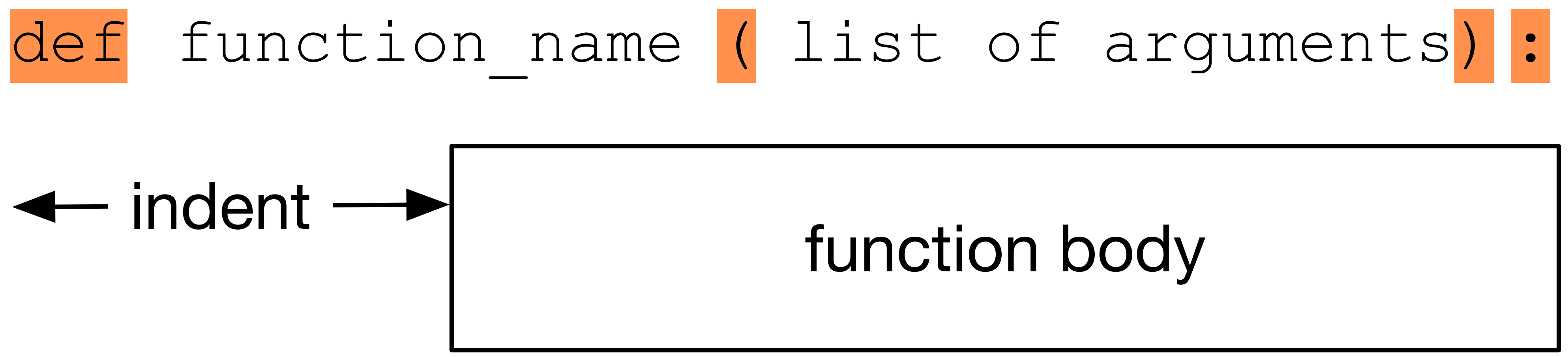
Functions in Python

Python Functions

- Functions defined by keyword def
- Can return value with keyword return



```
def function_name ( list of arguments):
```

Python Functions

- Without return:
  - Function returns when code is exhausted
- Peculiarities:
  - Neither argument nor return types are specified

Python Functions

- This is weird, but legal

```
def example(x):
    if x == 1:
        return 1
    if x == 2:
        return "two"
    if x == 3:
        return 3.0
```

- Returns a None value for x = 4
- Returns int for x=1, string for x=2, float for x=3

Functions of Functions

- Functions are full-fledged objects in Python
  - This means you can pass functions as parameters
  - Example: Calculate the average of the values of a function at -n, -n+1, -n+2, …, -2, -1, 0, 1, 2, … , n-2, n-1, n
    - The function needs to be a function of one integer variable
  - Example:
    - *n* = 2, function is squaring
    - Return value is $((-2)^2 + (-1)^2 + 0^2 + 1^2 + 2^2)/5 = 2$

Functions of Functions

- We first define the averaging function with two arguments
    - The number *n*
    - The function over which we average, called `func`

```
def averaging(n, func):
```

Functions of Functions

- Inside the function, we create an accumulator and a loop index, running from *-n* to *n.*

```
def averaging(n, func):
    accu = 0
    for i in range(-n, n+1):
```

Functions of Functions

- Inside the loop, we modify the accumulator `accu` by adding the value of the function at the loop variable.

```
def averaging(n, func):
    accu = 0
    for i in range(-n, n+1):
        accu += func(i)
```

Functions of Functions

- There are 2*n*+1 points at which we evaluate the function.
- We then return the average as the accumulator over the number of points

```
def averaging(n, func):
    accu = 0
    for i in range(-n, n+1):
        accu += func(i)
    return accu/(2*n+1)
```

Functions of Functions

- In order to try this out, we need to use a function
- We can just define one in order to try out our averaging function

```
def square(number):
   return number*number

def averaging(n, func):
    accu = 0
    for i in range(-n, n+1):
        accu += func(i)
return accu/(2*n+1)

print(averaging(2, square))
```

