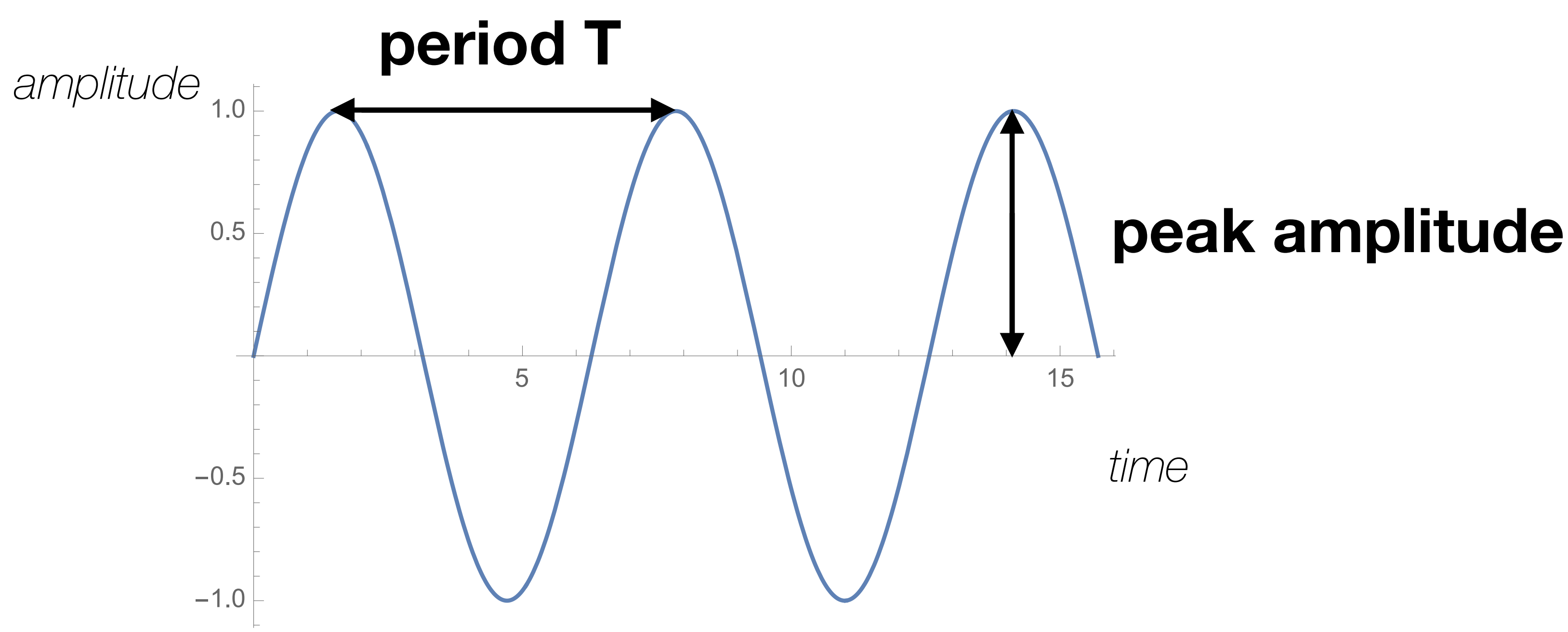
Networking

Marquette University
Fall 2021

Waves

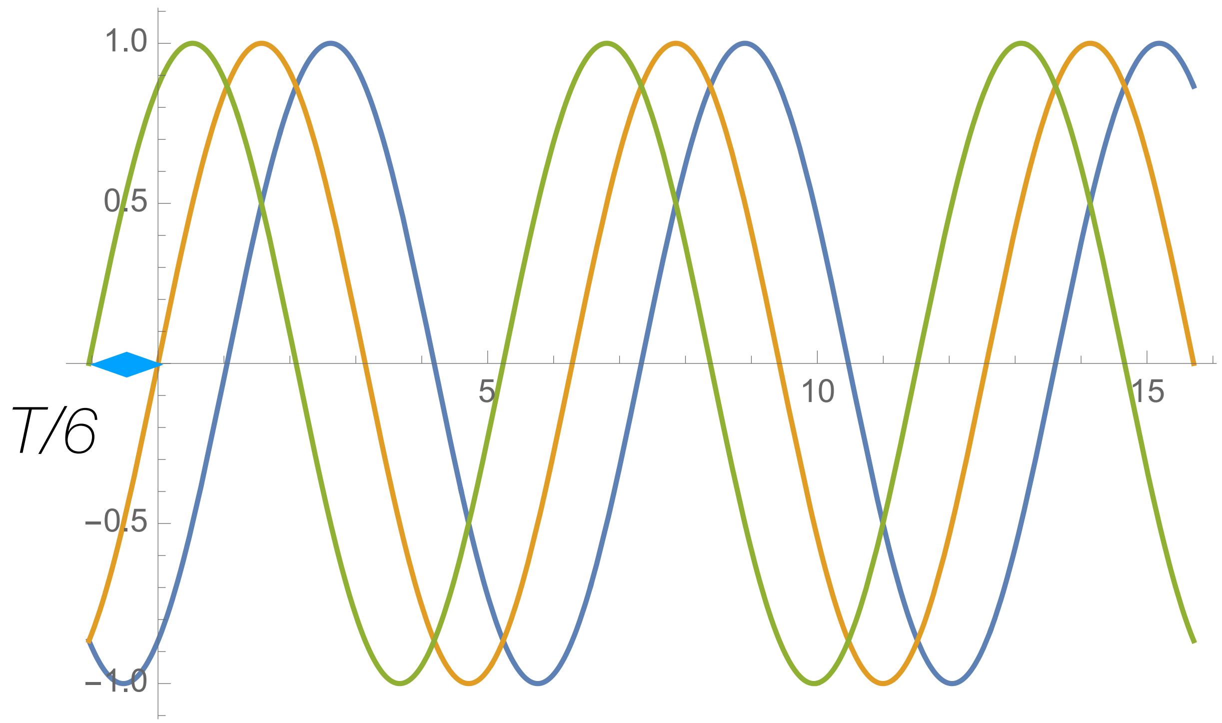
- Dynamic (sine-shaped) wave



- Period T
- Frequency $f = \frac{1}{T}$ measured in herz = 1/second

Waves

- Phase:
  - Amount of shift in forward direction needed to align to sine wave
  - Orange: Phase is zero
  - Blue: Phase is -T/6 or -60° = 300°
  - Green: Phase is T/6 or 60°



Exercises

- The period of a sine wave is 60 msec. What is the frequency?

$$f = \frac{1}{60 \text{ msec}} = \frac{1}{0.060 \text{ sec}} = 16.666667 \text{ herz}$$

- Electricity in the US has a frequency of 60 hz. What is the period?

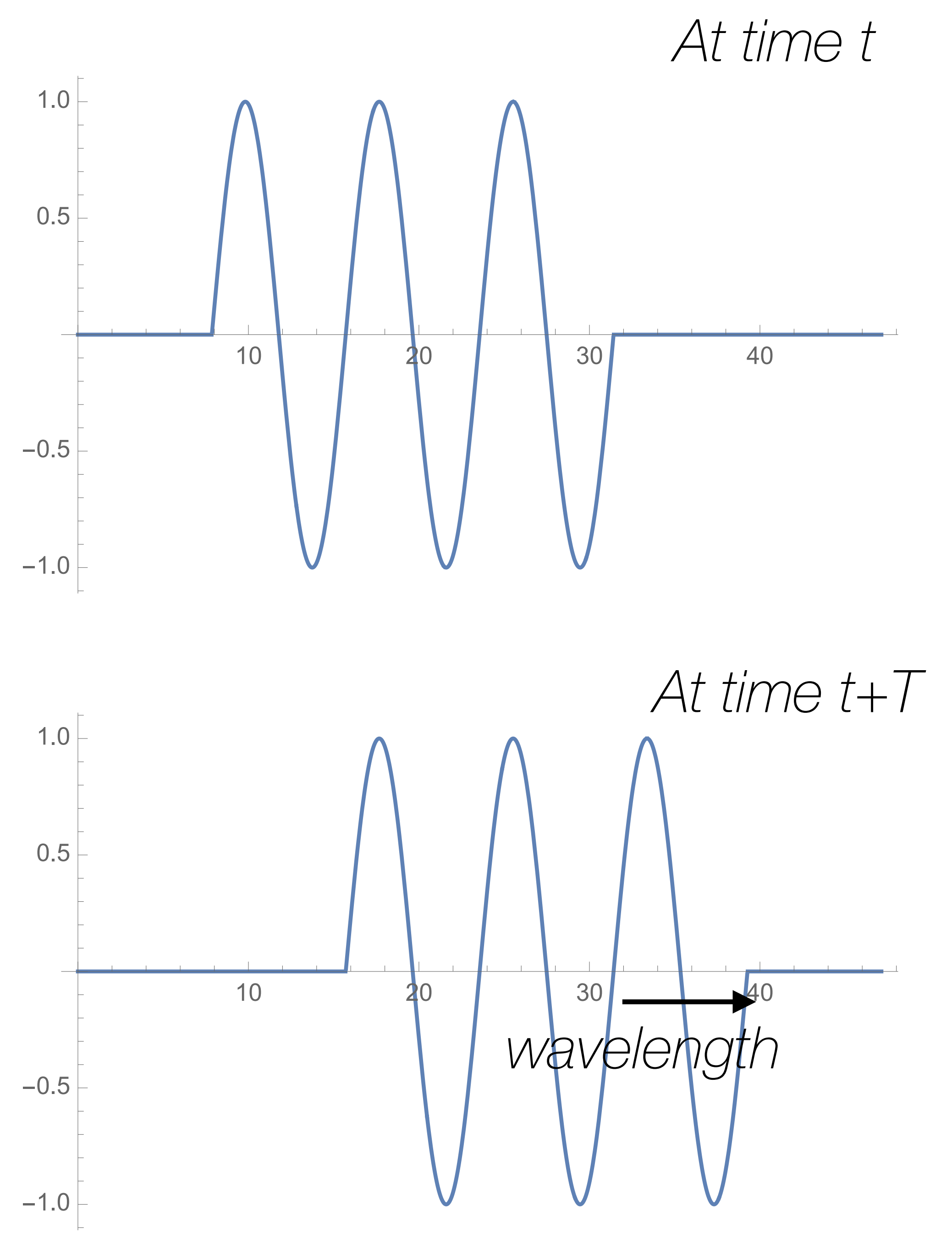
$$f = \frac{1}{60 \text{ herz}} = 0.016666667 \text{ sec} = 16.666667 \text{ msec}$$

Waves

- Waves propagate in a medium
- During one period, the wave progresses by a **wavelength** λ
- In vacuum, propagation speed is the speed of light, or

$$c \approx 3 \times 10^8 \mathrm{m/sec}$$

- General formula:
- $$\lambda = \frac{c}{f}$$



$$\text{wavelength} = (\text{propagation speed}) \times \text{period}$$

Waves

- Period: $T = \dfrac{1}{f}$
- Wavelength λ given by speed times travel time: $c \times T$
- Therefore
    - $\lambda = \dfrac{c}{f}$

Exercises

- Blue light has a frequency of about 650 Thz = 6.5×10^{14} hz
- What is its wavelength in micrometers?

$$\begin{aligned}\lambda &= \frac{3 \times 10^8 \text{ m/sec}}{6.5 \times 10^{14} \text{ 1/sec}} \\ &= 0.462 \times 10^{-6} \text{m} \\ &= 0.462 \ \mu\text{m}\end{aligned}$$

Exercises

- Blue light has a frequency of about 650 Thz $= 6.5 \times 10^{14}$ hz.
- What is its wavelength in micrometers in a coaxial cable. The propagation speed in a coax is two thirds of that in a vacuum.

$$\begin{aligned}\lambda = \frac{2 \times 10^8 \text{ m/sec}}{6.5 \times 10^{14} \text{ 1/sec}} \\ = 0.3 \times 10^{-6} \text{m} \\ = 0.3 \ \mu\text{m}\end{aligned}$$

