = \{A, B, C, D\}$, so $\{A, D\}$ is a superkey that is not a key.
 $\{B, C\}^+ = \{B, C, D\}$,
 $\{B, D\}^+ = \{B, C, D\}$,
 $\{C, D\}^+ = \{C, D\}$
 $\{A, B, C\}^+ = \{A, B, C, D\}$, so $\{A, B, C\}$ is a superkey that is not a key.
 $\{A, B, D\}^+ = \{A, B, C, D\}$, so $\{A, B, D\}$ is a superkey that is not a key.
 $\{A, C, D\}^+ = \{A, B, C, D\}$, so $\{A, C, D\}$ is a superkey that is not a key.
 $\{B, C, D\}^+ = \{A, B, C, D\}$, so $\{B, C, D\}$ is a superkey that is not a key.
 $\{A, B, C, D\}^+ = \{A, B, C, D\}$, so $\{A, B, C, D\}$ is a superkey that is not a key.

Subproblem C:

We first calculate the closures:

$\{A\}^+ = \{A\}$,
 $\{B\}^+ = \{B\}$,
 $\{C\}^+ = \{C\}$,
 $\{D\}^+ = \{D\}$,
 $\{A, B\}^+ = \{A, B, C, D\}$, so $\{A, B\}$ is a key.
 $\{A, C\}^+ = \{A, C\}$.
 $\{A, D\}^+ = \{A, B, C, D\}$, so $\{A, D\}$ is a key.
 $\{B, C\}^+ = \{A, B, C, D\}$, so $\{B, D\}$ is a key
 $\{B, D\}^+ = \{B, D\}$,
 $\{C, D\}^+ = \{A, B, C, D\}$, so $\{C, D\}$ is a key.
 $\{A, B, C\}^+ = \{A, B, C, D\}$, so $\{A, B, C\}$ is a superkey that is not a key.
 $\{A, B, D\}^+ = \{A, B, C, D\}$, so $\{A, B, D\}$ is a superkey that is not a key.
 $\{A, C, D\}^+ = \{A, B, C, D\}$, so $\{A, C, D\}$ is a superkey that is not a key.
 $\{B, C, D\}^+ = \{A, B, C, D\}$, so $\{B, C, D\}$ is a superkey that is not a key.
 $\{A, B, C, D\}^+ = \{A, B, C, D\}$, so $\{A, B, C, D\}$ is a superkey that is not a key.

We then determine the non-trivial functional dependencies from there:

$A, B \rightarrow C$, $A, B \rightarrow D$,
 $A, D \rightarrow B$, $A, D \rightarrow C$
 $B, C \rightarrow A$, $B, C \rightarrow D$
 $C, D \rightarrow A$, $C, D \rightarrow B$

Subproblem D:

We first calculate the closures:

$\{A\}^+ = \{A, B, C, D\}$, so $\{A\}$ is a key

$\{B\}^+ = \{B, C, D, A\}$, so $\{B\}$ is a key

$\{C\}^+ = \{C, D, A, B\}$, so $\{C\}$ is a key.

$\{D\}^+ = \{D, A, B, C\}$, so $\{D\}$ is a key.

Therefore all sets with two or more attributes have as closure the set of all four attributes and are superkeys that are not keys.

Problem 2:

Functional Dependencies:

Battle \rightarrow Date is possible, but not guaranteed

Battle, Date, Ship \rightarrow Outcome because American and English names form almost disjoint sets

Battle, Ship \rightarrow Outcome is possible, but not guaranteed

Navy, Ship \rightarrow Type is almost guaranteed, but cannot be presumed

Ship \rightarrow Navy is possible in the data set but for captured ships, unless the captured ship is renamed.

Type \rightarrow Guns is not a FD