Local Functions

- Can have a function definition inside a function
    - Not many use cases

```
def factorial(number):
    if not isinstance(number, int):
        raise TypeError("sorry", number, "must be an integer")
    if not number >= 0:
        raise ValueError("sorry", number, "must be positive")

    def inner_factorial(number):
        if number <= 1:
            return 1
        return number * inner_factorial(number-1)

    return inner_factorial
```

Local and Global Variables

- A Python function is an independent part of a program
  - It has its own set of variables
    - Called local variables
  - It can also access variables of the environment in which the function is called.
    - These are global variables
  - The space where variables live is called their scope
  - We will revisit this issue in the future

Examples

```
a=3
b=2
def foo(x):
    return a+x
def bar(x):
    b=1
    return b+x

print(foo(3), bar(3))
```

- a and b are two global variables
- In function foo:
  - *a* is global, its value remains 3
- In function bar:
  - *b* is local, since it is redefined to be 1

The global keyword

- In the previous example, we generated a local variable *b* by just assigning a value to it.
- There are now two variables with name *b*
- In bar, the global variable is hidden
- If we want to assign to the global variable, then we can use the keyword global to make *b* refer to the global variable. An assignment then does not create a new local variable, but rather changes the value of the old one.

Example

- In foo:
  - A local variable *b*
  - A global variable *a*
  - The value of *a* changes by executing *foo*( )

```
a = 1
b = 2

def foo():
    global a
    a = 2
    b = 3
    print("In foo:" , "a=", a, " b=", b)

print("Outside foo: " ,"a=", a, " b=", b)
foo()
print("Outside foo: " ,"a=", a, " b=", b)

##Outside foo:  a= 1  b= 2
##In foo: a= 2  b= 3
##Outside foo:  a= 2  b= 2
```

Scoping

- Global scope:
    - Names that we define are visible to all our code
- Local scope:
    - Names that we define are only visible to the current function

Scoping

- LEGB — rule to resolve names
    - Local
    - Enclosed (e.g. enclosing function)
    - Global
    - Built-in

Functions with Default Arguments

- We have created functions that have *positional* arguments
  - Example:

```
def fun(foo, bar):
    print(2*foo+bar)

fun(2, 3)
```

  - When we invoke this function, the first argument (2) gets plugged into variable foo and the second argument (3) get plugged into variable bar

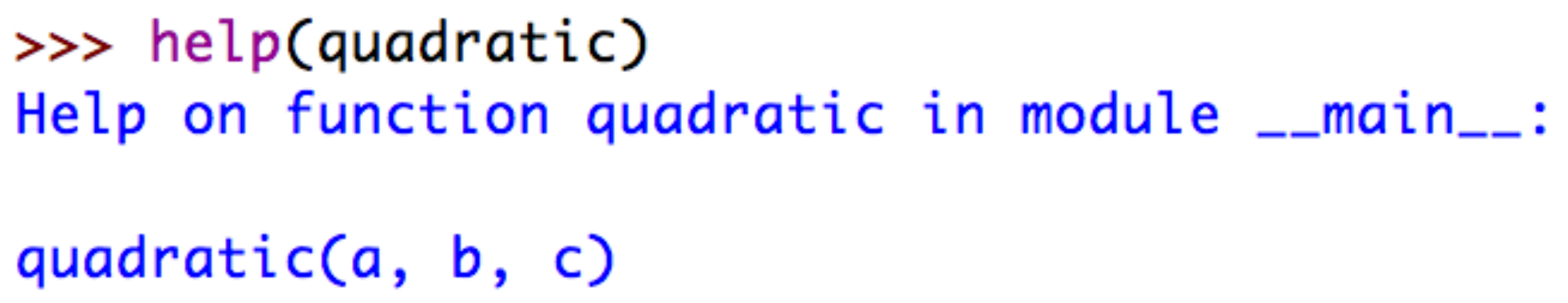
Keyword (Named) Arguments

- We can also use the names of the variables in the function definition.
- Example: (we soon learn how to deal better with errors)

```
def quadratic(a, b, c):
    if b**2-4*a*c >= 0:
        return -b/(2*a) + math.sqrt(b**2-4*a*c)/(2*a)
    else:
        print("Error: no solution")


print(quadratic(1, -4, 4))  #CALL BY POSITION
print(quadratic(c=4, a=1, b=-4)  #CALL BY KEYWORD
```