Exercises

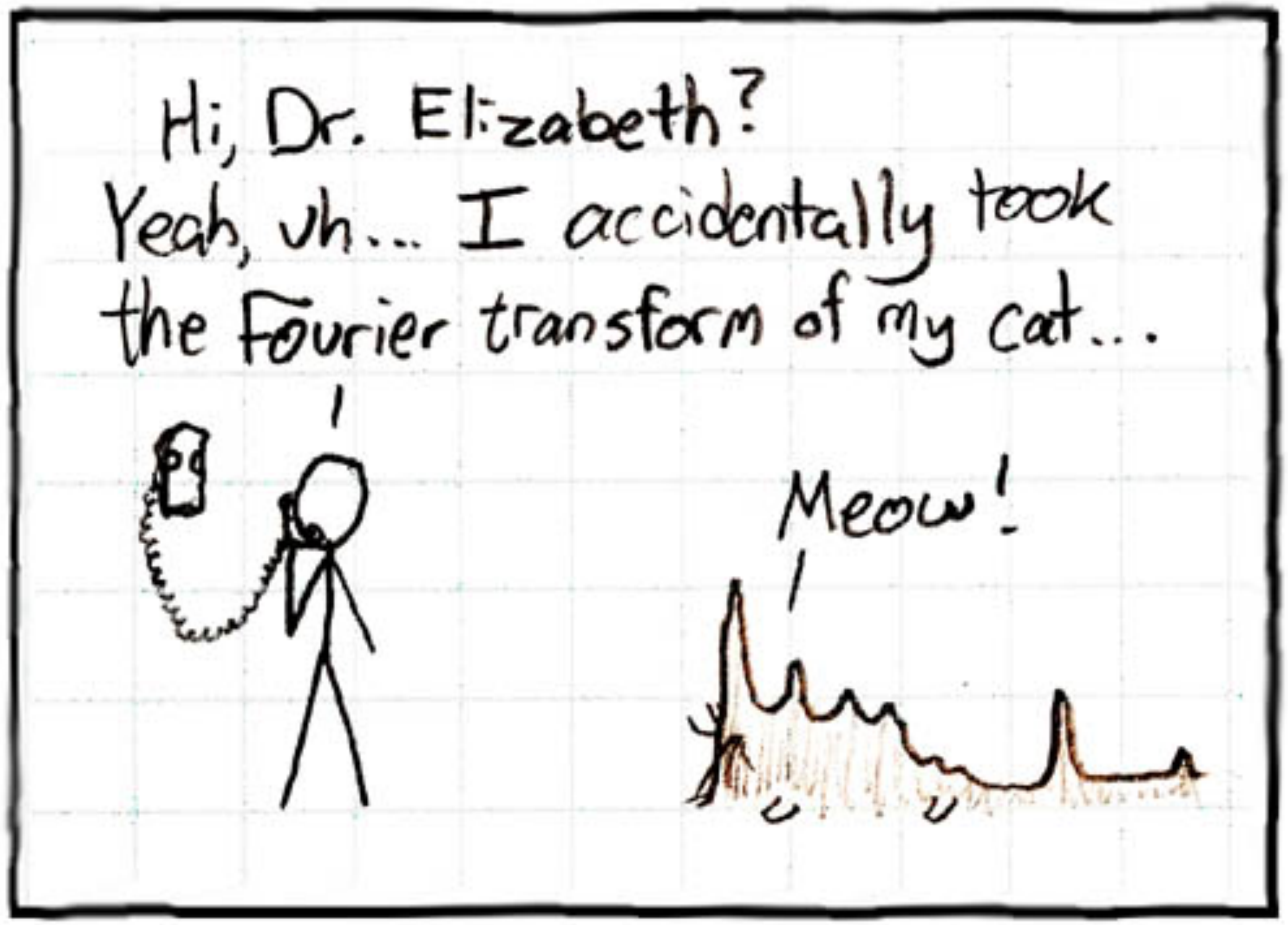
- The speed of sound is 343 m / sec (on a nice day). The standard A is 440 hz. What is the wavelength?

$$\lambda = \frac{343 \text{ m/sec}}{440 \text{ 1/sec}}$$
$$= 0.780\text{m}$$
$$= 78 \text{ cm}$$

Half a wavelength is used for wind instruments

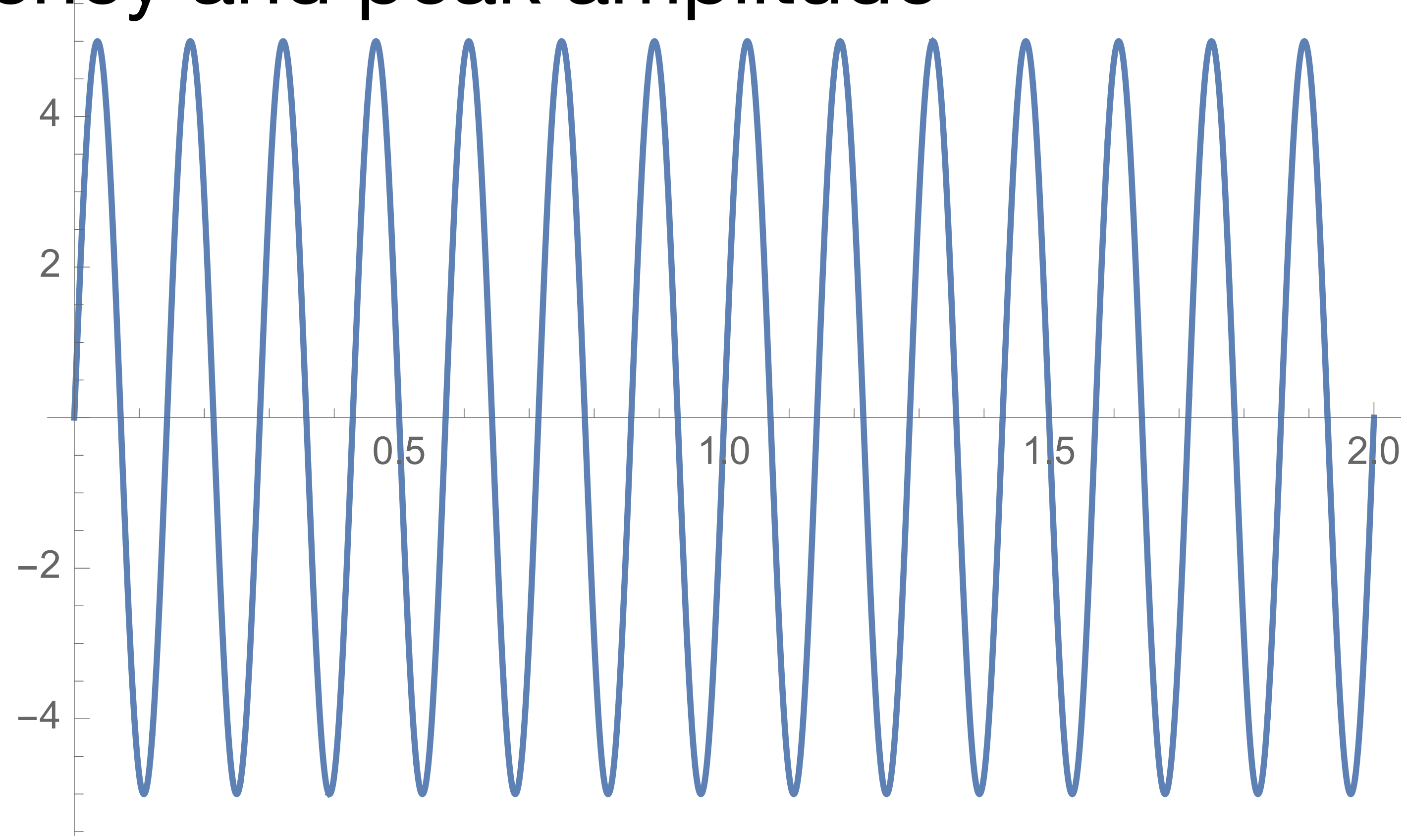
Time Frequency Domain

Fourier Analysis


Hi, Dr. Elizabeth?
Yeah, uh... I accidentally took the Fourier transform of my cat...
Meow!

Time - Frequency Domain

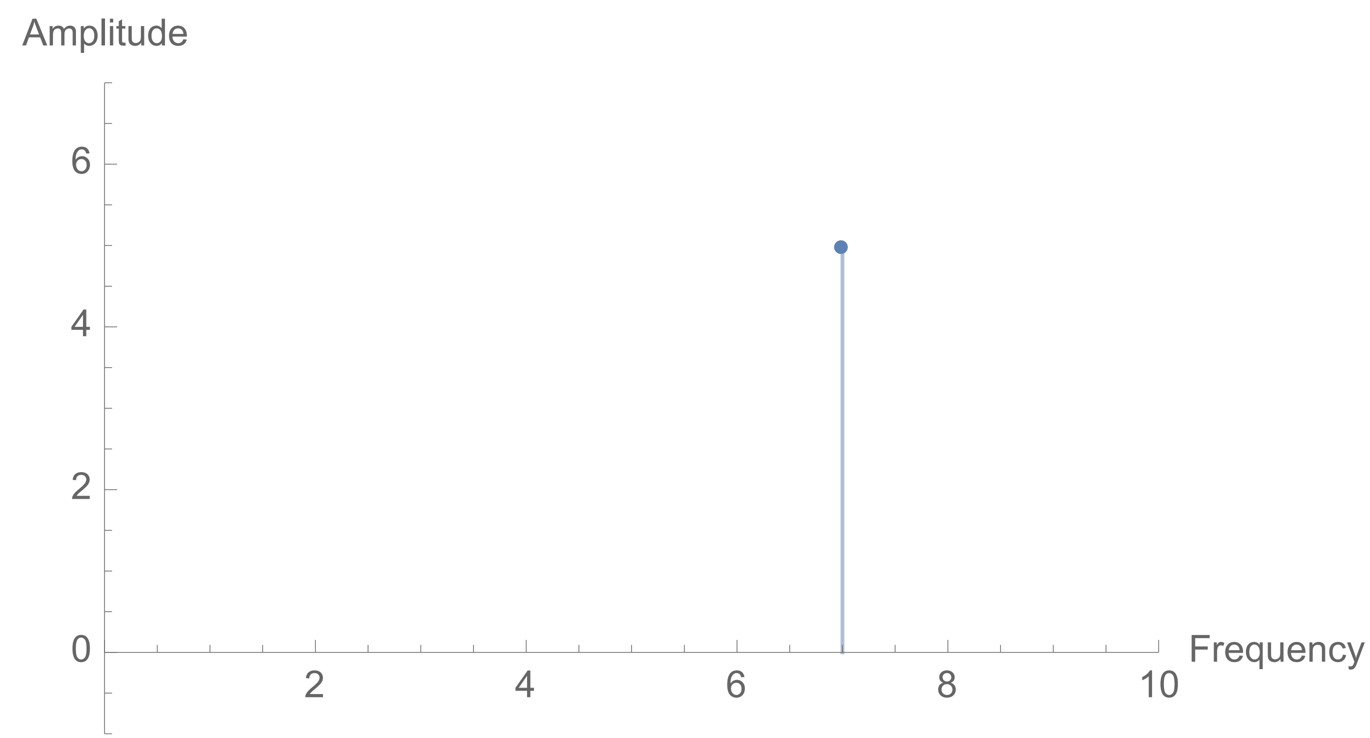
- If we disregard phase, then a sine-wave is determined by its frequency and peak amplitude



- Peak amplitude is 5, Frequency is 7 hz. (Interval is from 0 to 2.0)