Keyword (Named) Arguments

- Keyword arguments have advantages
  - If you have a function with many positional arguments, then you need to carefully match them up
  - At least, you can use the help function in order to figure out what each argument does, if you named them well in the function definition



```
>>> help(quadratic)
Help on function quadratic in module __main__:

quadratic(a, b, c)
```

Keyword (Named) Arguments

- You can force the user of a function to use keywords by introducing an asterisk into the definition of the function:
  - All arguments after the asterisk need to be passed by keyword
  - The arguments before the asterisk can be positional

```
def  function  (  posarg1,  * ,   keywarg1  ):
```

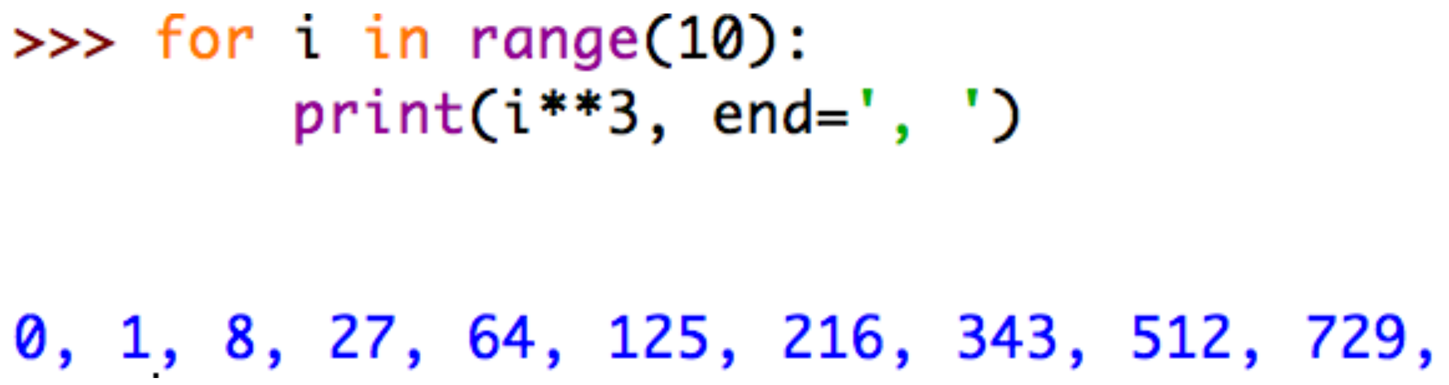
```
def fun(a, b, *, c):
             …

print(fun(2, 3, c=5))
```

Pythonic Tip

- If you want to write understandable code:
    - Use keyword arguments

Default arguments

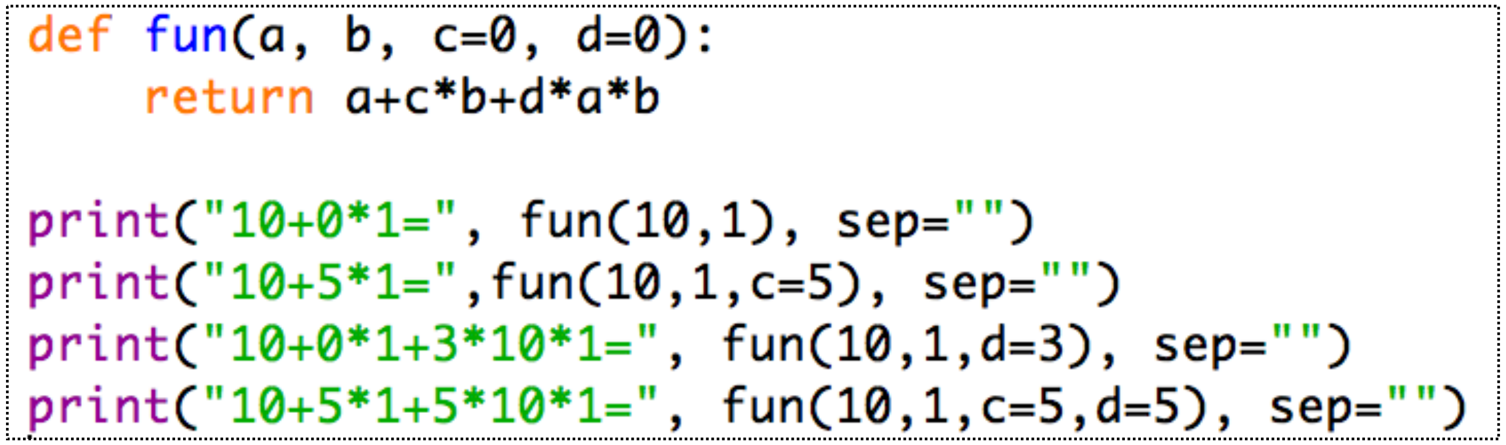
- You have already interacted with built-in functions that use default arguments
  - Print:
    - end: How the string is terminated (default is new-line character)
    - sep: What comes between different outputs (default is space)
    - file: Location of output (default is “standard output”)



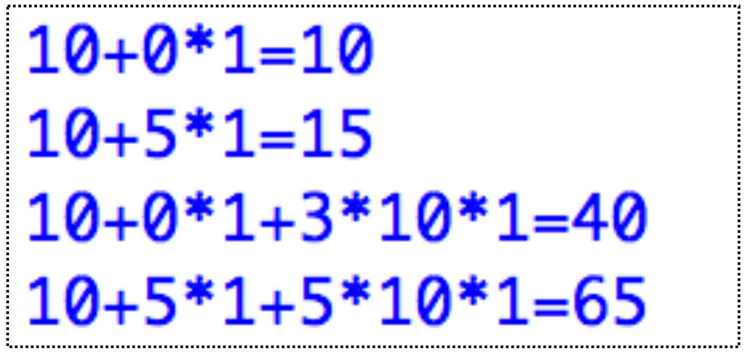
```
>>> for i in range(10):
        print(i**3, end=', ')


0, 1, 8, 27, 64, 125, 216, 343, 512, 729,
```

Default Arguments

- Defining default arguments is easy
    - Just use the arguments with default arguments last and assign default values in the function definition



```python
def fun(a, b, c=0, d=0):
    return a+c*b+d*a*b

print("10+0*1=", fun(10,1), sep="")
print("10+5*1=",fun(10,1,c=5), sep="")
print("10+0*1+3*10*1=", fun(10,1,d=3), sep="")
print("10+5*1+5*10*1=", fun(10,1,c=5,d=5), sep="")
```



```
10+0*1=10
10+5*1=15
10+0*1+3*10*1=40
10+5*1+5*10*1=65
```