Time - Frequency Domain

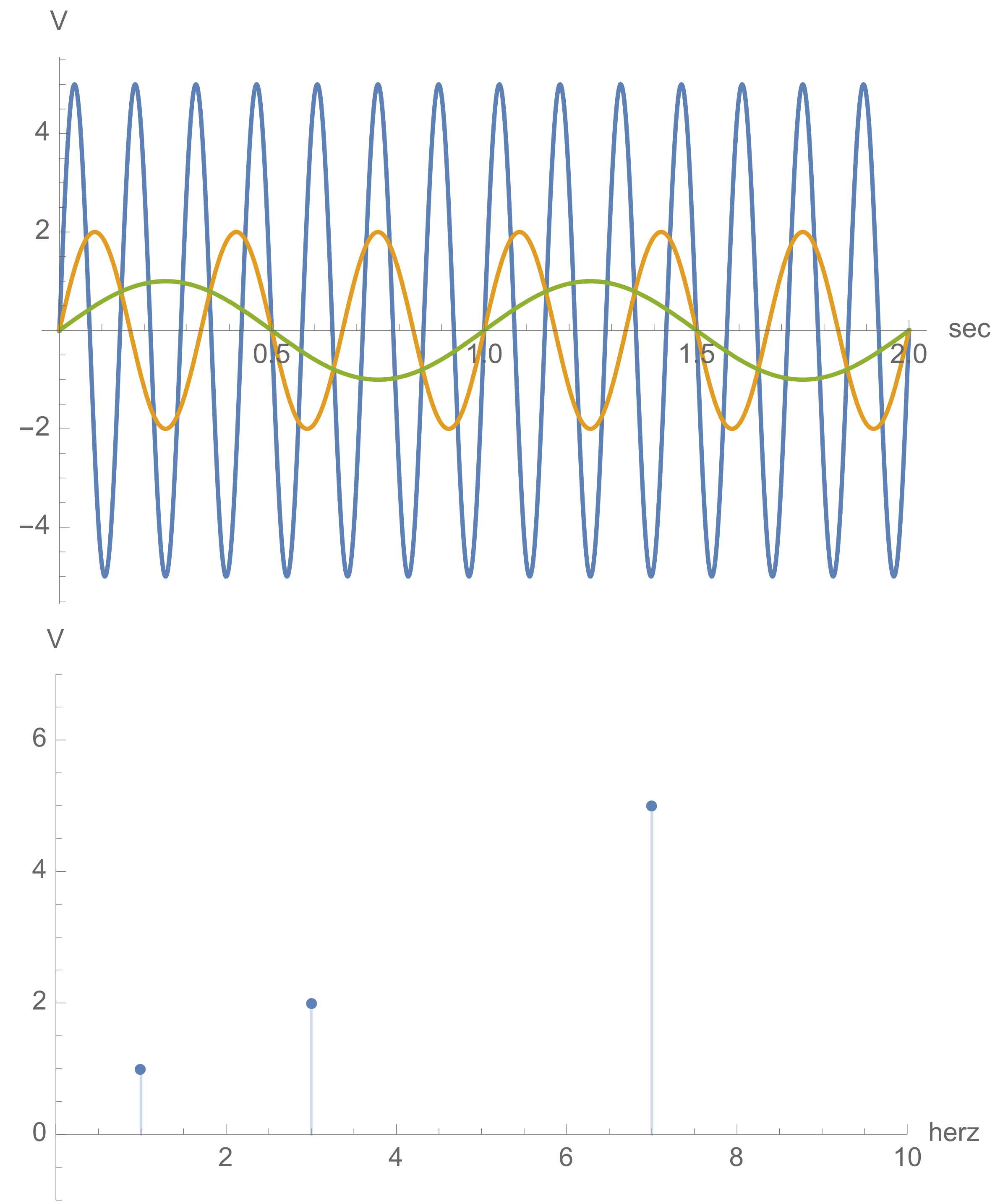
- Frequency Domain
- Gives the amplitude for the frequency



- A simple sine wave corresponds to a single spike

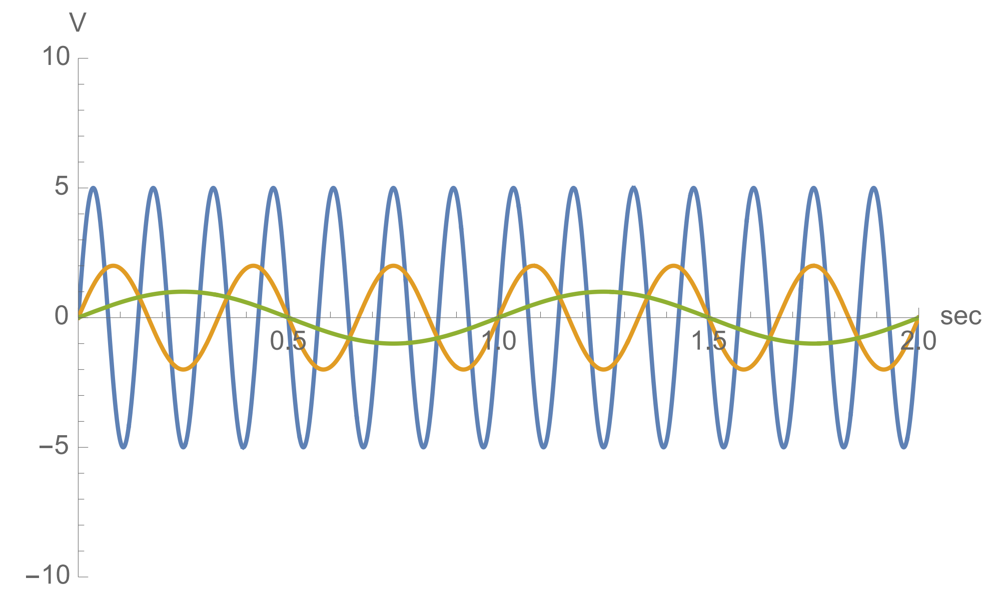
Time-Frequency Domain

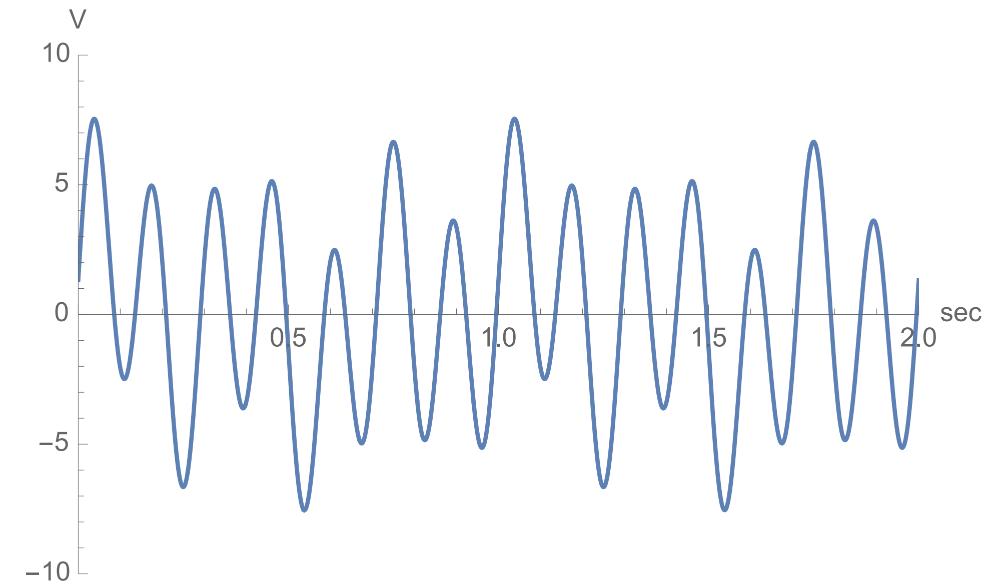
- Three functions with different frequencies and their frequency graph

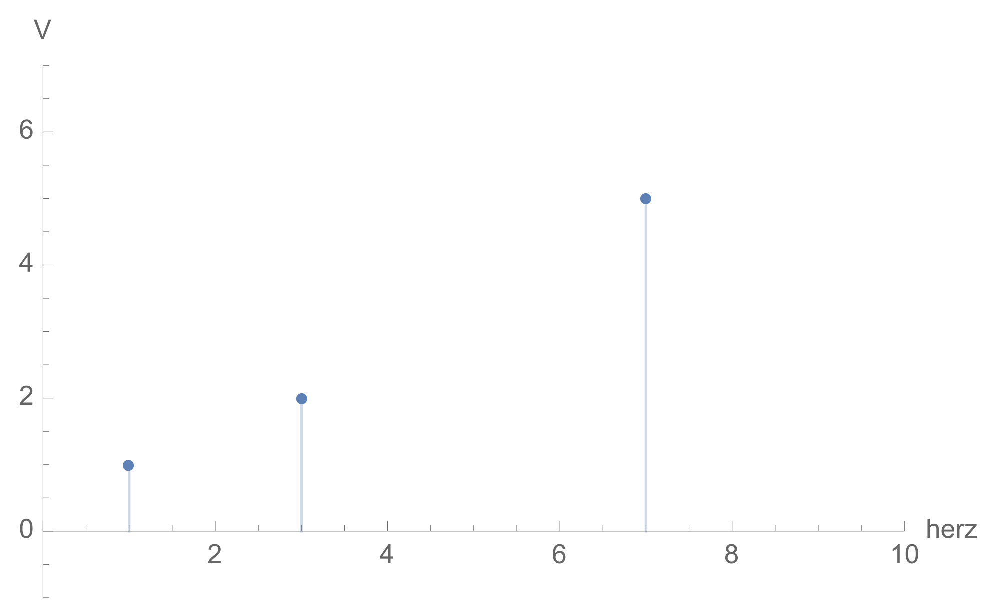


Time Frequency Domain

- We superimpose these functions







Fourier Analysis

- Any well-behaved periodic function can be written as a sum of sine functions with different frequencies, amplitudes, and phases
  - Assume a periodic function $f : \mathbb{R} \to \mathbb{R}$
    - Periodic means

$$\exists x_0 \in \mathbb{R} : \quad \forall x \in \mathbb{R} : \quad f(x + x_0) = f(x)$$

    - We pick $x = 2\pi$ without loss of generality

Fourier Analysis

- Then $f(x) = \frac{a_0}{2} + \sum_{i=1}^{\infty} a_i \cos(ix) + \sum_{i=1}^{\infty} b_i \sin(ix)$
  - with
    - $a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)dx$
    - $a_i = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\cos(ix)dx$
    - $b_i = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\sin(ix)dx$

Fourier Analysis

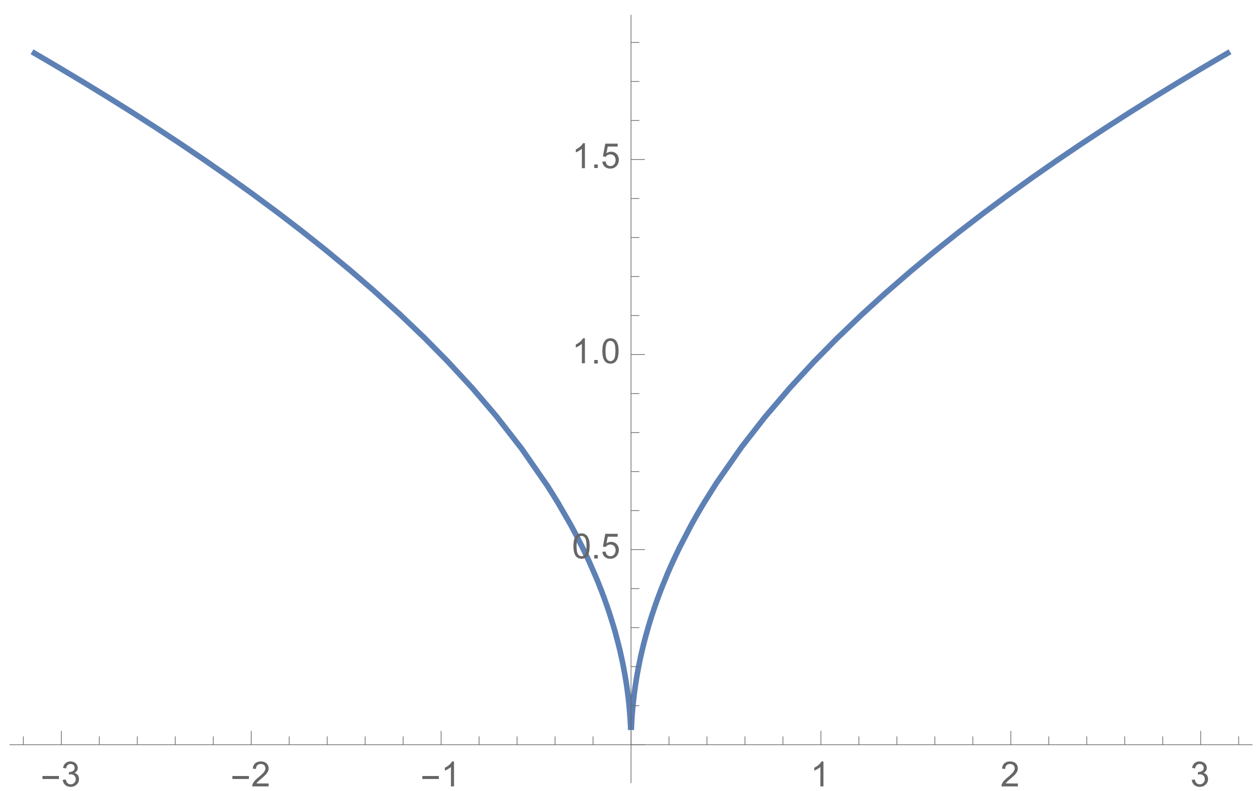
- Note that the sine is just the cosine with a phase difference of 90°
  - Peak amplitude is therefore just $\max(a_i + b_i)$

Fourier-Analysis Example

- Example:

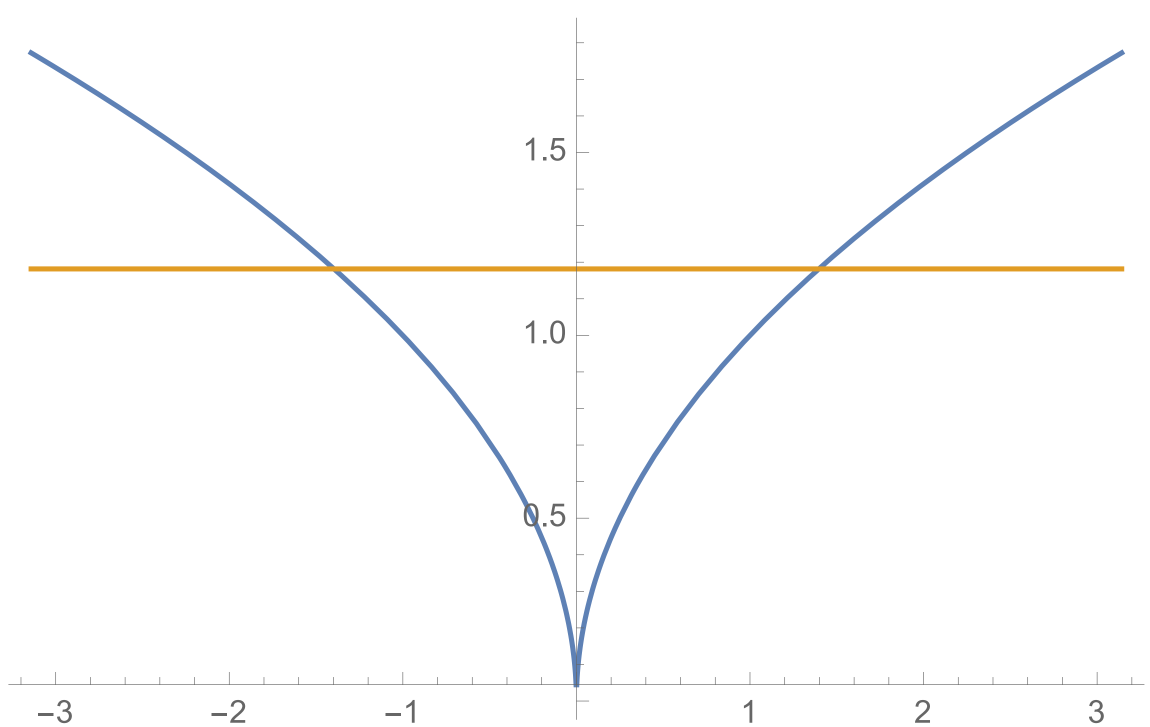
$$f(x) := \sqrt{|x|}; \quad x \in [-\pi, \pi]$$

- Use periodicity to extend definition to all real numbers



Fourier Analysis Example

- Constant member of Fourier series is $a_0 = \frac{2\sqrt{\pi}}{3}$
  - Of course, frequency 0 means a constant current
  - (This is actually a problem for signaling from one node to another because you are using up energy.)



Fourier Analysis Example

- The sine components are all zero

$$b_i = 0$$

- This is because the function is even:

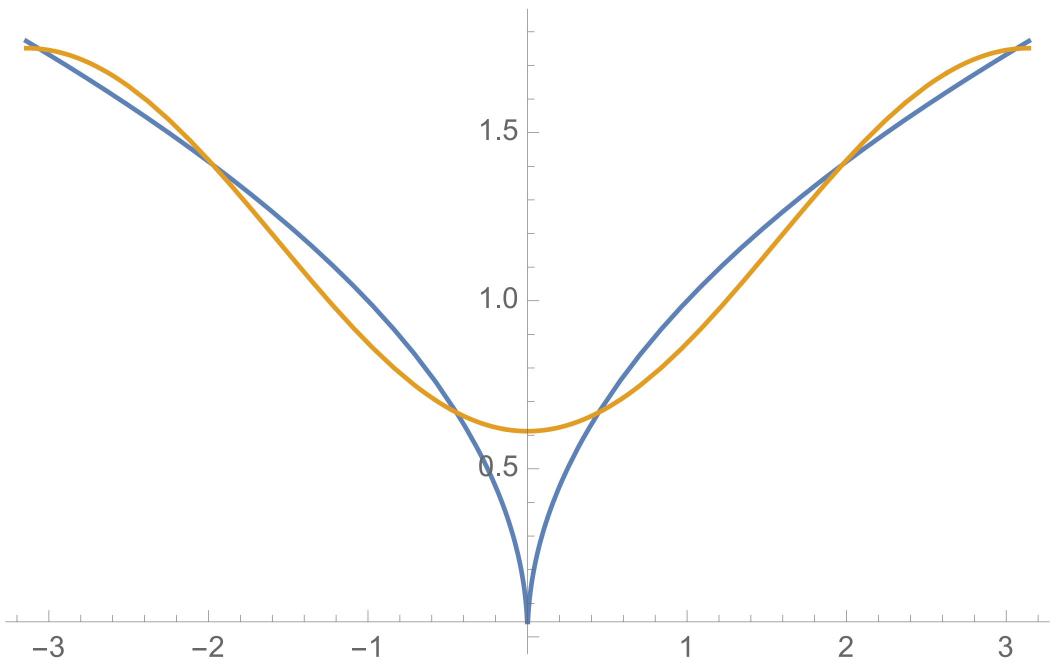
$$f(x) = f(-x)$$

Fourier Analysis Example

- First cosine coefficient is

$$a_1 = -0.569667$$

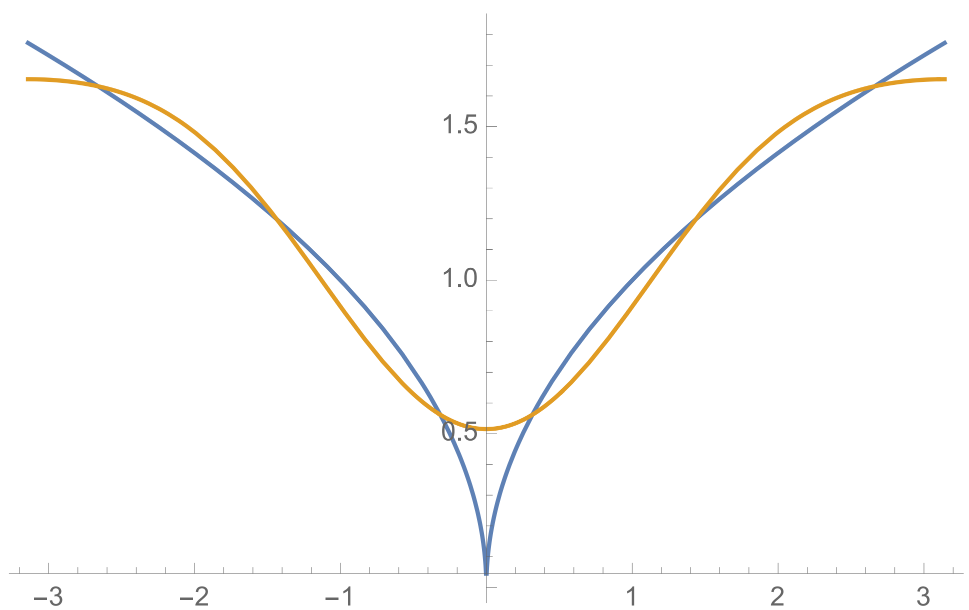
- First partial Fourier series is $a_0 + a_1 \cos(x)$
  - (Negative cos is just the cos at a phase of 180°)



Fourier Analysis Example

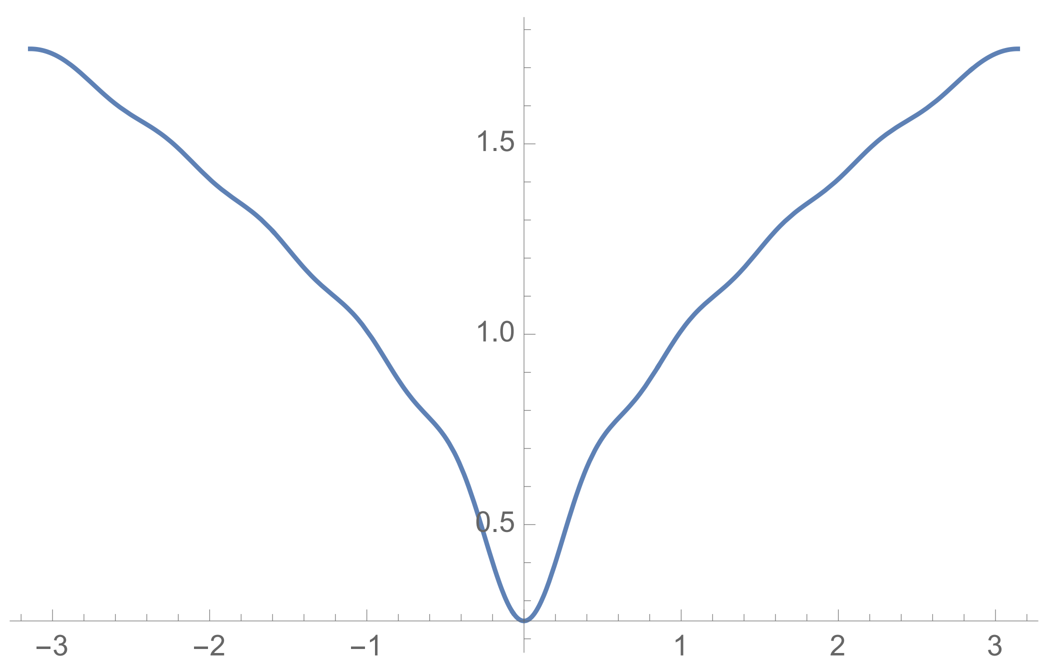
- Second cosine coefficient is $a_2 = -0.0968758$
- Second partial Fourier series is

$$a_0 + a_1 \cos(x) + a_2 \cos(2x)$$



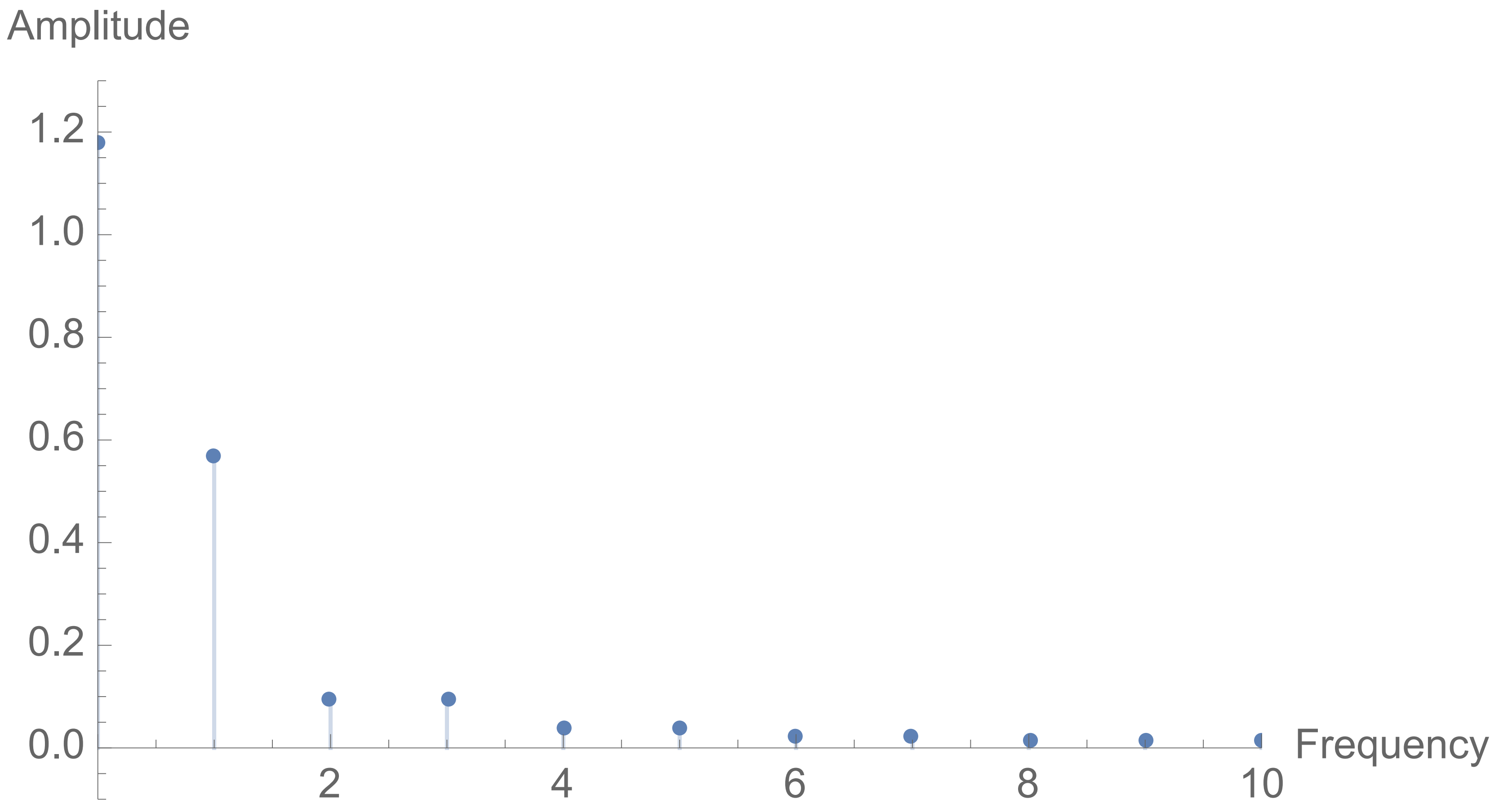
Fourier Analysis Example

- List of cosine coefficients is -0.569667, -0.0968758, -0.0965755, -0.0386943, -0.0428448, -0.0221663, -0.0252003, -0.0148282, -0.0169907, -0.0108211, …
- 10th partial Fourier sum gives a somewhat shaky approximation