Default Arguments

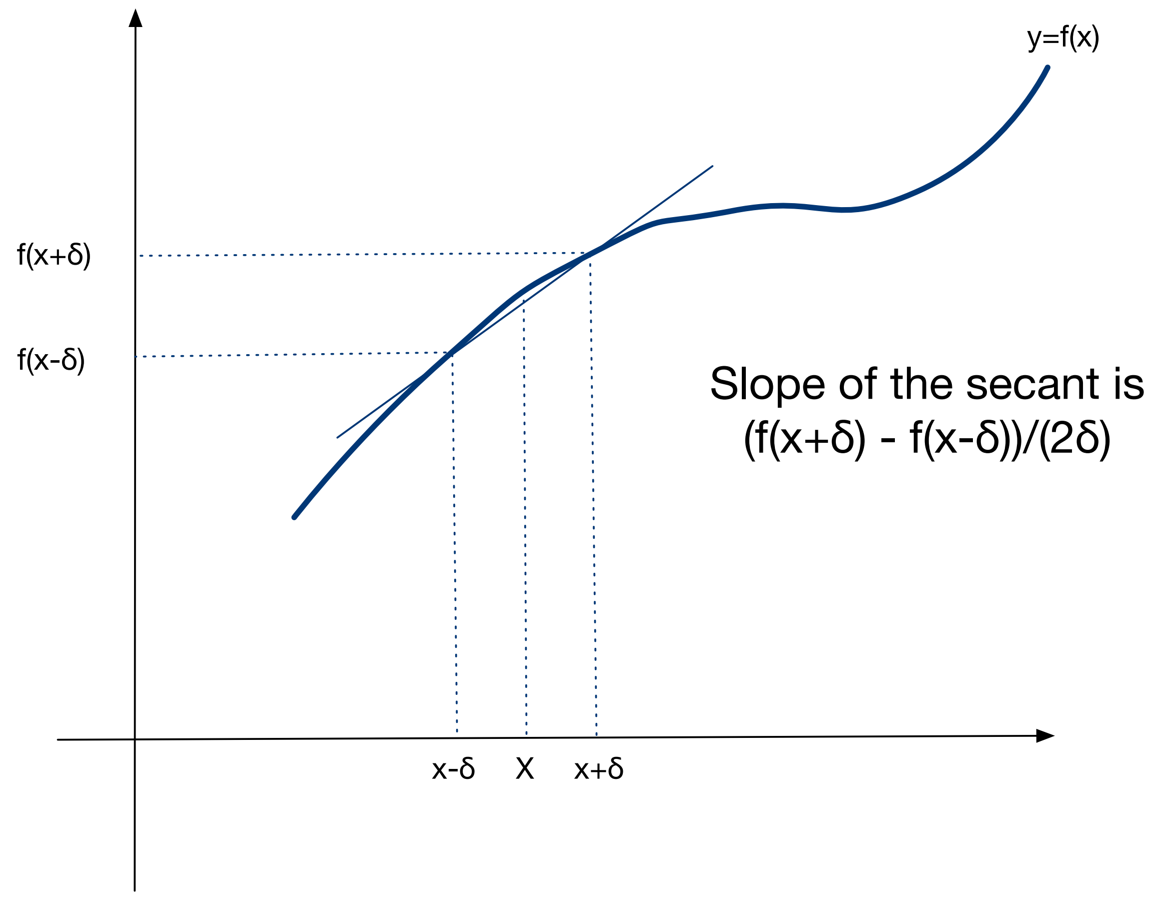
- How to write readable code:
    - Named arguments and default arguments with well-chosen names make code more readable
    - Most effort in software engineering goes towards maintaining code

Anonymous Functions

- Up till now, we used the def-construct in order to define functions
- Sometimes it is necessary to pass functions to another function, but not necessary to define the argument for future uses

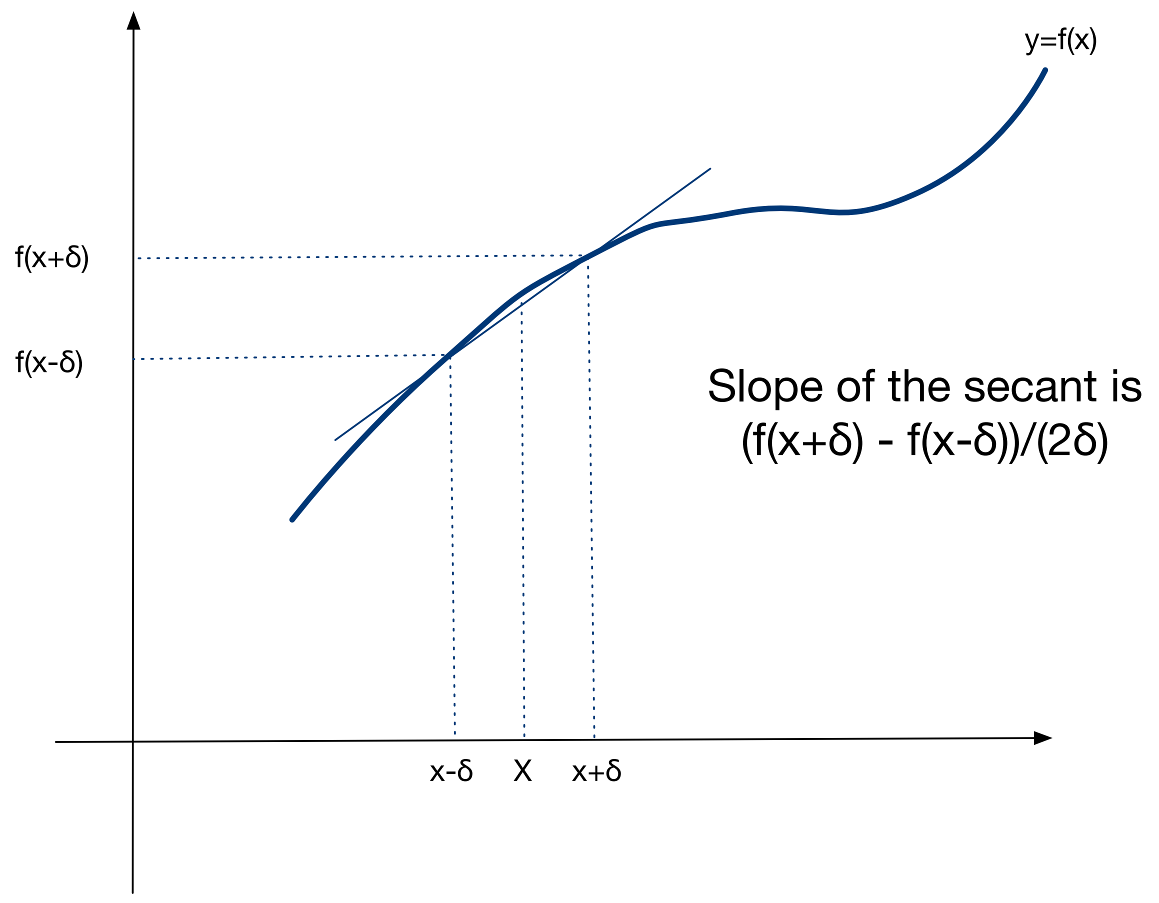
Anonymous Function

- Example:
  - Numerical Differentiation
    - Derivative of a function *f* at a point is the slope of the tangent
    - Approximated by a secant



Slope of the secant is
(f(x+δ) - f(x-δ))/(2δ)

Anonymous Functions



- The slope of the secant is the difference of values over the difference of arguments:

$$\frac{f(x+\delta)-f(x-\delta)}{x+\delta-(x-\delta)} = \frac{f(x+\delta)-f(x-\delta)}{2\delta}$$

- If δ is small, then this is a good approximation of the derivative

Anonymous Functions

- A simple method for derivation uses a fixed, but small value for δ.

```
def derivative(function, x):
    delta = 0.000001
    return (function(x+delta)-function(x-delta))/(2*delta)
```

- To test this, we try it out with sine, whose derivative is cosine

```
for i in range(20):
    x = i/20
    print(x, math.cos(x), derivative(math.sin, x))
```