Fourier Analysis Example

- We can use the coefficients to translate this function to the frequency domain



Fourier Analysis Example

- A simpler example is the square wave

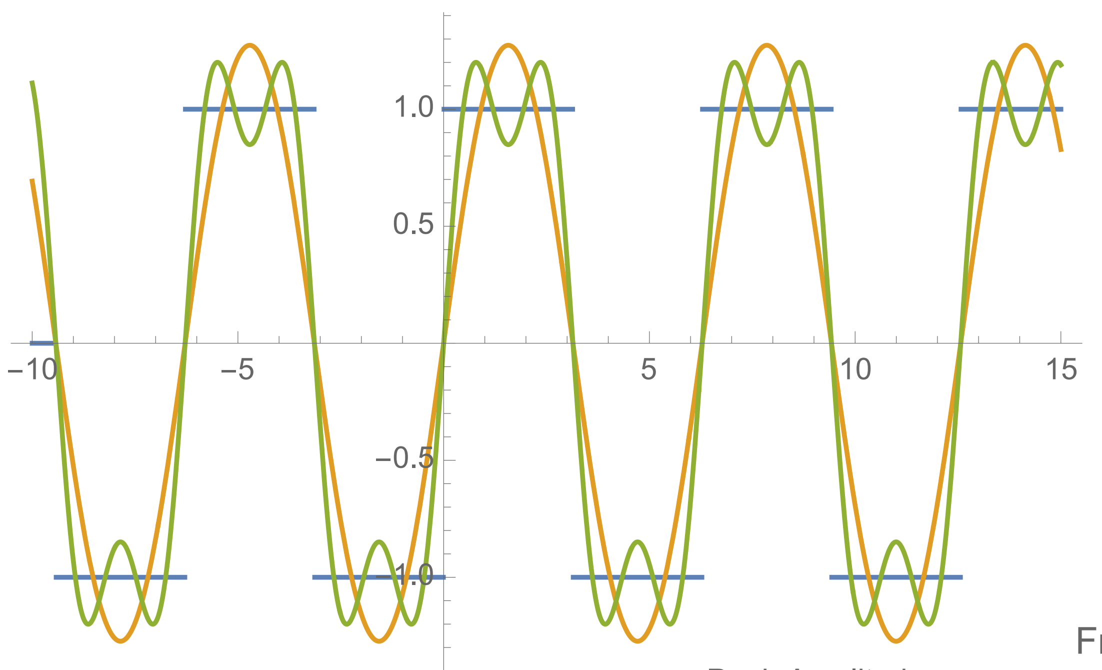
$$g(x) = \begin{cases} -1, & \text{if } 2n-1 < x < 2n \text{ for } n \in \mathbb{Z} \\ 1, & \text{if } 2n < x < 2n+1 \text{ for } n \in \mathbb{Z} \end{cases}$$

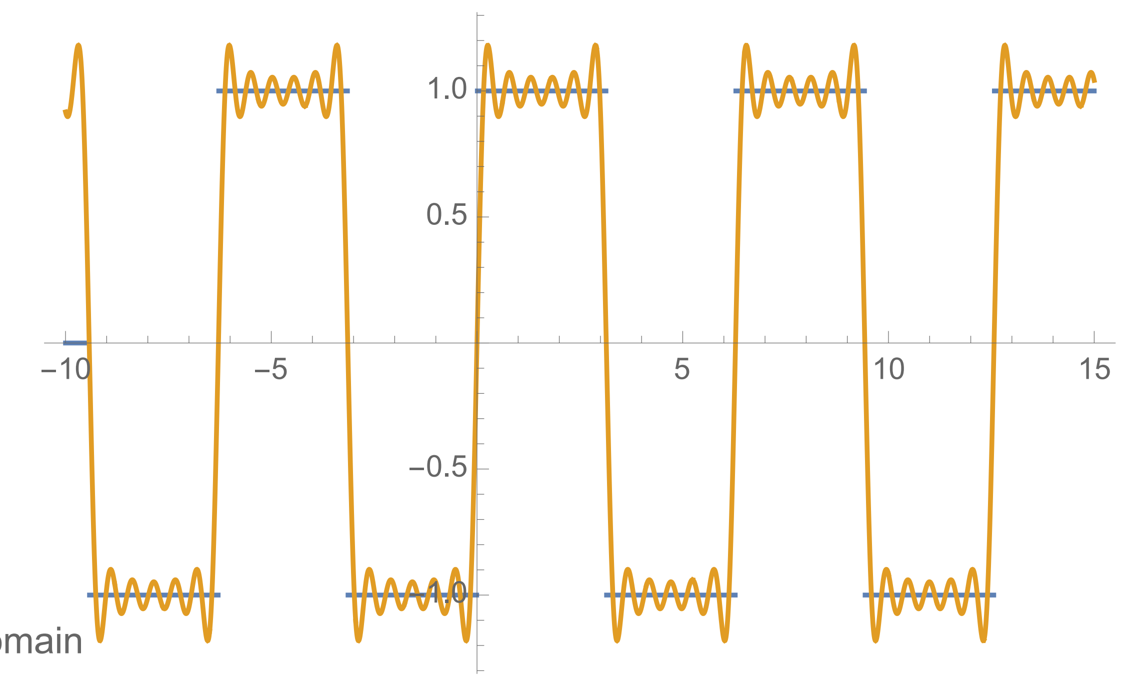
- Evaluating the integrals, we obtain

$$b_i = \begin{cases} 0, & \text{if } i \text{ is even} \\ \frac{4}{i\pi}, & \text{if } i \text{ is even} \end{cases}$$

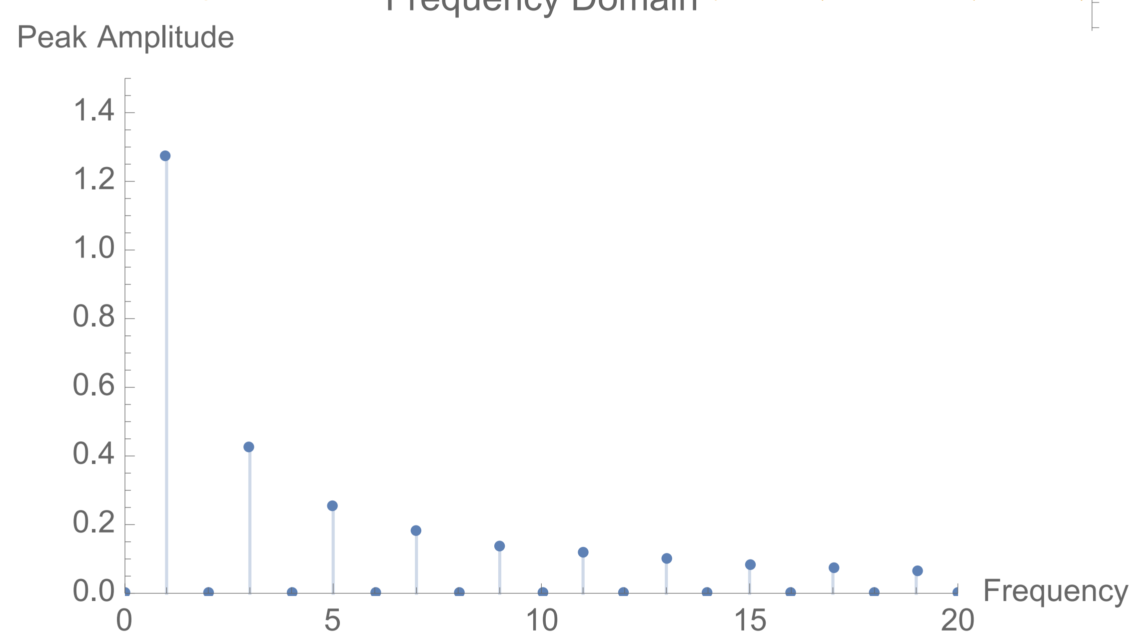
Fourier Analysis Example

- Notice Gibb's phenomenon: Ringing near discontinuities



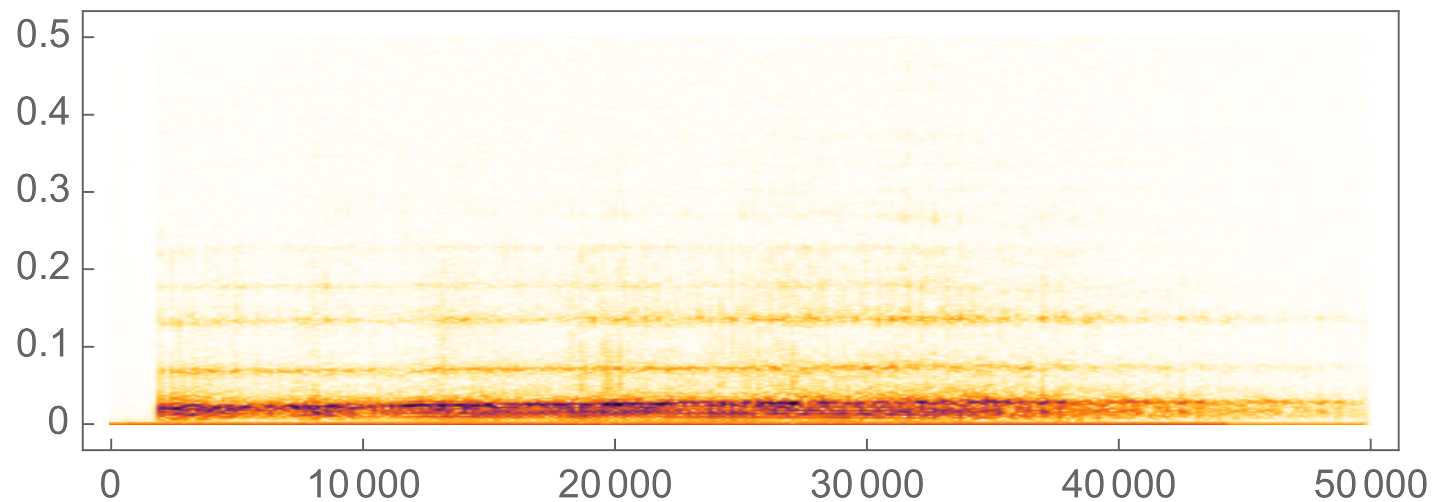


Frequency Domain



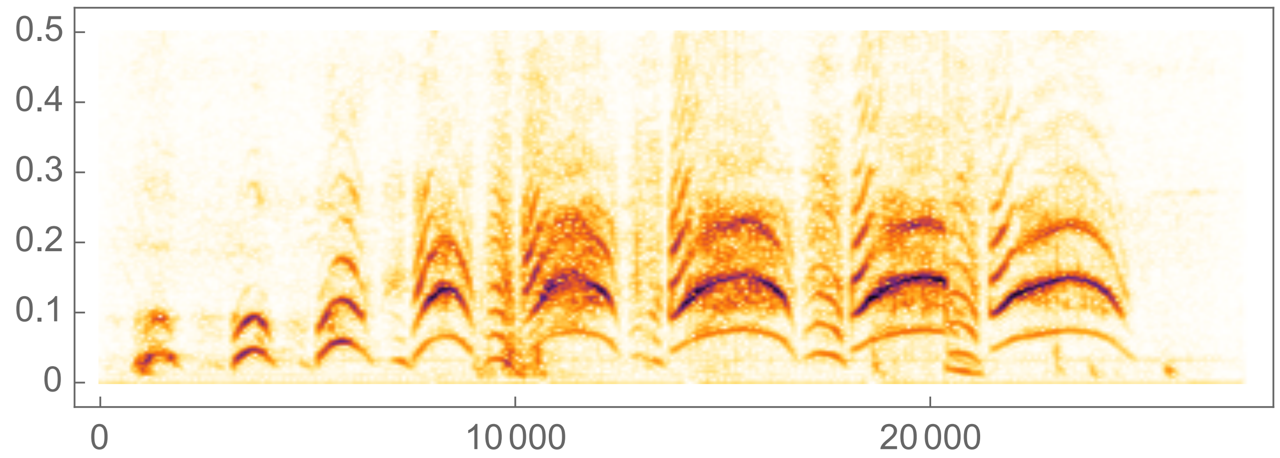
Fourier Analysis Examples

- Spectograms:
  - Audio signal
    - Break into short intervals
    - Display Fourier transform per interval
    - Bear growling



Fourier Analysis Examples

- Spectograms:
  - Audio signal
    - Break into short intervals
    - Display Fourier transform per interval
    - Monkey being a monkey



Fourier Analysis

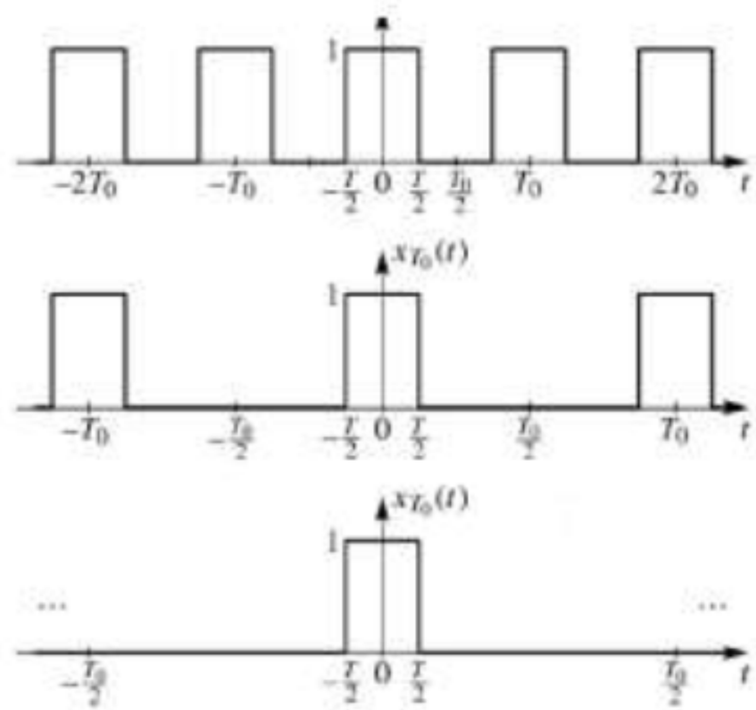
- To transmit signals, we need more than a single sine wave
- A composite signal is a combination of simple sine waves with different frequencies
  - Periodic signals can be decomposed into series of simple sign waves with discrete frequencies
  - An aperiodic signal can be decomposed into a combination of an infinite number of simple sine waves

Fourier Analysis

- Why is this important:
  - Carriers offer different attenuation to different frequencies
- We want to understand how carriers can subdivide their spectrum

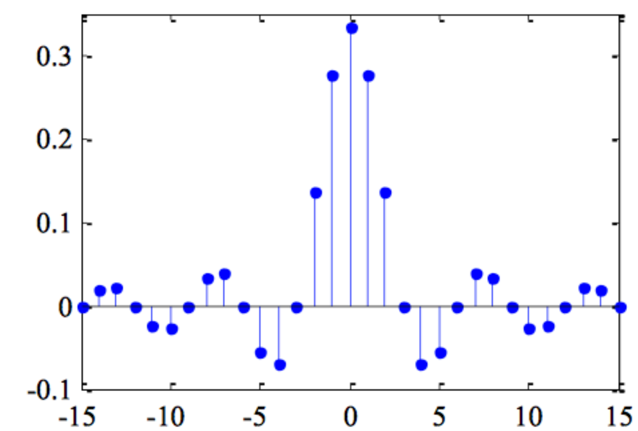
Non-periodic Signals

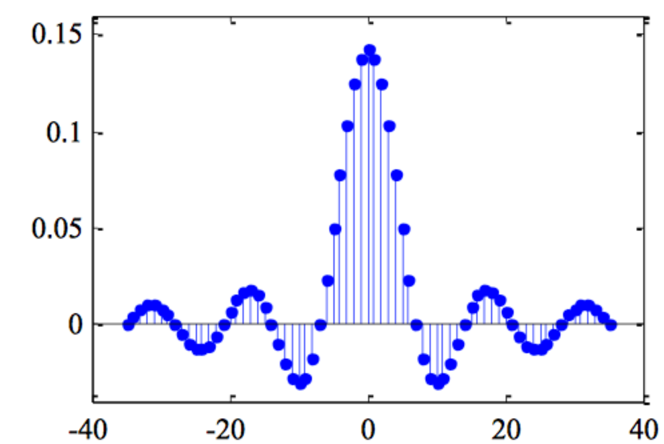
- In reality, signals are not periodic
  - We can make them periodic by repeating the pattern at larger and larger distances

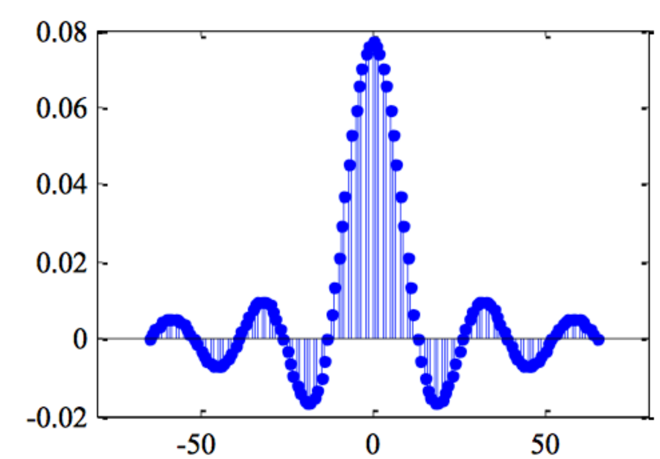


Non-periodic signals

- As the period gets bigger and bigger
  - Frequency domain graph becomes closer and closer to continuous function

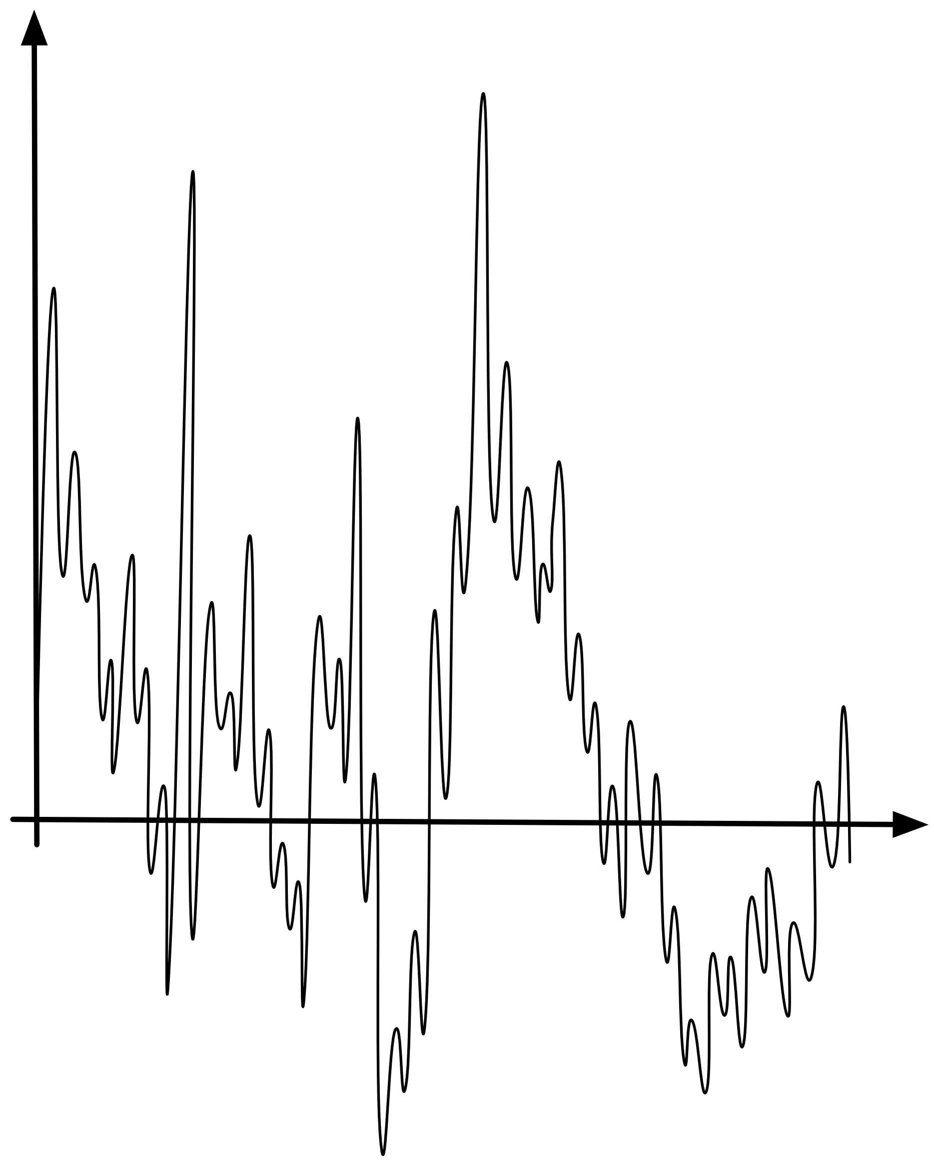




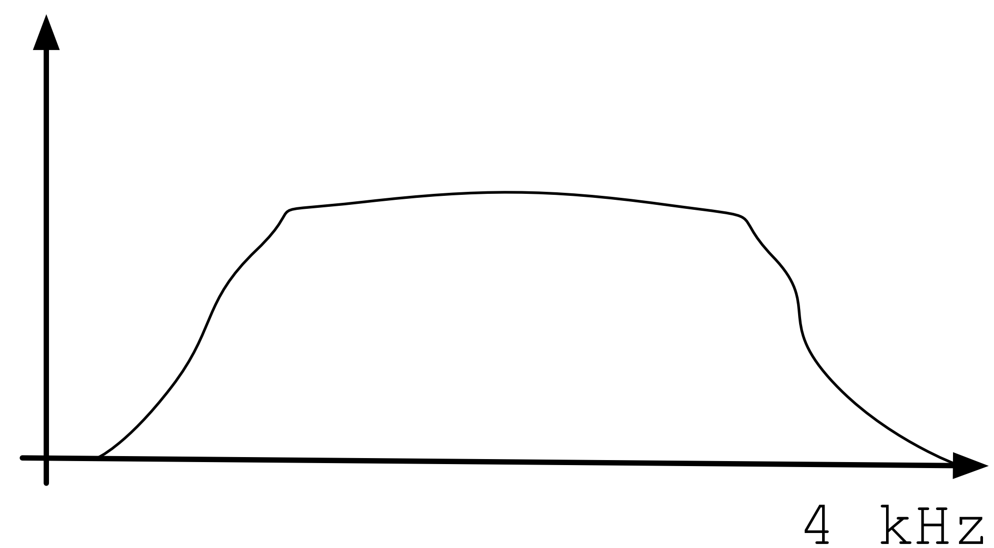


Non-periodic signal

Time Domain

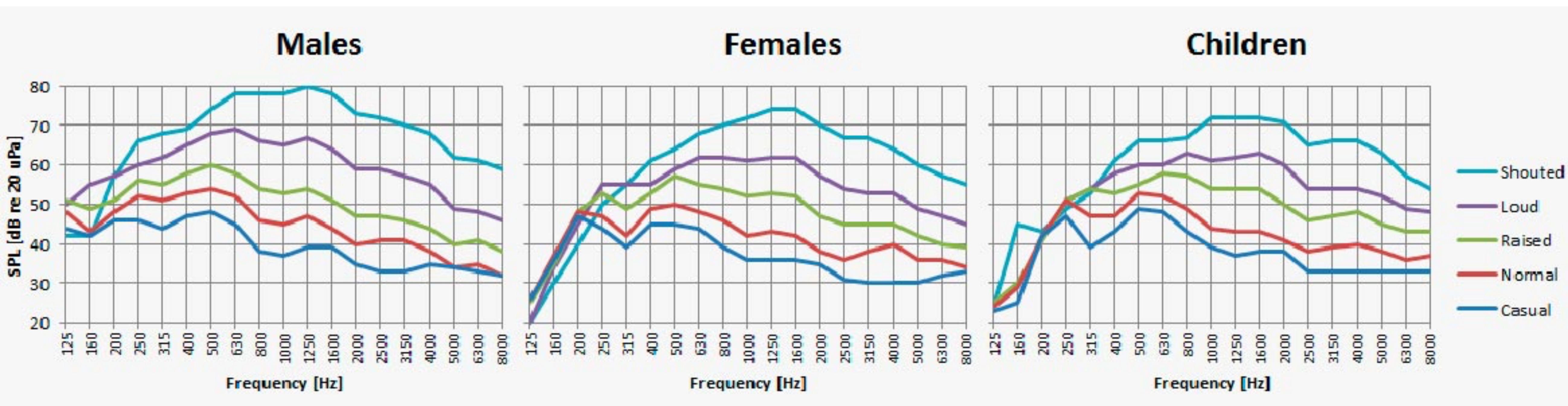


Frequency Domain



Non-periodic Signal

- Human voice has a continuous range of 20 to 10,000 Hz
  - Needs 100 to 4000 Hz to be understandable