Anonymous Functions

- It turns out that the numerical derivative is quite close in this test

```
0.0 1.0 0.9999999999998334
0.05 0.9987502603949663 0.9987502603940601
0.1 0.9950041652780257 0.9950041652759256
0.15 0.9887710779360422 0.9887710779310499
0.2 0.9800665778412416 0.9800665778519901
0.25 0.9689124217106447 0.9689124216977207
0.3 0.955336489125606 0.9553364891112803
0.35 0.9393727128473789 0.9393727128381713
0.4 0.9210609940028851 0.9210609939747094
0.45 0.9004471023526769 0.9004471023255078
0.5 0.8775825618903728 0.8775825618978494
0.55 0.8525245220595057 0.8525245220880606
0.6 0.8253356149096783 0.8253356149623414
0.65 0.7960837985490559 0.7960837985487856
0.7 0.7648421872844885 0.7648421873063249
0.75 0.7316888688738209 0.7316888688824186
0.8 0.6967067093471655 0.6967067094354462
0.85 0.6599831458849822 0.6599831459119798
0.9 0.6216099682706645 0.6216099682765375
0.95 0.5816830894638836 0.5816830894733727
```

Anonymous Functions

- Notice that in the test, we specified math.sin and not math.sin(x),
- The former is a function (which we want)
- The latter is a value (which we do not want)

```
for i in range(20):
    x = i/20
    print(x, math.cos(x), derivative(math.sin, x))
```

Anonymous Functions

- To specify a function argument, I can use a **lambda-expression**
  - Lambda-expressions were used in Mathematical Logic to investigate the potential of formal calculations

```
lambda  x :  5*x*x-4*x+3
```

  - Lambda expression consists of a keyword lambda
    - followed by one or more variables
    - followed by a colon
    - followed by an expression for the function
  - This example implements the function $x \to 5x^2 - 4x + 3$

Anonymous Functions

- To test our numerical differentiation function, we pass it the function $x \rightarrow x^2$, which has derivative $2x$

```
for i in range(20):
    x = i/20
    print("{:5.3f} {:5.3f} {:5.3f}”.format(
      x,
      derivative(lambda x: x*x, x),
      2*x))
```

Anonymous Functions

- Since we are rounding to only three digits after the decimal point, we get perfect results

```
0.000 0.000 0.000
0.050 0.100 0.100
0.100 0.200 0.200
0.150 0.300 0.300
0.200 0.400 0.400
0.250 0.500 0.500
0.300 0.600 0.600
0.350 0.700 0.700
0.400 0.800 0.800
0.450 0.900 0.900
0.500 1.000 1.000
0.550 1.100 1.100
0.600 1.200 1.200
0.650 1.300 1.300
0.700 1.400 1.400
0.750 1.500 1.500
0.800 1.600 1.600
0.850 1.700 1.700
0.900 1.800 1.800
0.950 1.900 1.900
```

Anonymous Functions

- I can even use lambda expressions as an alternative way of defining functions:

```
norm = lambda x, y: math.sqrt(x*x+y*y)
```

- Since there are two variables, norm is a function of two arguments:

```
print(norm(2.3, 1.7))
```

Annotations

- Completely optional way to make function definitions easier to read
    - Uses swift language convention
        - for arguments: name colon type
            - where type is either a Python type or a string
        - for return value: use ->

```
def quadratic(a: 'number', b: 'number', c: 'number') -> float :
    disc = (b**2-4*a*c)**0.5
    return (-b+disc)/(2*a)
```

Decorators

- Functions are also return values
  - One way to use this are decorators (for the future)
    - A decorator is put on top of a function
    - The decorator then takes the function and replaces it with another function

Decorators

- This is an example of a function factory!

```
def my_decorator(func):
    def wrapper():
        print("Something is happening before the function is called.")
        func()
        print("Something is happening after the function is called.")
    return wrapper

def say_namaste():
    print("Namaste!")

say_when = my_decorator(say_whee)
```

- We can automatically apply the decorator

```
@my_decorator
def say_namaste():
    print("Namaste!")
```

Decorators

- Some decorators are provided in modules
  - lru_cache in functools
    - stores the result of functions in an lru cache

Future topics on functions

- Memoization
- Decorators