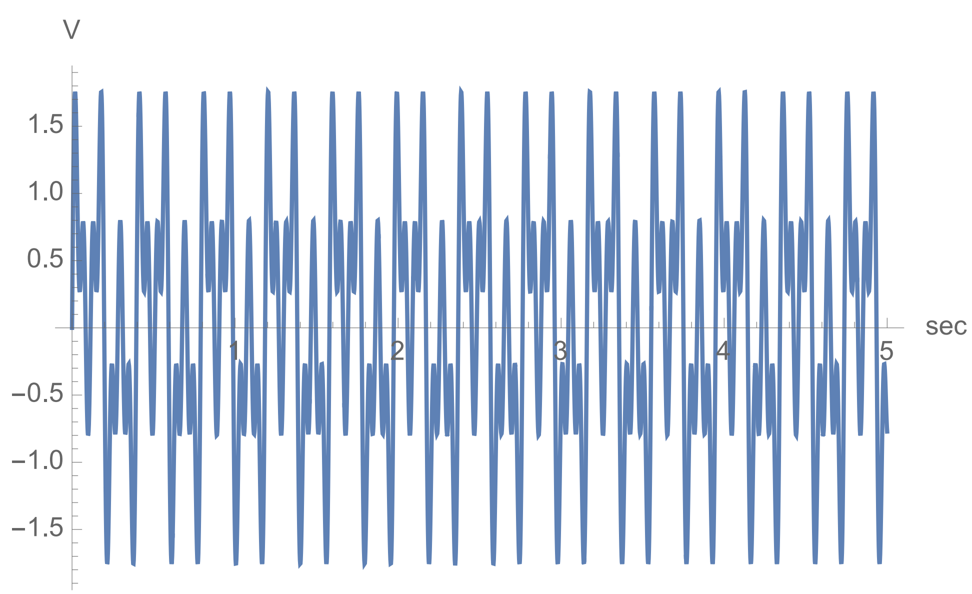
Bandwidth of Physical Links

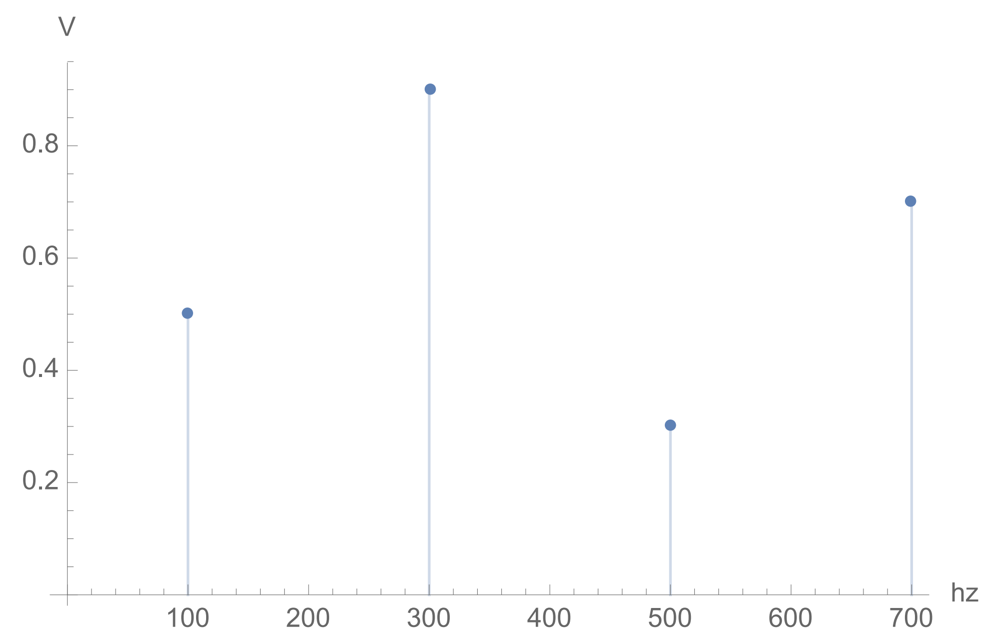
Physical Links

- Any analog signal can be represented with a Fourier series for a limited time
    - Fourier series technique important because medium or electronics can limit frequencies transmitted

Bandwidth

- Range of frequencies in a composite signal is ***bandwidth***
  - Difference between highest and smallest frequency
    - Example:





    - Here: Bandwidth is 600 Hz, not 700 Hz

Bandwidth

- Periodic signal
  - Bandwidth contains all integer frequencies between the lowest and the highest frequency
- Non-periodic signal
  - Bandwidth contains all frequencies between the lowest and the highest frequency

Bandwidth

- AM radio station in US has a bandwidth of 10 kHz
  - Total AM radio range is from 530 kHz to 1700 kHz
  - Bandwidth chosen to allow for good human speech transmission
- FM radio station in US has a bandwidth of 200 kHz
  - FM spectrum ranges from 88 to 108 Mhz

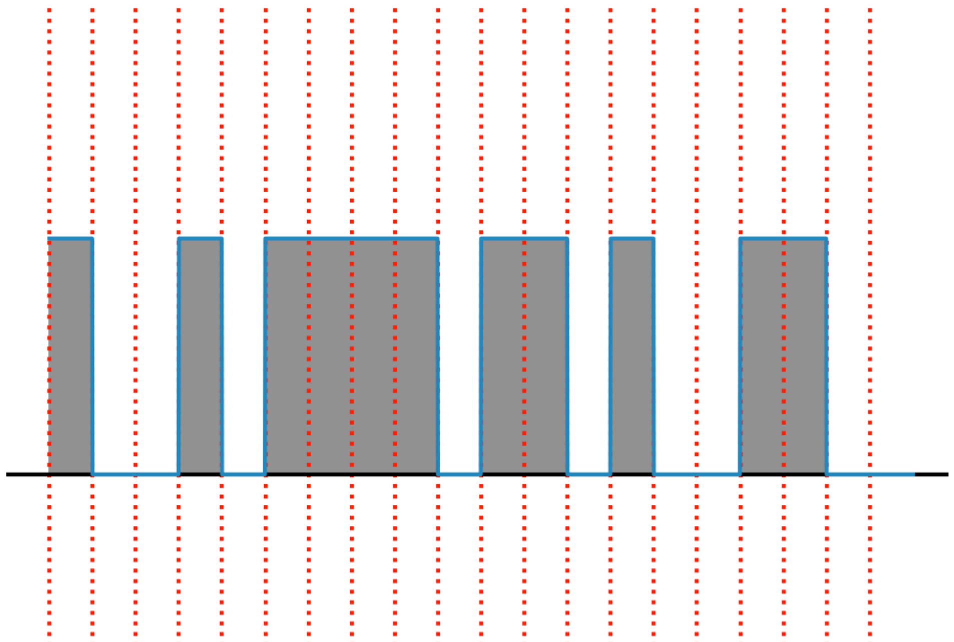
Bandwidth

- Analog black-and-white TV
  - Screen made up of pixels that are either black or white
  - Screen is scanned 30 times / sec
  - Resolution is 525 $\times$ 700 for 367,500 pixels
  - Need 367,500 $\times$ 30 = 11,025,000 pixels
  - Worst case scenario is alternating black and white pixels
  - Represent one color by minimum, other by maximum amplitude
  - Allows us to send 2 pixels per cycle
  - Need 11,025,000/2 = 5,512,500 cycles per second or 5.512 MHz
  - Best case is all one color, with frequency 0
  - Therefore, bandwidth needed is 5.512 MHz

Digital Signals

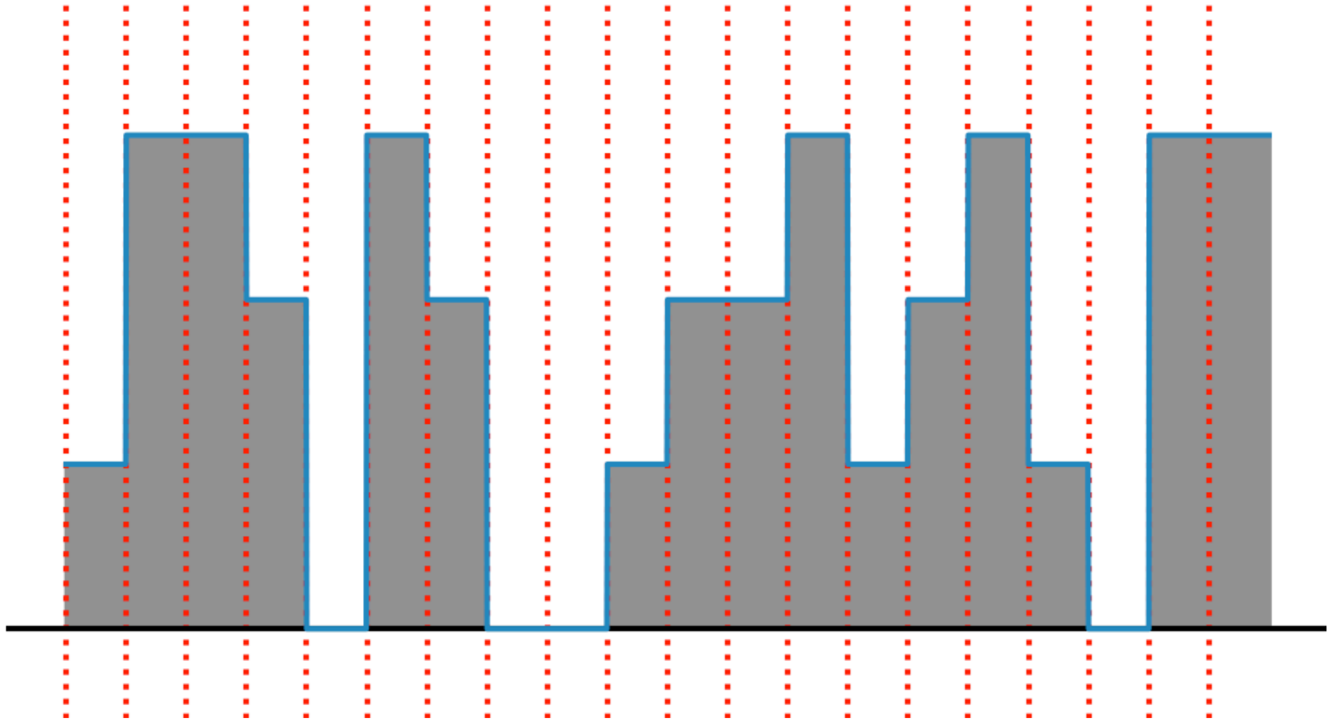
Digital Signals

- Digital signals have few *signal levels*
  - Two signal levels zero and one
  - 20 bits sent per second
  - bit-rate is 20bps



Digital Signals

- Four signal levels
- 40 bits sent per second
  - 40 bps



Digital Signals

- The number of signal levels needed to send b bits per time unit is
  - 2^b
- Reversely: With k signal levels, can send $\log_2(k)$ bits per time unit

Digital Signals

- Group exercise:
  - We download text documents at 100 pages per second
  - What is the bit rate needed?

Digital Signals

- Group exercise answer:
  - Assume 30 lines with 80 characters per line
  - Each character requires 8b
  - Per second, we download
    - $30 \times 80 \times 8 \times 100 = 1920000$ bits
    - Or 1.92 Mbps

Digital Signals

- To be recognizable, human voice needs a 4-kHz bandwidth analog signal
- What is the bit rate?
    - We need to sample at *twice* the highest frequency
    - Each sample takes 8 bits

Digital Signals

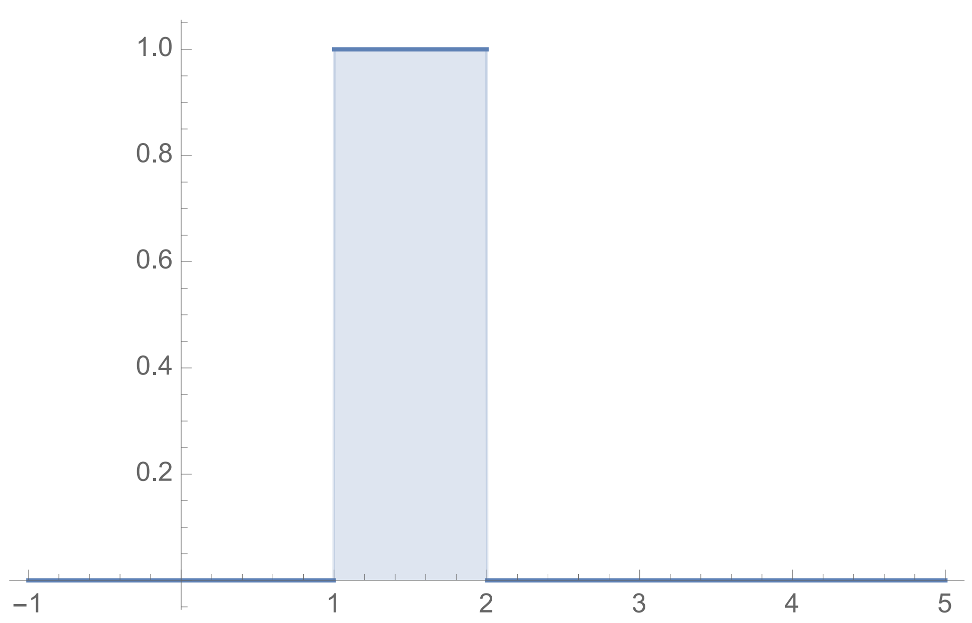
- To be recognizable, human voice needs a 4-kHz bandwidth analog signal
- What is the bit rate?
    - We need to sample at *twice* the highest frequency
    - Each sample takes 8 bits
- $2 \times 4000 \times 8 = 64000$ bps

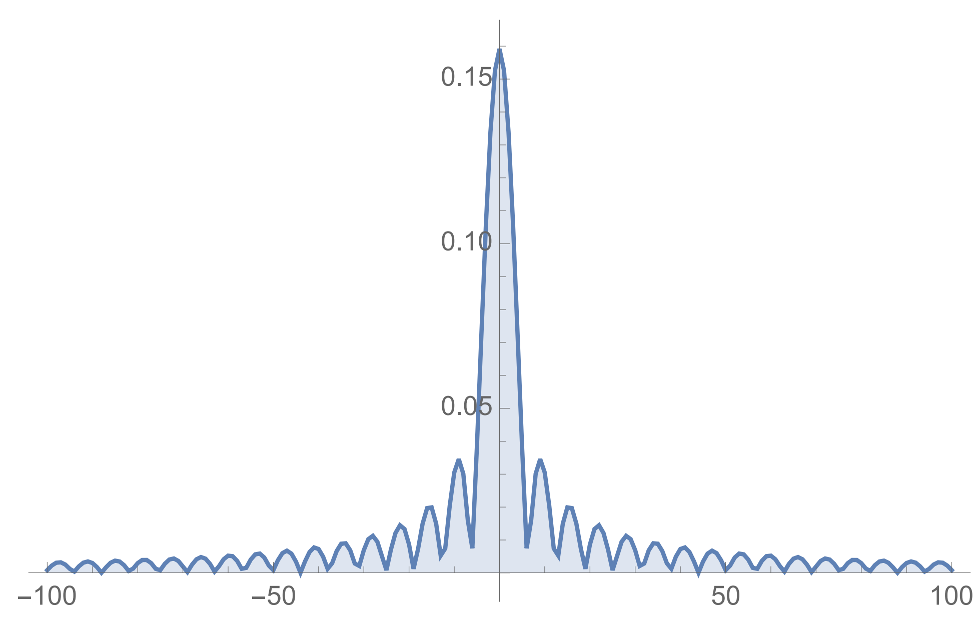
Bit Length

- Digital equivalent to wavelength
  - Distance one bit occupies on the transmission medium
  - $l = c \times d$
    - l bit length
    - c propagation speed
    - d bit duration

Nature of Digital Signals

- Digital signal is non-periodic, so it has an infinite bandwidth (using Fourier Analysis)
- Example: A single pulse





Transmission of Digital Signals

- Baseband transmission:
  - Sends the digital signal over a channel without changing the digital signal to an analog signal
  - Needs a ***low-pass channel***
    - A channel with bandwidth that starts from zero
    - Example: Use a dedicated cable between two computers
    - Example: Connect several computers to a bus, but have only one computer send at a time

Transmission of Digital Signals

- Baseband transmission:
  - The digital signal is not distorted if the low-pass channel has infinite bandwidth
  - Otherwise:
    - Signal is distorted

Transmission of Digital Signals

- Broadband transmission:
  - We change the digital signal to an analog signal for transmission
  - This allows us to use a ***bandpass channel***
    - A channel where bandwidth does not start from zero

Transmission Impairment

Causes of Impairment

- Attenuation: Loss of energy
- Distortion: Signal changes its form
- Noise: Other sources add to the signal

Attenuation

- Measured in decibel (dB)
  - Compares the relative strength of signals
  - Signal power at two different locations

$$db = 10\log_{10}(\frac{P_2}{P_1})$$

  - If we use voltage (power is proportional to the square of voltage)

$$db = 20\log_{10}(\frac{V_2}{V_1})$$

Attenuation

- Group Activity
- A signal suffers an attenuation of -15 decibel. The original peak amplitude is 5V. What is the peak amplitude when it is received.

$$-15 = 20\log_{10}(\frac{V_2}{5V})$$

$$-0.75 = \log_{10}(\frac{V_2}{5V})$$

$$10^{-0.75} = \frac{V_2}{5V}$$

$$0.177828 = \frac{V_2}{5V}$$

$$0.88914V = V_2$$

Attenuation

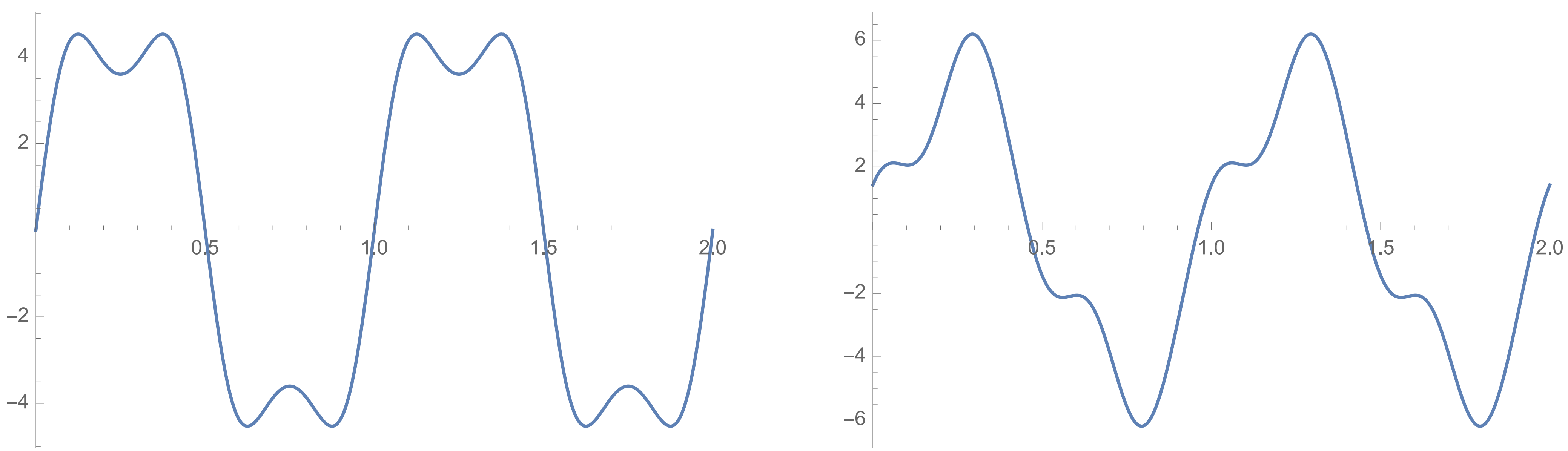
- Group activity
  - A cable suffers an attenuation of -0.5 db per km. What is the power of a signal that is original 0.7 mW after 5 km.
  - (This is why decibels use a logarithm, you can just multiply the length of the cable with the attenuation per km to get the attenuation of the whole cable)

Attenuation

$$-2.5 = 10\log_{10}(\frac{P_2}{0.5\text{mW}})$$
$$-.25 = \log_{10}(\frac{P_2}{0.5\text{mW}})$$
$$10^{-.25} = \frac{P_2}{0.5\text{mW}}$$
$$0.562341 = \frac{P_2}{0.5\text{mW}}$$
$$P_2 = 0.281171\text{mW}$$

Distortion

- Signal changes its form or shape.
  - Signal Fourier components can have their own propagation speed and the received composite signal can be different.



Result of big phase shift in the higher harmonics

Noise

- Noise: “random" noise is added to signal
  - Thermal, induced, crosstalk, impulse (from lightning)
  - Signal to Noise Ratio

$$\text{SNR} = \frac{\text{average signal power}}{\text{average noise power}}$$

$$\text{SNR}_{\text{db}} = 10\log_{10}(\text{SNR})$$

Noise

- Activity
  - The power of a signal is 10 mW and the power of the noise is 1 µW. What are the values for SNR and SNR-decibel?
  - SNR $= \dfrac{10^4 \mu \text{w}}{1 \mu \text{w}} = 10^4$
  - SNR$_{\text{db}} = 40$

Data Rate Limits

Nyquest Bit Rate

- For a noiseless channel:
  - Levels: number of signal levels
  - Maximum bit rate = $2 \times$ bandwidth $\times \log_2$(levels)

- Can increase the maximum bit rate by increasing the number of levels
  - But there is a practical limit to how many levels can be sustained

Nyquest Bit Rate

- What is the maximum bit rate for two signal levels for a noiseless channel with a bandwidth of 3000 Hz?
    - $2 \times 3000 \times \log_2 2$bps $= 6000$bps

Shannon Capacity

- In reality, all channels are noisy
  - Shannon capacity for highest data rate
    - Depends on the Signal Noise Ratio
    - bandwidth $\times \log_2(1 + SNR)$

Shannon Capacity

- If $SNR = 0$, then Shannon capacity is
  - bandwidth $\times \log_2(1) = 0$

Shannon Capacity

- Regular telephone line
  - Has bandwidth of 3000 Hz (300 Hz to 3300 Hz)
  - Signal to Noise ratio is > 3000.
  - Shannon capacity is
    - $3000 \times \log_2(3001)$bps $= 34652$bps
- To increase the bit rate, we need to increase the bandwidth of the line or improve the signal to noise ratio

Shannon Capacity

- Signal to Noise ratio is often given in decibel
- If channel bandwidth is 2 MHz and SNR is 36 decibel:
    - $36 = 10\log_{10}(SNR)$, so $SNR = 10^{36/10}$
    - Maximum bandwidth is
        - $2 \times 10^6 \times \log_2(1 + 10^{3.2})$bps
        - $= 23.918 \times 10^6$bps $= 23.918$Mbps

Shannon Capacity

- We have a channel with bandwidth 1-MHz. SNR is 60.
- What are appropriate bit rate and signal level?
  - We use Shannon capacity to find the upper limit for the bit-rate:
    - $10^6 \log_2(61)$bps $= 5.931$Mbps
  - For reasonable performance, we try to achieve 4Mbps.
  - Nyquist tells us:
    - 4Mbps $\leq 2 \times 1$Mhz $\times \log_2(L)$
    - So, we choose $L = 4$ signal levels

Performance

Performance

- Bandwidth
  - For analog data: in Hz
  - For digital data: in bps
  - Relationship will be discussed next week
  - E.g.: Telephone subscriber line
    - has bandwidth of 4kHz
    - and 56 000 bps

Performance

- Throughput
  - How fast can we actually send data through a network
  - Limited by bandwidth

Performance

- Latency (Delay)
  - Latency consists of
    - propagation time = distance / propagation speed
    - transmission time = message size / bandwidth
    - queuing time : time needed for each device to hold message before it can be processed
    - processing delay

Performance

- Example:
  - Interoceanic copper cable for a 5 MB message
    - Assume bandwidth is 1Mbps
    - Distance = 12000 km
    - Propagation speed is 1.8×10^8msec

Performance

- Propagation time is
- $\dfrac{12000 \times 10^3}{1.8 \times 10^8}$sec
- $= 0.667 \times 10^{-2}$msec $= 66.666$msec
- Transmission time is
- $5 \times 10^6 \times 8/10^6$sec $= 40$sec